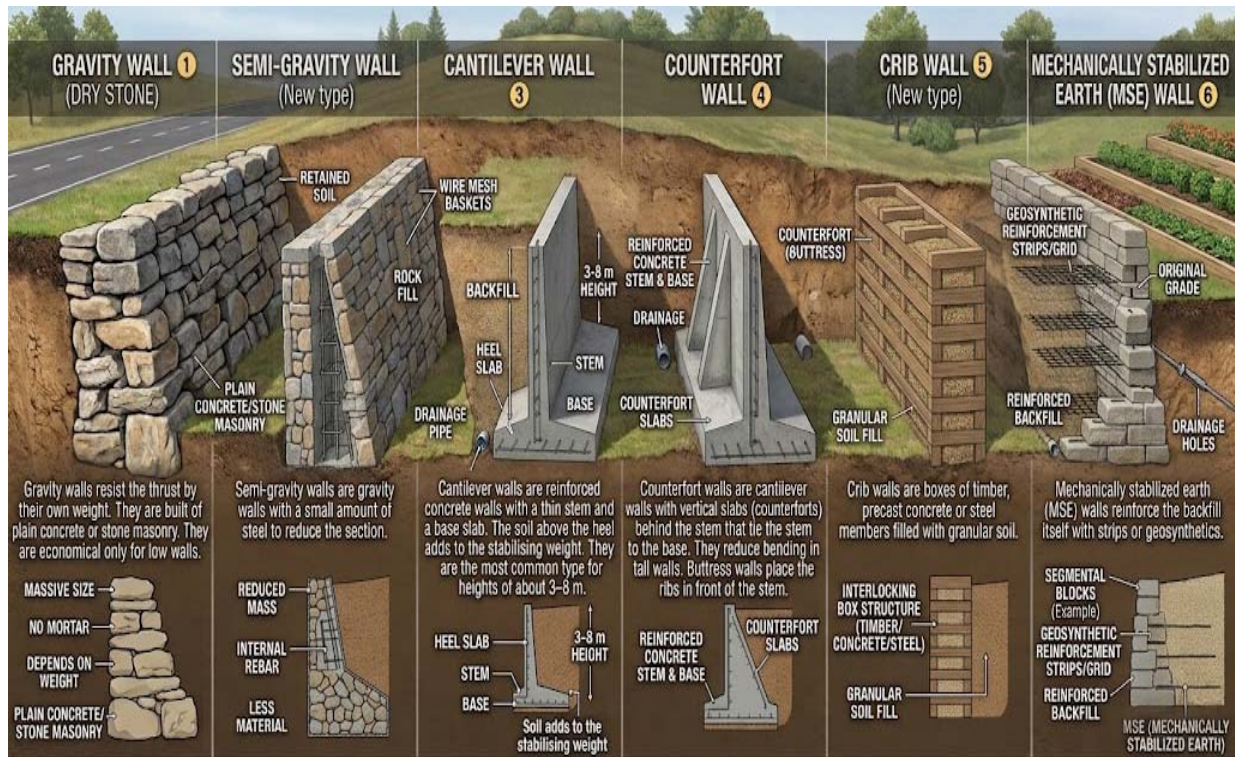

Retaining Walls I: Types, Proportioning, Overturning and Sliding

1 Retaining Walls in Water Resources Engineering

A retaining wall supports soil (and sometimes water) at a slope steeper than the soil can stand alone. In water resources projects they appear as canal and drain side walls, wing walls and abutments of regulators and bridges, flood walls along rivers, walls of pumping-station sumps (wet well) and intake structures, and walls of navigation locks. These walls often carry unbalanced water pressure and are exposed to seepage, so drainage and uplift receive special attention.

2 Types of Retaining Walls

1. **Gravity walls** resist the thrust by their own weight. They are built of plain concrete or stone masonry. They are economical only for low walls.
2. **Semi-gravity walls** are gravity walls with a small amount of steel to reduce the section.
3. **Cantilever walls** are reinforced concrete walls with a thin stem and a base slab. The soil above the heel adds to the stabilising weight. They are the most common type for heights of about 3–8 m.
4. **Counterfort walls** are cantilever walls with vertical slabs (counterforts) behind the stem that tie the stem to the base. They reduce bending in tall walls. Butress walls place the ribs in front of the stem.
5. **Crib walls** are boxes of timber, precast concrete or steel members filled with granular soil.
6. **Mechanically stabilized earth (MSE) walls** reinforce the backfill itself with strips or geosynthetics.



Types of retaining walls.

Guide to the selection of the wall type (heights are indicative only).

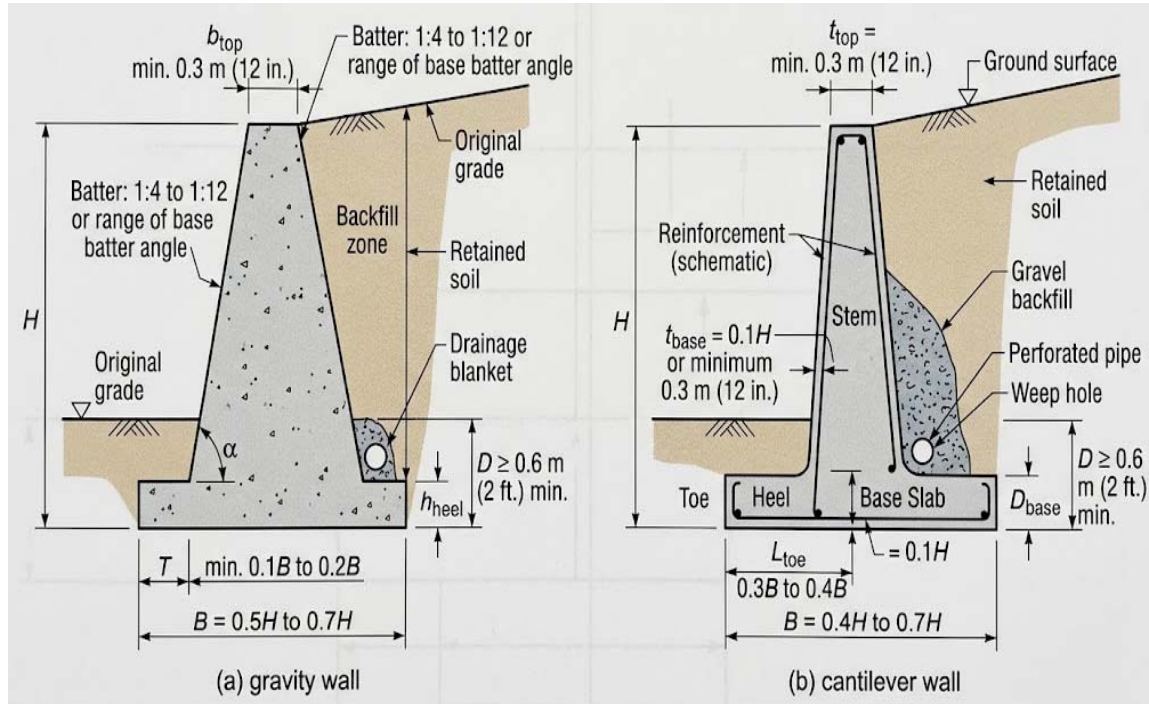
Type	Typical economical height	Main advantage	Main limitation
Gravity	up to about 3 m	Simple, no reinforcement	Large volume; high base pressure
Cantilever	about 3–8 m	Uses soil weight on heel	Stem bending grows with H^3
Counterfort	above about 7–8 m	Stem spans between ribs	Complex formwork
Crib	up to about 6 m	Fast, flexible, drains freely	Needs good granular fill
MSE	up to 10–20 m and more	Tolerates settlement; economical	Large reinforced zone width

3 Proportioning Retaining Walls

Design begins with trial dimensions (proportioning). The trial section is checked for stability. If a check fails, the dimensions are changed and the section is checked again. Below figure gives the usual starting proportions. **Two practical limits apply to all walls:**

1. the top of the stem should be at least 0.3 m wide for proper placement of concrete;

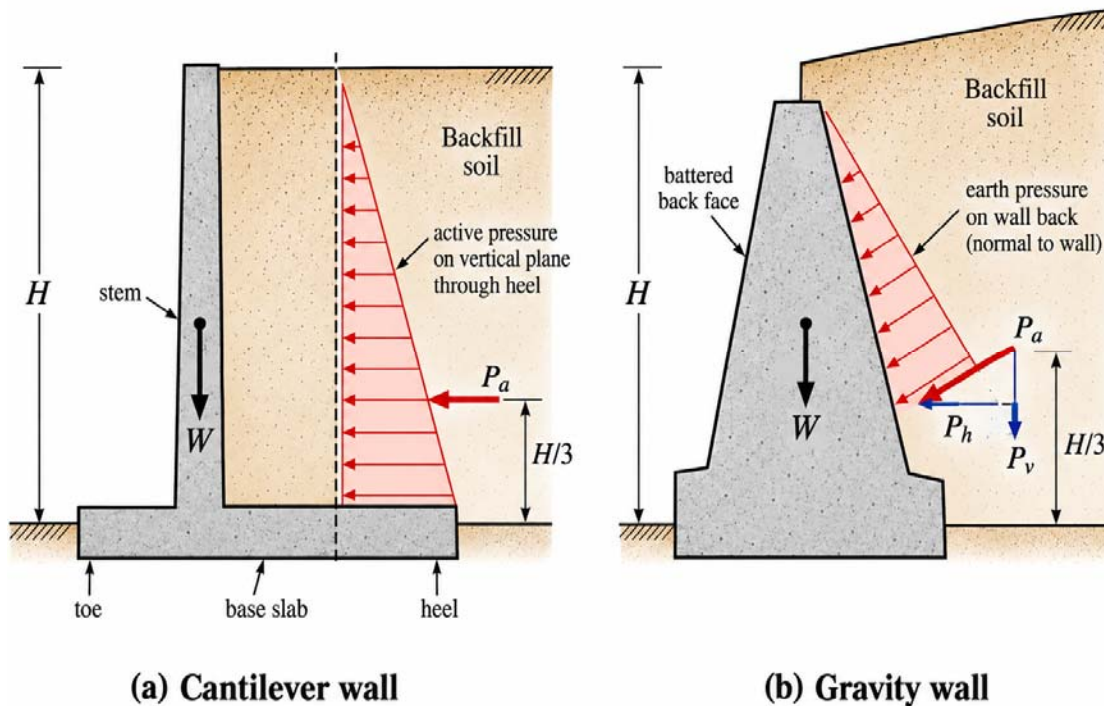
2. the depth D to the bottom of the base slab should be at least 0.6 m, and below the zone of seasonal moisture change and scour. For counterfort walls the ribs are about 0.3 m thick and spaced at $0.3H$ to $0.7H$.



Approximate dimensions for initial stability checks: (a) gravity wall; (b) cantilever wall.

4 Applying Earth Pressure Theories to Design

For a **cantilever wall**, the Rankine active pressure is computed on a vertical plane AB through the end of the heel. The soil between the stem and this plane moves with the wall and is treated as part of it. H' is the height of this plane. For a sloping backfill, P_a is inclined at α and has a vertical component $P_v = P_a \sin \alpha$. For a **gravity wall**, Coulomb theory is applied directly to the back face with wall friction δ . Rankine can also be used on a vertical plane through the heel when the back has a large inclination.

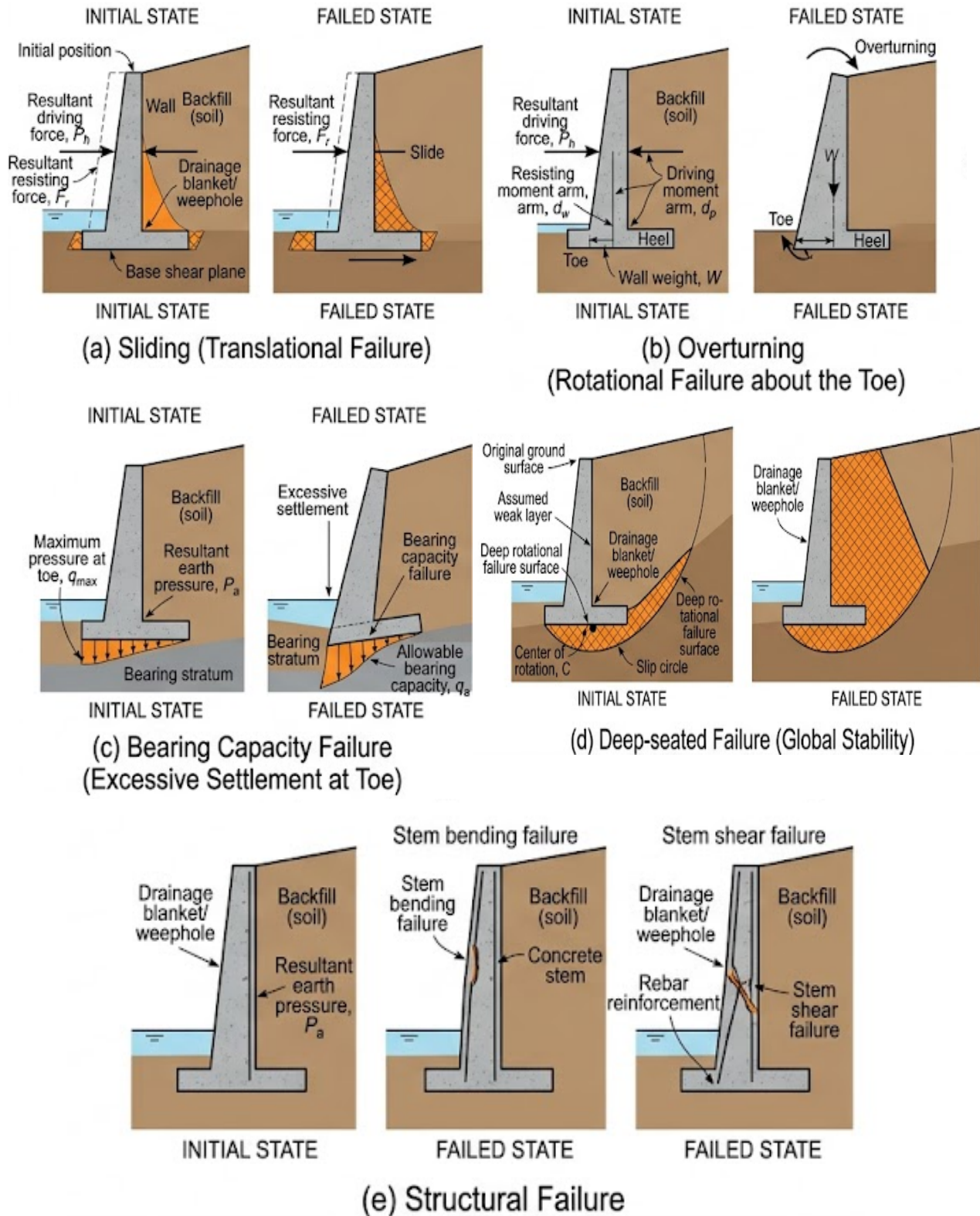


Earth pressure for stability checks: (a) cantilever wall; (b) gravity wall.

5 Stability Checks

With the lateral pressure known, the wall is checked as a rigid body for overturning, sliding and bearing capacity (the overall check). Each structural component is then checked for strength and its reinforcement is designed. A retaining wall can fail in four ways:

1. it may overturn about its toe;
2. it may slide along its base;
3. it may fail by loss of bearing capacity of the soil under the base;
4. it may undergo deep-seated shear failure or excessive settlement (soft soil below the base).



Failure modes of a retaining wall.

Usual minimum factors of safety for retaining walls.

Check	Minimum factor of safety
Overturning	1.5 (granular backfill) to 2.0 (cohesive backfill)
Sliding, passive resistance neglected	1.5
Sliding, passive resistance included	2.0
Bearing capacity	3.0
Deep-seated shear	1.5 (minimum)

6 Check for Overturning

The factor of safety against overturning about the toe (point O) is:

$$FS_{\text{overturning}} = \frac{\sum M_R}{\sum M_O} \quad (1)$$

The **overturning moment** comes from the horizontal component of the thrust:

$$\sum M_O = P_h \frac{H'}{3} + \text{moments of other horizontal forces (water, surcharge),} \quad P_h = P_a \cos \alpha \quad (2)$$

- The **resisting moment** comes from the weights of the concrete parts and the soil above the heel and toe, and from the vertical component of the thrust, $P_v = P_a \sin \alpha$, acting at the heel.
- The passive force is usually neglected in this check.
- The calculation is organized in a table: divide the section into rectangles and triangles, compute each area, weight and arm from the toe, and sum.

Tabular procedure for $\sum M_R$ ($\gamma_c \approx 23.6\text{--}24 \text{ kN/m}^3$ for reinforced concrete).

Section	Area	Weight per metre	Arm from toe	Moment about toe
1, 2 (soil)	A_1, A_2	$W = \gamma_1 A$	x_1, x_2	$M = Wx$
3–6 (concrete)	$A_3 \dots A_6$	$W = \gamma_c A$	$x_3 \dots x_6$	$M = Wx$
Thrust	—	P_v	B	$M_v = P_v B$
Sum		$\sum V$		$\sum M_R$

7 Check for Sliding along the Base

The factor of safety against sliding is the ratio of the horizontal resisting forces to the horizontal driving forces:

$$FS_{sliding} = \frac{\sum F_R}{\sum F_d} \quad (3)$$

The shear strength between the base and the soil is $\tau = c_a + \sigma \tan \delta$, where c_a is the adhesion and δ is the base friction angle. Over a base of width B the resistance is:

$$S = c_a B + (\sum V) \tan \delta, \quad c_a = k_2 c_2, \quad \delta = k_1 \phi_2 \quad (4)$$

- k_1 and k_2 are commonly taken in the range $\frac{1}{2}$ to $\frac{2}{3}$.
- For concrete cast directly on the soil, $\delta = \phi_2$ is often acceptable.
- The passive resistance P_p of the soil in front of the toe may be added, but only if the soil will not be removed (by excavation, scour or erosion) during the life of the wall:

$$FS_{sliding} = \frac{S}{P_h} \geq 1.5 \quad \text{or} \quad FS_{sliding} = \frac{S + P_p}{P_h} \geq 2.0 \quad (5)$$

Friction angles δ between foundation materials and soil.

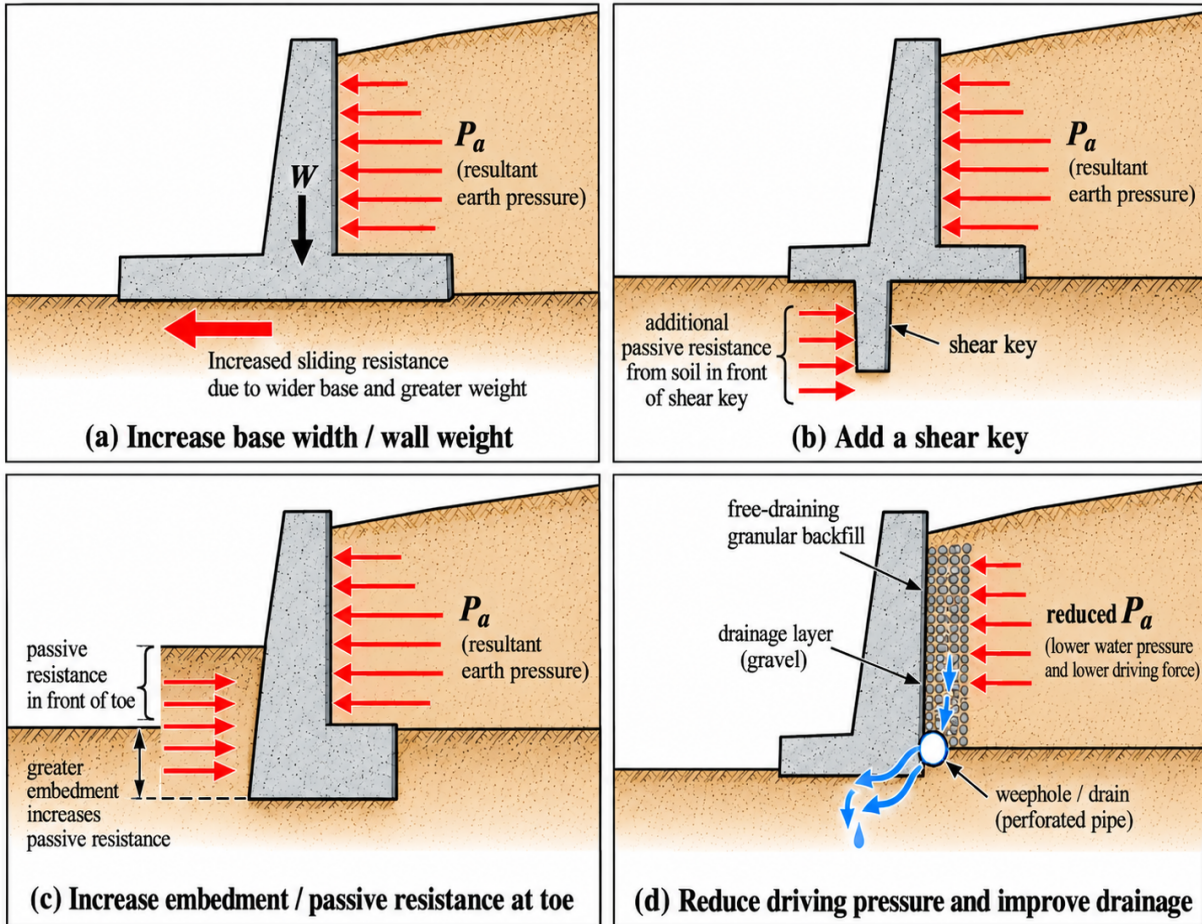
Interface	Friction angle δ
Mass concrete on clean sound rock	35°
Mass concrete on gravel, sand, silt or clay	ϕ of the soil (concrete poured on soil)
Formed concrete on clean gravel, gravel-sand	22–26°
Formed concrete on clean sand, silty sand-gravel	17–22°
Formed concrete on silty sand, sand mixed with silt or clay	17°
Formed concrete on fine sandy silt	14°
Steel sheet piles on clean gravel / clean sand	22° / 17°

Measures When the Sliding Check Fails

If the factor of safety is not achieved, the following alternatives are investigated:

1. increase the width of the base slab, mainly the heel, to add weight and friction;
2. add a key below the base slab; the passive wedge then starts at the bottom of the key and the sliding plane passes through soil instead of along the interface;

3. tie the wall to a deadman anchor behind the active zone;
4. incline the base slightly upward toward the heel, so that part of the thrust acts normal to the base.



Alternatives for increasing the factor of safety against sliding.

Tutorial: Worked Examples**Example 1**

Propose trial dimensions for a cantilever retaining wall that retains 7.0 m of soil above the toe level (total $H = 7.0$ m from the bottom of the base to the top of the stem).

Solution:

Element	Rule	Computed	Adopted
Stem top width	≥ 0.3 m	0.30 m	0.40 m
Base thickness	0.1H	0.70 m	0.70 m
Stem bottom thickness	0.1H	0.70 m	0.70 m
Base width B	0.5H–0.7H	3.5–4.9 m	4.20 m
Toe length	$\approx 0.1H$ to $B/3$	0.7–1.4 m	1.00 m
Heel length	$B - \text{toe} - \text{stem}$		2.50 m
Depth of base D	≥ 0.6 m		1.50 m

These values are only a starting point. The stability checks below decide whether B must change.

Example 2

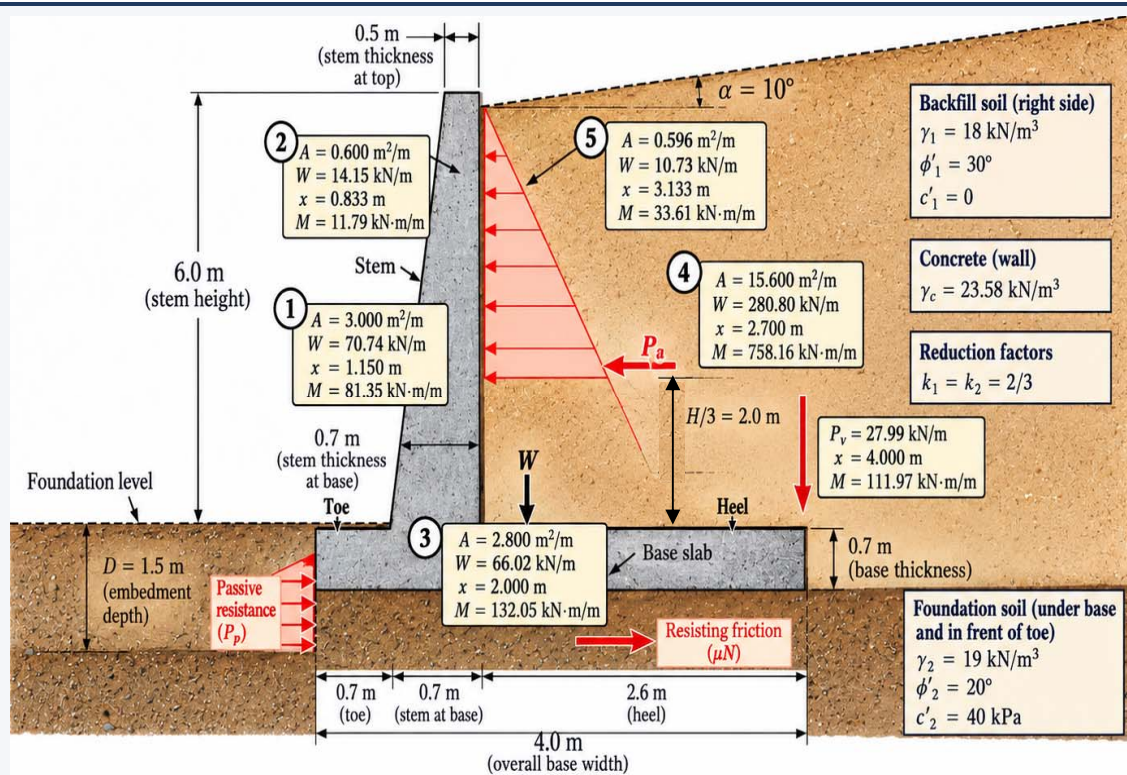
The cantilever wall shown below retains a backfill sloping at $\alpha = 10^\circ$ ($\gamma_1 = 18$ kN/m³, $\phi'_1 = 30^\circ$, $c'_1 = 0$). The base rests on soil with $\gamma_2 = 19$ kN/m³, $\phi'_2 = 20^\circ$, $c'_2 = 40$ kPa. Toe 0.7 m, stem 0.7 m at the base and 0.5 m at the top (front inclination), heel 2.6 m, base thickness 0.7 m, stem height 6 m, $D = 1.5$ m, $\gamma_c = 23.58$ kN/m³. Find the factors of safety against overturning and sliding. Use $k_1 = k_2 = \frac{2}{3}$.

Solution:

$$H_1 = 2.6 \tan 10^\circ = 0.458 \text{ m}, \quad H' = 0.7 + 6.0 + 0.458 = 7.158 \text{ m}$$

$$K_a(\alpha = 10^\circ, \phi' = 30^\circ) = 0.3495, \quad P_a = \frac{1}{2}(18)(7.158)^2(0.3495) = 161.20 \text{ kN/m}$$

$$P_h = 161.20 \cos 10^\circ = 158.75, \quad P_v = 161.20 \sin 10^\circ = 27.99 \text{ kN/m}$$



Wall section and numbered parts.

Section	Material	Area (m ² /m)	Weight W (kN/m)	Arm from toe (m)	Moment (kN·m/m)
1	Concrete	3.000	70.74	1.150	81.35
2	Concrete	0.600	14.15	0.833	11.79
3	Concrete	2.800	66.02	2.000	132.05
4	Soil	15.600	280.80	2.700	758.16
5	Soil	0.596	10.73	3.133	33.61
P _v	Earth thrust	—	27.99	4.000	111.97
	Σ		$\Sigma V = 470.43$		$\Sigma M_R = 1128.93$

$$\Sigma M_O = P_h \frac{H'}{3} = 158.75 \times 2.386 = 378.79 \text{ kN}\cdot\text{m/m}$$

$$FS_{\text{overturning}} = 1128.93 / 378.79 = 2.98 > 1.5 \text{ OK}$$

$$S = \frac{2}{3}(40)(4.0) + 470.43 \tan\left(\frac{2}{3} \times 20^\circ\right) = 218.16 \text{ kN/m}$$

$$FS_{\text{sliding}} = 218.16 / 158.75 = 1.37 < 1.5 \text{ not adequate without passive resistance}$$

$$K_p = \tan^2(55^\circ) = 2.040, P_p = \frac{1}{2}(19)(1.5^2)(2.040) + 2(40)(1.5)\sqrt{2.040} = 215.0 \text{ kN/m}$$

$$FS_{\text{sliding}} = (218.16 + 215.0) / 158.75 = 2.73 > 2.0 \text{ OK}$$

The earlier course notes used $K_a = 0.353$ and obtained $FS = 2.95$ and 1.37 . The small differences come from rounding of K_a (the exact value is 0.3495).

Example 3

A gravity wall 6 m high has a vertical back, a top width of 1.5 m, and a base width of 2.0 m (front inclination 0.5 m). It retains a level sand ($\gamma = 18 \text{ kN/m}^3$, $\phi' = 32^\circ$, $\delta = 0$). $\gamma_c = 22 \text{ kN/m}^3$. The base soil has $\phi' = 28^\circ$, $c' = 0$, and $\delta_{\text{base}} = \frac{2}{3}\phi'$. Check overturning and sliding.

Solution:

$$K_a = 0.307, \quad \sigma'_a(6) = 18(6)(0.307) = 33.18 \text{ kPa}, \quad P_a = 99.55 \text{ kN/m at 2.0 m}$$

Section	Material	Area (m ² /m)	Weight W (kN/m)	Arm from toe (m)	Moment (kN·m/m)
1	Concrete	9.000	198.00	1.250	247.50
2	Concrete	1.500	33.00	0.333	11.00
	Σ		$\Sigma V = 231.00$		$\Sigma M_R = 258.50$

$$FS_{\text{overturning}} = 258.50 / (99.55 \times 2.0) = 1.30 < 1.5 \quad \text{not adequate}$$

$$FS_{\text{sliding}} = \frac{231 \tan(18.67^\circ)}{99.55} = 0.78 < 1.5 \quad \text{not adequate}$$

Note the arm of the front triangle: its centroid is at $\frac{2}{3} \times 0.5 = 0.333 \text{ m}$ from the toe, not 0.25 m. The section is inadequate in both checks. A wider base, a batter on the back face (which adds soil weight), or a base key is needed.

Example 4

For the cantilever wall of Example 2, scour in front of the wall (a canal wall) may remove the top 1.0 m of soil, and softening may destroy the cohesion of the front soil. Count only the passive resistance below the scour level with $c' = 0$ and require $FS_{\text{sliding}} \geq 2.0$. Find the depth D_1 below the original ground to the bottom of a base key.

Solution:

$$\text{Requirement: } S + P_p \geq 2.0P_h \Rightarrow P_p \geq 2(158.75) - 218.16 = 99.33 \text{ kN/m}$$

$$P_p = \frac{1}{2}\gamma_2(D_1 - 1.0)^2 K_p = \frac{1}{2}(19)(D_1 - 1.0)^2(2.040) \geq 99.33$$

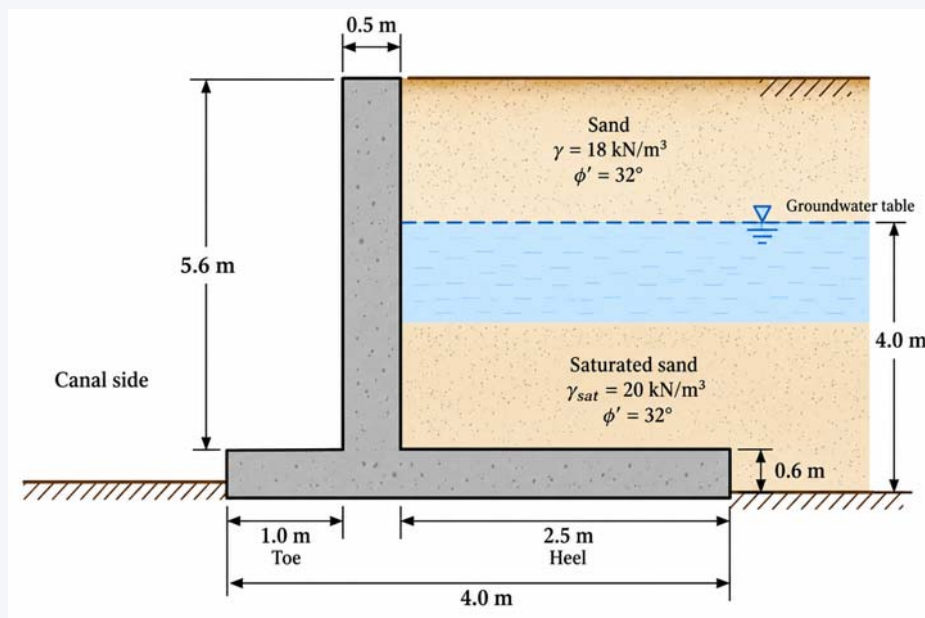
$$D_1 - 1.0 \geq \sqrt{\frac{99.33}{19.38}} = 2.26 \text{ m} \Rightarrow D_1 \geq 3.26 \text{ m}$$

D_1 (m)	P_p (kN/m)	FS_{sliding}
2.0	19.4	1.50
2.5	43.6	1.65
3.0	77.5	1.86
3.5	121.1	2.14

Adopt $D_1 = 3.3$ m, which means a key 1.8 m deep below the underside of the base (base bottom at 1.5 m). Comparing with Example 2 shows how much the design depends on the assumptions about the front soil.

Example 5

A cantilever canal wall (base 4.0 m $\times$ 0.6 m, toe 1.0 m, stem 0.5 m, stem top at 5.6 m above the base bottom) retains sand ($\gamma = 18 \text{ kN/m}^3$ above and $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$ below the water table, $\phi' = 32^\circ$). The groundwater behind the wall stands 4.0 m above the base bottom. Compare the sliding factor of safety ($\delta = 26^\circ$, $c_a = 0$, passive neglected) for (a) canal full to the same level, and (b) canal suddenly emptied while the groundwater behind the wall stays high. Assume full uplift under the base in (a) and a triangular uplift (γ_{whw} at the heel to zero at the toe) in (b).



Solution:

$$K_a = 0.307; P_a \text{ (effective)} = 67.52 \text{ kN/m, water behind } P_w = \frac{1}{2}(9.81)(4^2) = 78.48 \text{ kN/m}$$

(a) Canal full: the water forces on the two faces cancel. Uplift $U = \gamma_w h_w B = 157.0 \text{ kN/m}$ reduces the normal force; the water above the toe adds 33.4 kN/m .

$$\sum V = 236.0 \text{ kN/m}, \quad FS = \frac{236.0 \tan 26^\circ}{67.52} = 1.70$$

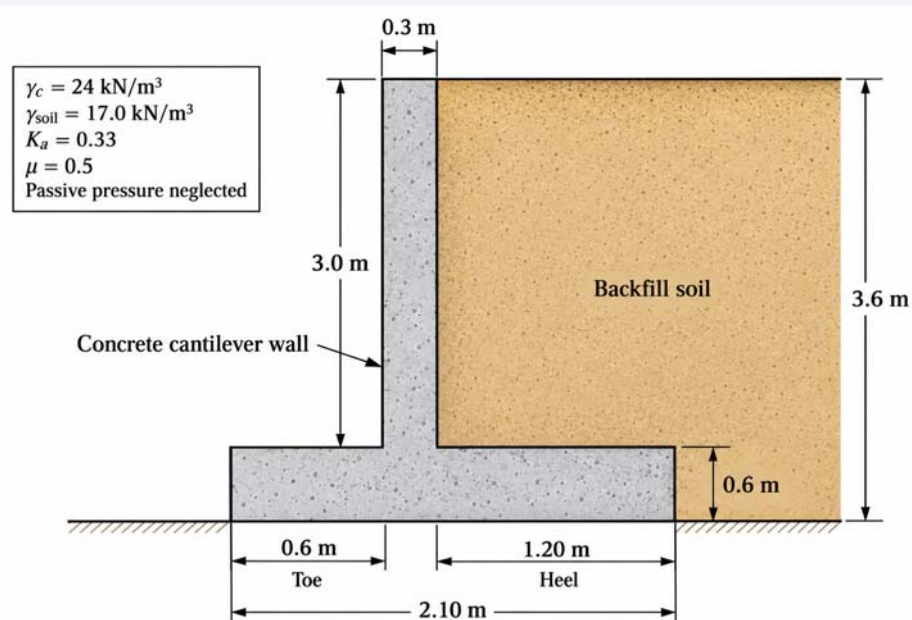
(b) Canal empty: the full water force acts from behind and the uplift is triangular (78.5 kN/m).

$$\sum V = 281.1 \text{ kN/m}, \quad FS = \frac{281.1 \tan 26^\circ}{67.52 + 78.48} = 0.94$$

Emptying the canal (for maintenance, or a sudden drawdown) is the critical case: the factor of safety drops from 1.70 to 0.94. Drainage behind canal walls (weep holes, drains) is essential.

Example 6

A cantilever wall: stem $0.3 \text{ m} \times 3.0 \text{ m}$, base $2.10 \text{ m} \times 0.6 \text{ m}$ (toe 0.6 m , heel 1.20 m), backfill height 3.6 m above the base bottom, $\gamma_c = 24 \text{ kN/m}^3$, $\gamma_{\text{soil}} = 17.0 \text{ kN/m}^3$, $K_a = 0.33$, $\mu = 0.5$, passive neglected. Check overturning and sliding.



Solution:

$$p_{\max} = 0.33(110)(12) = 435.6 \text{ psf}, \quad F_H = \frac{1}{2}(435.6)(12) = 2613.6 \text{ lb/ft}, \quad M_{OT} = 10454.4 \text{ lb}\cdot\text{ft/ft}$$

$$\sum W = 4400 + 1500 + 2100 = 8000 \text{ lb/ft}, \quad M_R = 4400(5) + 1500(2.5) + 2100(3.5) = 33100 \text{ lb}\cdot\text{ft/ft}$$

$$FS_{OT} = 33100/10454.4 = 3.17; \quad FS_S = 0.5(8000)/2613.6 = 1.53$$

Both checks pass ($3.17 > 2.0$ and $1.53 > 1.5$), but the sliding margin is small.

Key Terms (English – Arabic)

English term	المصطلح العربي
Retaining wall	الجدار الساند
Gravity wall	جدار ساند وزني
Cantilever wall	جدار ساند ظفري
Counterfort	دعامة خلفية
Stem	الساق (جذع الجدار)
Toe	مقدمة القاعدة
Heel	مؤخرة القاعدة (العقب)
Overturning	الانقلاب
Sliding	الانزلاق
Base key (shear key)	المفتاح القصي
Factor of safety	معامل الأمان
Deadman anchor	مرساة ميتة

References for Further Reading

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- Bowles, J.E. (1996). *Foundation Analysis and Design*, 5th ed. McGraw-Hill, New York.
- Coduto, D.P., Kitch, W.A. and Yeung, M.R. (2016). *Foundation Design: Principles and Practices*, 3rd ed. Pearson, Boston.
- Knappett, J.A. and Craig, R.F. (2019). *Craig's Soil Mechanics*, 9th ed. CRC Press, Boca Raton.

Homework No. 4:

- Q1. Proportion a cantilever wall to retain 5.5 m of level backfill above the toe level. Present the dimensions in a table and a sketch.
- Q2. For the wall of Q1 with $\gamma = 18 \text{ kN/m}^3$, $\phi' = 32^\circ$ (backfill) and $\gamma = 19 \text{ kN/m}^3$, $\phi' = 26^\circ$, $c' = 20 \text{ kPa}$ (base soil), $\gamma_c = 24 \text{ kN/m}^3$, compute FS against overturning and sliding ($k_1 = k_2 = \frac{2}{3}$, $D = 1.2 \text{ m}$).
- Q3. Homework from the earlier notes (Q1): $H = 8.0 \text{ m}$, $x_1 = 0.4 \text{ m}$, $x_2 = 0.6 \text{ m}$, $x_3 = 1.5 \text{ m}$, $x_4 = 3.5 \text{ m}$, $x_5 = 0.96 \text{ m}$, $D = 1.75 \text{ m}$, $\alpha = 10^\circ$; $\gamma_1 = 16.5 \text{ kN/m}^3$, $\phi'_1 = 32^\circ$; $\gamma_2 = 17.6 \text{ kN/m}^3$, $\phi'_2 = 28^\circ$, $c'_2 = 30 \text{ kPa}$. Find FS against overturning and sliding.
- Q4. Repeat Example 3 with a base width of 3.2 m (the extra width added at the heel side as a back inclination). Do the checks pass?
- Q5. Explain why the passive resistance in front of a canal wall should be treated with caution. Propose a design value.
- Q6. A wall has $FS_{\text{sliding}} = 1.25$. List three ways to raise it to 1.5 and estimate, for one of them, the required change in dimensions.
- Q7. A cantilever wall retains sand ($\gamma = 17 \text{ kN/m}^3$, $\delta = 0$) with $q = 34 \text{ kN/m}$; stem 6 m high and 1 m thick, base 1 m thick on clay ($c_u = 63.75 \text{ kPa}$, $\phi_u = 0$), $\gamma_{\text{conc}} = 24 \text{ kN/m}^3$. If $FS(\text{overturning}) = FS(\text{sliding})$, find B and the least ϕ that keeps the wall stable against sliding.

