

Ex/ determine whether $f(x) = \sqrt[3]{x-2}$ is the inverse of $g(x) = x^3 + 2$

$$f \circ g(x) = f(g(x))$$

$$= f(x^3 + 2)$$

$$= \sqrt[3]{(x^3 + 2) - 2}$$

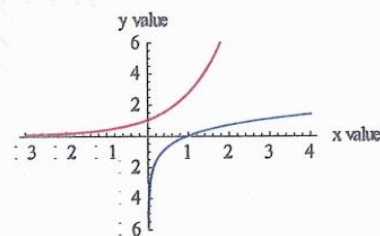
$$= \sqrt[3]{x^3}$$

$$= x$$

$$\therefore f \circ g(x) = x$$

$$\therefore g(x) = f^{-1}(x)$$

Logarithms :



Def : $\text{Log}_a x = y \Leftrightarrow x = a^y$

Properties:

$$1. \text{Log}_a (x \cdot y) = \text{Log}_a x + \text{Log}_a y$$

$$2. \text{Log}_a \left(\frac{x}{y}\right) = \text{Log}_a x - \text{Log}_a y$$

$$3. \text{Log}_a x^n = n \text{Log}_a x$$

$$4. \text{Log}_a a = 1, \text{Log}_a 1 = 0$$

Ex/ Solve for x

$$\text{Log}_5 (5^{2x}) = 8$$

$$2x \text{Log}_5 5 = 8$$

$$2x (1) = 8 \Rightarrow x = 4$$

Ex/ Solve for x $\text{Log}_{10} x^2 + \text{Log}_{10} x = 30$

$$2 \text{Log}_{10} x + \text{Log}_{10} x = 30$$

$$3 \text{Log}_{10} x = 30 \Rightarrow \text{Log}_{10} x = 10 \Rightarrow x = 10^{10}$$

Note: if $y = \text{Log}_a u$ then $\frac{dy}{dx} = \frac{1}{u} \text{Log}_a e \frac{du}{dx}$ where $e \approx 2.7$

Ex/ Find dy/dx if $y = \text{Log}_2 x$

$$\frac{dy}{dx} = \frac{1}{x} \text{Log}_2 e$$

Ex / Find dy/dx if $y = \text{Log}_3(2x^2 + 5x + 1)$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2x^2 + 5x + 1} \text{Log}_3 e \cdot (4x + 5) \\ &= \frac{4x + 5}{2x^2 + 5x + 1} \text{Log}_3 e \end{aligned}$$

Note: $b^{\text{Log}_b x} = x$ for $x > 0$

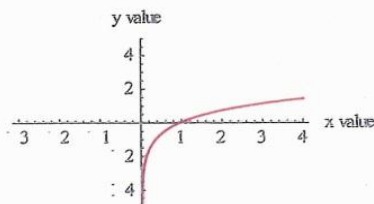
The Natural Logarithm

If $a=e$ then $\text{Log}_a x = \text{Log}_e x = \ln x$

$$y = f(x) = \ln x$$

$$D_f = \{x: x > 0\}$$

$$R_f = \{R\}$$



Ex / Find domein of $f(x) = \log_3(x - 2)$

$$x - 2 > 0$$

$$x > 2$$

$$\therefore D_f = \{x: x > 2\}$$

Properties

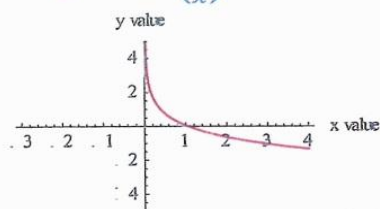
$$1. \ln(x \cdot y) = \ln x + \ln y$$

$$2. \ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$3. \ln x^n = n \ln x$$

$$4. \ln 1 = 0, \ln e = 1$$

ex/ draw the graph of $y = \ln\left(\frac{1}{x}\right)$



$$\therefore \ln\left(\frac{1}{x}\right) = \ln 1 - \ln x$$

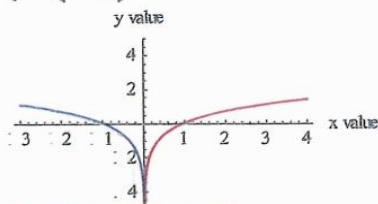
$$= 0 - \ln x$$

$$= -\ln x$$

$$\therefore \ln\left(\frac{1}{x}\right) = -\ln x$$

ex/ draw the graph of $y = \ln|x|$

$$y = \ln|x| = \begin{cases} \ln x & x \geq 0 \\ \ln(-x) & x < 0 \end{cases}$$



if $y = \ln u$ then $\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$

ex/ let $y = \ln(\sin x)$ Find dy/dx

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$$

Ex/ Find $\frac{dy}{dx}$ if $y = \ln \left(\frac{x^2 \sin^3 x}{\cos x^4 \sqrt{1+x}} \right)$

$$y = \ln \left(\frac{x^2 \sin^3 x}{\cos x^4 \sqrt{1+x}} \right)$$

$$y = \ln(x^2 \sin^3 x) - \ln(\cos x^4 \sqrt{1+x})$$

$$y = \ln x^2 + \ln \sin^3 x - [\ln \cos x^4 + \ln(1+x)^{1/2}]$$

$$y = 2 \ln x + 3 \ln \sin x - \ln \cos x^4 - \frac{1}{2} \ln(1+x)$$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{x} + 3 \cdot \frac{1}{\sin x} \cos x - \frac{1}{\cos x^4} \cdot (-\sin x^4 (4x^3)) - \frac{1}{2} \cdot \frac{1}{1+x}$$

$$\frac{dy}{dx} = \frac{2}{x} + 3 \cot x + 4x^3 \cot x^2 - \frac{1}{2(1+x)}$$

Ex/ Let $y = x^{\sin x}$ Find $\frac{dy}{dx}$

$$y = x^{\sin x} \Rightarrow \ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$$

$$\frac{dy}{dx} = y \left[\frac{\sin x}{x} + \ln x \cos x \right] = x^{\sin x} \left[\frac{\sin x}{x} + \ln x \cos x \right]$$

Ex/ Find $\frac{dy}{dx}$ if $y = \frac{x^3 \sqrt{x-5}}{\sin^2 x \cos^3 x}$

$$\ln y = \ln \left(\frac{x^3 \sqrt{x-5}}{\sin^2 x \cos^3 x} \right)$$

$$\ln y = \ln [x^3 \sqrt{x-5}] - \ln [\sin^2 x \cos^3 x]$$

$$\ln y = \ln x^3 + \ln(x-5)^{1/2} - [\ln \sin^2 x + \ln \cos^3 x]$$

$$\ln y = 3 \ln x + \frac{1}{2} \ln(x-5) - 2 \ln \sin x - 3 \ln \cos x$$

$$\frac{1}{y} \frac{dy}{dx} = 3 \frac{1}{x} + \frac{1}{2} \frac{1}{x-5} - 2 \frac{1}{\sin x} \cos x - 3 \frac{1}{\cos x} (-\sin x)$$

$$\frac{dy}{dx} = y \left[\frac{3}{x} + \frac{1}{2(x-5)} - 2 \cot x + 3 \tan x \right]$$

$$\frac{dy}{dx} = \frac{x^3 \sqrt{x-5}}{\sin^2 x \cos^3 x} \left[\frac{3}{x} + \frac{1}{2(x-5)} - 2 \cot x + 3 \tan x \right]$$