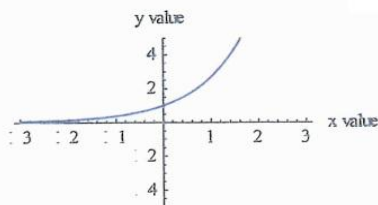


The exponential function

$$y = f(x) = e^x$$



$$D_f = R$$

$$R_f = \{y: y > 0\}$$

$$\lim_{x \rightarrow \infty} e^x = \infty, \quad \lim_{x \rightarrow -\infty} e^x = 0$$

Properties:

$$1. e^x \cdot e^y = e^{x+y}$$

$$2. \frac{e^x}{e^y} = e^{x-y}$$

$$3. (e^x)^n = e^{nx}$$

Ex/ Sketch the graph of $x = 2 + \ln(y - 1)$

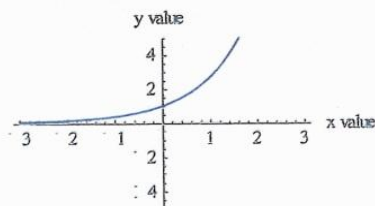
$$x = 2 + \ln(y - 1)$$

$$x - 2 = \ln(y - 1)$$

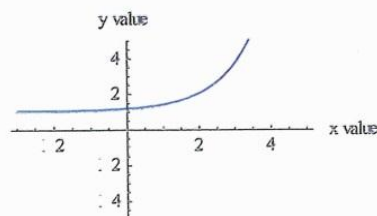
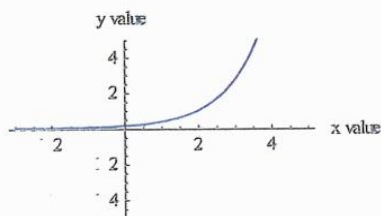
$$e^{x-2} = y - 1$$

$$1 + e^{x-2} = y$$

$$y = 1 + e^{x-2}$$



→



$$\text{if } y = e^u \text{ then } \frac{dy}{dx} = e^u \cdot \frac{du}{dx}$$

ex / Find $\frac{dy}{dx}$ if $y = e^{x \tan x}$

$$\frac{dy}{dx} = e^{x \tan x} \cdot [x \cdot \sec^2 x + \tan x \cdot 1]$$

$$\int e^u du = e^u + c$$

ex / $\int \frac{e^{\tan x}}{\cos^2 x} dx = \int e^{\tan x} \sec^2 x dx$

$$\begin{aligned} \text{Let } u &= \tan x \quad du = \sec^2 x dx \\ &= \int e^u du = e^u + c = e^{\tan x} + c \end{aligned}$$

ex / $\int e^{2 \ln x} dx = \int e^{\ln x^2} dx = \int x^2 dx = \frac{x^3}{3} + c$

Ex / $\int e^{x+e^x} dx = \int e^x \cdot e^{e^x} dx$ let $u = e^x \Rightarrow du = e^x dx$

$$\begin{aligned} &= \int e^u du = e^u + c \\ &= e^{e^x} + c \end{aligned}$$

The function $y = f(x) = a^x$, $a > 0$

1) $a^x \cdot a^y = a^{x+y}$

2) $\frac{a^x}{a^y} = a^{x-y}$

3) $(a^x)^n = a^{nx}$

if $y = a^u$ then $\frac{dy}{dx} = a^u \cdot \frac{du}{dx} \cdot \ln a$

Ex/ find $\frac{dy}{dx}$ if $y = 2^{\sin x}$

$$\frac{dy}{dx} = 2^{\sin x} \cdot \cos x \cdot \ln 2$$

Ex/ Find $\frac{dy}{dx}$ if $y = \pi^{x^2} + e^x \cdot x^e$

$$\frac{dy}{dx} = \pi^{x^2} \cdot (2x) \cdot \ln \pi + [e^x \cdot e x^{e-1} + x^e \cdot e^x]$$

Ex/ Find $\lim_{x \rightarrow \infty} \frac{2+e^x}{1+3e^x} \left(\frac{\infty}{\infty} \right)$

$$= \lim_{x \rightarrow \infty} \frac{2+e^x}{1+3e^x} \cdot \frac{e^{-x}}{e^{-x}}$$

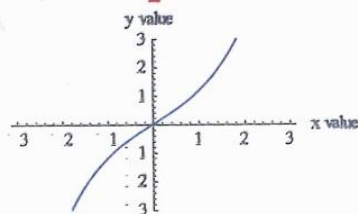
$$= \lim_{x \rightarrow \infty} \frac{2e^{-x}+1}{e^{-x}+3} = \frac{0+1}{0+3} = \frac{1}{3}$$

note : $e^{\infty} \rightarrow \infty$, $e^{-\infty} \rightarrow 0$

The hyperbolic functions

The hyperbolic sine and hyperbolic cosine functions define as

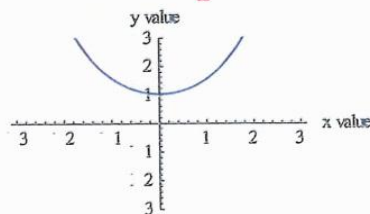
1) $\sinh x = \frac{e^x - e^{-x}}{2}$



$$f(x) = \sinh x$$

$$D_f = R_f = R$$

2) $\cosh x = \frac{e^x + e^{-x}}{2}$



$$f(x) = \cosh x$$

$$D_f = R$$

$$R_f = [1, \infty)$$