

Ex/ find $\frac{dy}{dx}$ if $y = 2^{\sin x}$

$$\frac{dy}{dx} = 2^{\sin x} \cdot \cos x \cdot \ln 2$$

Ex/ Find $\frac{dy}{dx}$ if $y = \pi^{x^2} + e^x \cdot x^e$

$$\frac{dy}{dx} = \pi^{x^2} \cdot (2x) \cdot \ln \pi + [e^x \cdot e^{x^{e-1}} + x^e \cdot e^x]$$

Ex/ Find $\lim_{x \rightarrow \infty} \frac{2+e^x}{1+3e^x} \left(\frac{\infty}{\infty} \right)$

$$= \lim_{x \rightarrow \infty} \frac{2+e^x}{1+3e^x} \cdot \frac{e^{-x}}{e^{-x}}$$

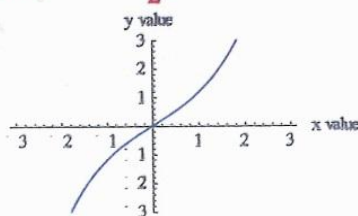
$$= \lim_{x \rightarrow \infty} \frac{2e^{-x}+1}{e^{-x}+3} = \frac{0+1}{0+3} = \frac{1}{3}$$

note : $e^{\infty} \rightarrow \infty$, $e^{-\infty} \rightarrow 0$

The hyperbolic functions

The hyperbolic sine and hyperbolic cosine functions define as

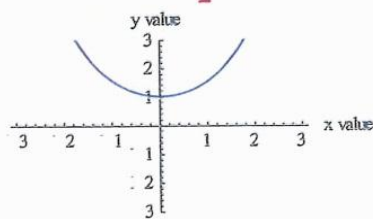
1) $\sinh x = \frac{e^x - e^{-x}}{2}$



$$f(x) = \sinh x$$

$$D_f = R_f = R$$

2) $\cosh x = \frac{e^x + e^{-x}}{2}$

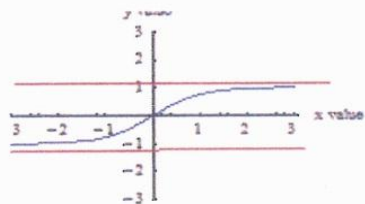


$$f(x) = \cosh x$$

$$D_f = R$$

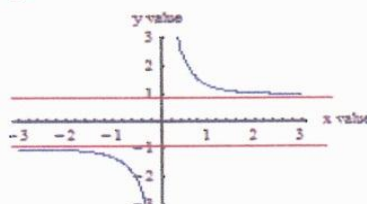
$$R_f = [1, \infty)$$

$$3) f(x) = \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



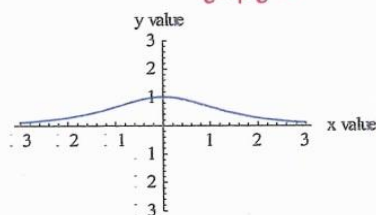
$$D_f = \mathbb{R}, R_f = (-1, 1)$$

$$4) f(x) = \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$



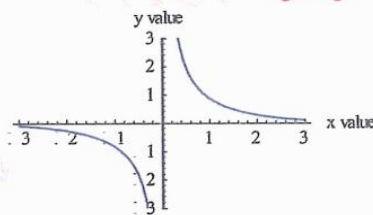
$$D_f = \mathbb{R} \setminus \{0\}, R_f = \mathbb{R} \setminus [-1, 1]$$

$$5) f(x) = \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$



$$D_f = \mathbb{R}, R_f = (0, 1]$$

$$6) f(x) = \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$



$$D_f = \mathbb{R} \setminus \{0\}, R_f = \mathbb{R} \setminus \{0\}$$

- 1) $\cosh^2 x - \sinh^2 x = 1$
- 2) $1 - \tanh^2 x = \operatorname{sech}^2 x$
- 3) $\coth^2 x - 1 = \operatorname{csch}^2 x$
- 4) $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
- 5) $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
- 6) $\cosh x + \sinh x = e^x$
- 7) $\cosh x - \sinh x = e^{-x}$
- 8) $\sinh 2x = 2 \sinh x \cosh x$
- 9) $\cosh 2x = \begin{cases} \cosh^2 x + \sinh^2 x \\ 2 \sinh^2 x + 1 \\ 2 \cosh^2 x - 1 \end{cases}$

Derivative

$$\frac{d}{dx} (\sin hx) = \cos hx$$

$$\frac{d}{dx} (\cos hx) = -\sin hx$$

$$\frac{d}{dx} (\tan hx) = \sec^2 hx$$

$$\frac{d}{dx} (\cot hx) = -\operatorname{csc}^2 hx$$

$$\frac{d}{dx} (\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx} (\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Ex/ prove that $\cosh(3x) = 4 \cosh^3 x - 3 \cosh x$

$$\begin{aligned} \cosh(3x) &= \cosh(x + 2x) \\ &= \cosh x \cosh 2x + \sinh x \sinh 2x \\ &= \cosh x (2 \cosh^2 x - 1) + \sinh x (2 \sinh x \cosh x) \\ &= 2 \cosh^3 x - \cosh x + 2 \sinh^2 x \cosh x \\ &= 2 \cosh^3 x - \cosh x + 2 (\cosh^2 x - 1) \cosh x \\ &= 2 \cosh^3 x - \cosh x + 2 \cosh^3 x - 2 \cosh x \\ &= 4 \cosh^3 x - 3 \cosh x \end{aligned}$$

Ex/ Solve $\sinh x = \frac{1}{4} \cosh x$ in terms of $\ln 3$, $\ln 5$

$$\begin{aligned} \sinh x &= \frac{1}{4} \cosh x \\ \left[\frac{e^x - e^{-x}}{2} = \frac{1}{4} \frac{e^x + e^{-x}}{2} \right] * 2 \\ \left[e^x - e^{-x} = \frac{1}{4} e^x + \frac{1}{4} e^{-x} \right] * e^x \\ e^{2x} - 1 &= \frac{1}{4} e^{2x} + \frac{1}{4} \\ e^{2x} - \frac{1}{4} e^{2x} &= 1 + \frac{1}{4} \\ \left[\frac{3}{4} e^{2x} = \frac{5}{4} \right] * \frac{4}{3} \\ e^{2x} &= \frac{5}{3} \Rightarrow 2x = \ln \frac{5}{3} \Rightarrow x = \frac{1}{2} [\ln 5 - \ln 3] \end{aligned}$$

Ex/show that $y = e^{ax} \sinh bx$ satisfies $y'' - 2a y' + (a^2 + b^2)y = 2b^2 e^{ax} \sinh bx$

$$y' = b e^{ax} \cosh bx + a e^{ax} \sinh bx$$

$$y'' = b^2 e^{ax} \sinh bx + ab e^{ax} \cosh bx + ab e^{ax} \cosh bx + a^2 e^{ax} \sinh bx$$

$$y'' = e^{ax} \sinh bx (a^2 + b^2) + 2ab e^{ax} \cosh bx$$

$$y'' - 2a y' + (a^2 + b^2)y = (\cancel{a^2} + b^2)e^{ax} \sinh bx + 2ab \cancel{e^{ax}} \cosh bx$$

$$- 2a [b \cancel{e^{ax}} \cosh x + a e^{ax} \sinh bx] + (\cancel{a^2} + b^2)e^{ax} \sinh bx = 2b^2 e^{ax} \sinh bx$$