Power Electronics and Special Machines

Lecture 5: Rectifying Circuits

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Lecture 5: Rectifying Circuits

1. Uncontrolled single-phase half-wave rectifier

The basic circuit diagram for the single-phase half-wave rectifiers is shown in Figure 1. Where the AC source is connected to the load (could be resistive or inductive) through a diode.

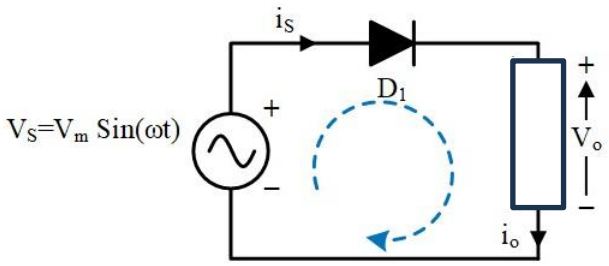


Figure 1: Basic circuit of single-phase half-wave rectifiers.

Figure 2 illustrate the rectifier waveforms in case of a pure resistive load, the supply voltage (v_s) being sinusoidal of peak value (V_{max}).

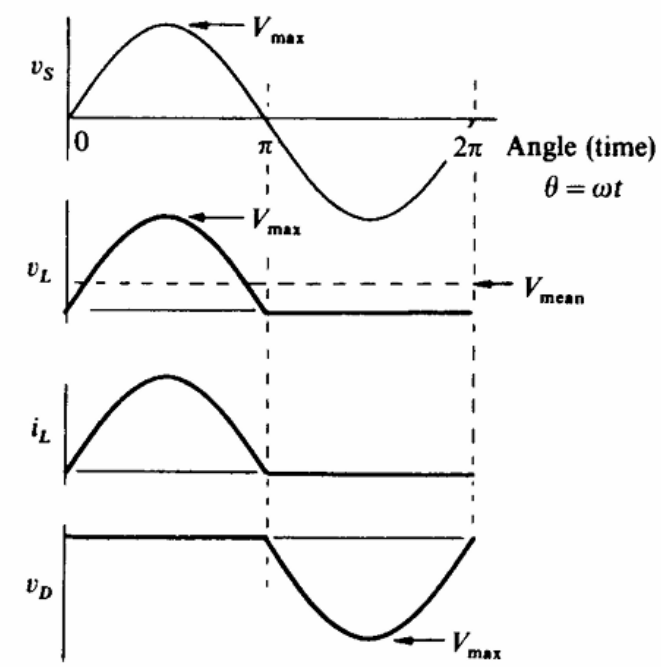


Figure 2: Rectifier waveforms for a pure resistive load.

Neglecting the small diode voltage-drop, the mean value of a half sinewave is given by:

$$V_{mean} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \; d(\omega t)$$

$$V_{mean} = \frac{V_m}{2\pi}\left(-\cos\omega t|_0^{\pi}\right) = \frac{V_m}{2\pi}(-\cos\pi + \cos 0)$$

$$V_{mean} = \frac{V_m}{\pi} \qquad (1)$$

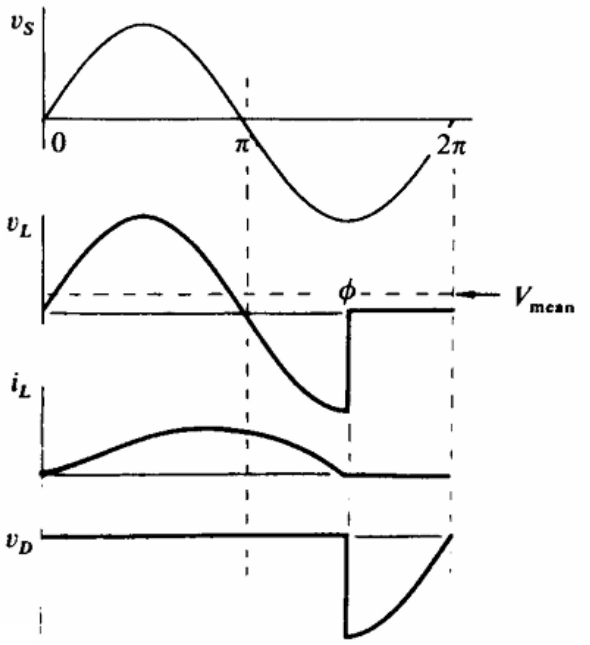
The load current (i_L)

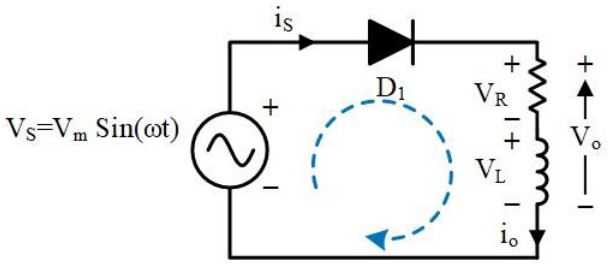
$$i_L = \frac{v_s}{R} = \frac{V_{max}\sin\omega t}{R} \qquad 0 \le \omega t \le \pi$$

The current is negative for the negative half cycle of the supply voltage. The mean value of load current can be calculated as:

$$I_{mean} = \frac{V_{mean}}{R} = \frac{V_m}{\pi\, R} \qquad (2)$$

Almost all DC loads contain some inductance; the waveforms shown in Figure 3a are for an inductive load having the equivalent circuit shown in Figure 3b.





(a) (b)

Figure 3: Rectifier waveforms for an inductive load (R-L load).

The current flow will start directly the supply voltage goes positive, but the presence of the inductance will delay current change, the current still flowing at the end of half cycle, the diode remain ON, and the load sees the negative supply until the current drops to zero. Referencing to Figure 4 below, the instantaneous load voltage will be:

$$v_L = V_R + V_L = R\, i_L + L\frac{di_L}{dt} \tag{3}$$

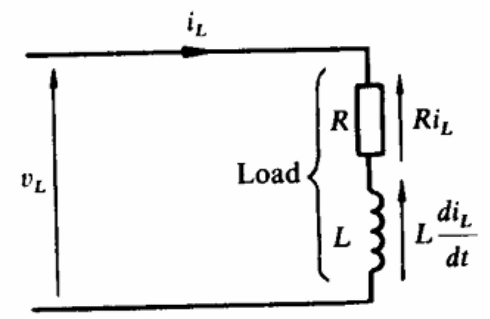


Figure 4: Load equivalent circuit.

This differential Equation (3) can be solved to determine the wave shape of the load current (i_L).

The mean voltage for Figure 3a is given by:

$$V_{mean} = \frac{1}{2\pi}\int_0^{\varphi} V_m \sin\omega t\; d(\omega t)$$

$$V_{mean} = \frac{V_m}{2\pi}\left(-\cos\omega t\big|_0^{\varphi}\right) = \frac{V_m}{2\pi}(-\cos\varphi + \cos 0)$$

$$V_{mean} = \frac{V_m}{2\pi}\,(1-\cos\varphi) \tag{4}$$

Figure 5a can be used to determine the extinction angle (φ), given any load impedance angle $\left(\theta = \tan^{-1}\frac{\omega L}{R}\right)$. Figure 5b shows the normalized output voltage $\frac{V_{mean}}{v_s}$ as a function of $\frac{\omega L}{R}$.

The mean value of load current is given by:

$$I_{mean} = \frac{V_{mean}}{R} = \frac{V_m}{2\pi\, R}\,(1-\cos\varphi) \tag{5}$$

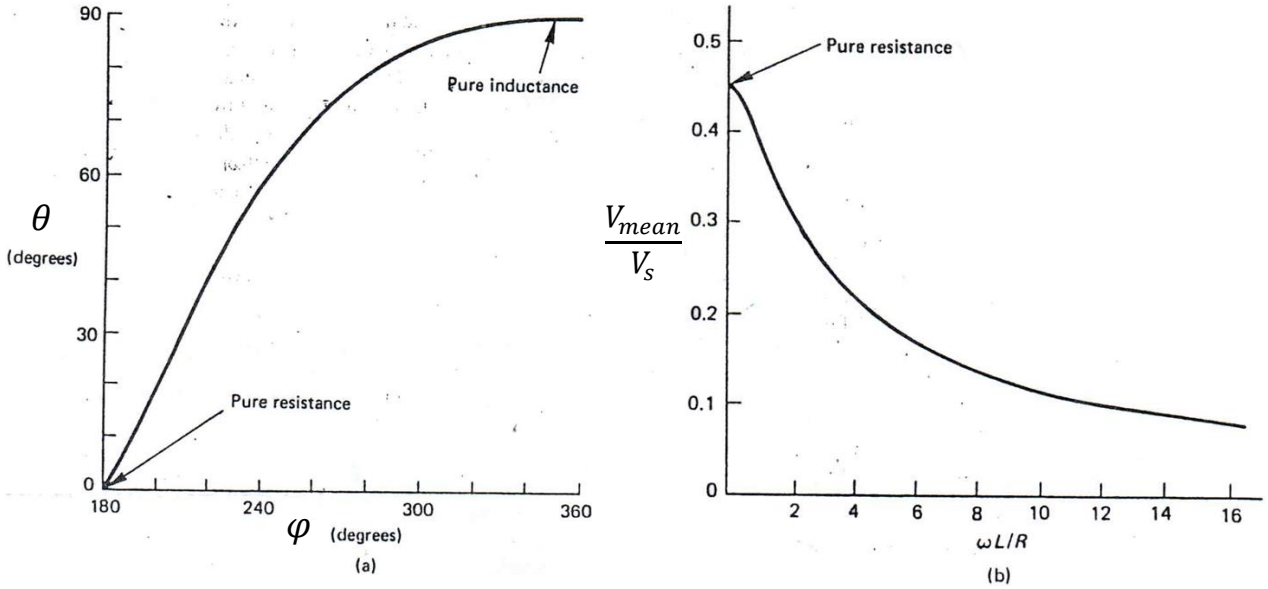


Figure 5: Single-phase half-wave converter characteristics: (a) load impedance angle (θ) versus current extinction angle (φ); (b) variation in mean output voltage versus $\left(\frac{\omega L}{R}\right)$.

Example-1: A circuit is connected as shown in Figure 1 to a 240 V, 50 Hz supply. Neglecting the diode volt-drop, determine the current waveform, the mean load voltage, and the mean load current for a load of **(i)** pure resistance of 10 Ω, **(ii)** an inductance of 0.1 H in series with a 10 Ω resistor.

Solution:

$V_m = 240\sqrt{2} = 339.4\ \text{V}$

(i) $I_m = \frac{V_m}{R} = \frac{339.4}{10} = 33.94\ \text{A}$

$$V_{mean} = \frac{V_m}{\pi} = \frac{339.4}{3.14} = 108\ \text{V}$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{108}{10} = 10.8\ \text{A}$$

(ii) Neglecting diode drop and at t=0 the source voltage will be equal to the load voltage

$$V_m \sin\omega t = R\, i_L + L\frac{di_L}{dt}$$

$$339.4 \sin\omega t = 10\, i_L + 0.1\frac{di_L}{dt}$$

Using the Laplace transform method of solution, where $\bar{\imath}(s)$ is the transform of $i(t)$, then:

$$339.4 \times \left[\frac{2\pi(50)}{\left(s^2 + \left(2\pi(50)\right)^2\right)}\right] = 0.1(s\bar{\imath} - i^0) + 10\bar{\imath}, where\ i^0 = 0$$

$$\bar{\imath} = 1066000 / [\,(s^2 + (2\pi(50))^2)(s + 100)\,]$$

Using the inverse Laplace transform to obtain the current as a function of time:

$$i = 9.81e^{-100t} + 10.29\,sin(2\pi(50)t - 1.262)\ \text{A}$$

Another method for the determination of the current expression:

$i = i_t + i_{ss}$, where i_t = transient current, and i_{ss} = steady-state AC current.

$$i_{ss} = \left[\frac{339.4}{\sqrt{10^2 + (2\pi \times 50 \times 0.1)^2}}\right] \sin\left(\omega t - tan^{-1}\left(\frac{(2\pi \times 50 \times 0.1)}{10}\right)\right)$$

$$,\theta = tan^{-1}\left(\frac{(2\pi \times 50 \times 0.1)}{10}\right) = 72.3°\,, or\ \theta = 1.262\,rad.$$

$$i_{ss} = \left(\frac{339.4}{31.4}\right)\sin(\omega t - 1.262) = 10.29\sin(2\pi(50)t - 1.262)$$

$$i_t = A\,e^{-\left(\frac{R}{L}\right)t} = A\,e^{-\left(\frac{10}{0.1}\right)t} = A\,e^{-100t}$$

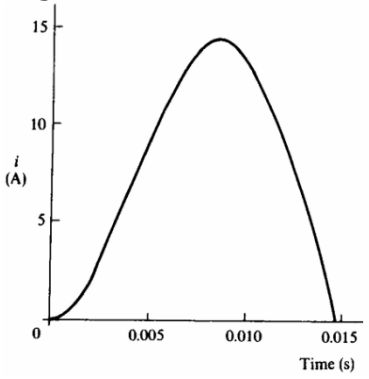
$$i = 10.29\,sin(2\pi(50)t - 1.262) + A\,e^{-100t}, at\ t = 0, i = 0$$

$$0 = 10.29\,sin(0 - 1.262) + A$$

$$A = -10.29\,sin(-1.262) = 9.81$$

$$\therefore\ i = 10.29\,sin(2\pi(50)t - 1.262) + 9.81e^{-100t}$$

The current will cease when $i = 0$ yielding a time $= 0.01472\ s$ which gives an angular duration of current of $\varphi = 265°$.



Hence,

$$V_{mean} = \left(\frac{1}{2\pi}\right)\int_{0°}^{265°} 339.4\sin(\omega t)\,d(\omega t) = 58.8\ \text{V}$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{58.8}{10} = 5.88\ \text{A}$$

The angle φ can be obtained either by trial and error or using a curve in Fig. 5-a.

2. Uncontrolled single phase half-wave rectifier with an R-L load and freewheeling diode (commutating diode):

The circuit in Figure 3.b, which has an R-L load, is characterized by discontinuous and high ripple current. Continuous load current can result when a diode is added across the load as shown in Fig. 6. This diode prevents the voltage across the load from reversing during negative half-cycle of the supply voltage. Diode D_f provides an alternative freewheeling path as indicated by the waveforms in Fig. 7 and 8.

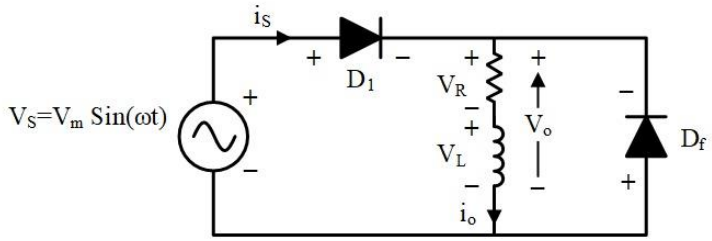


Figure 6: Half-wave rectifier with free-wheeling diode and an R-L load circuit.

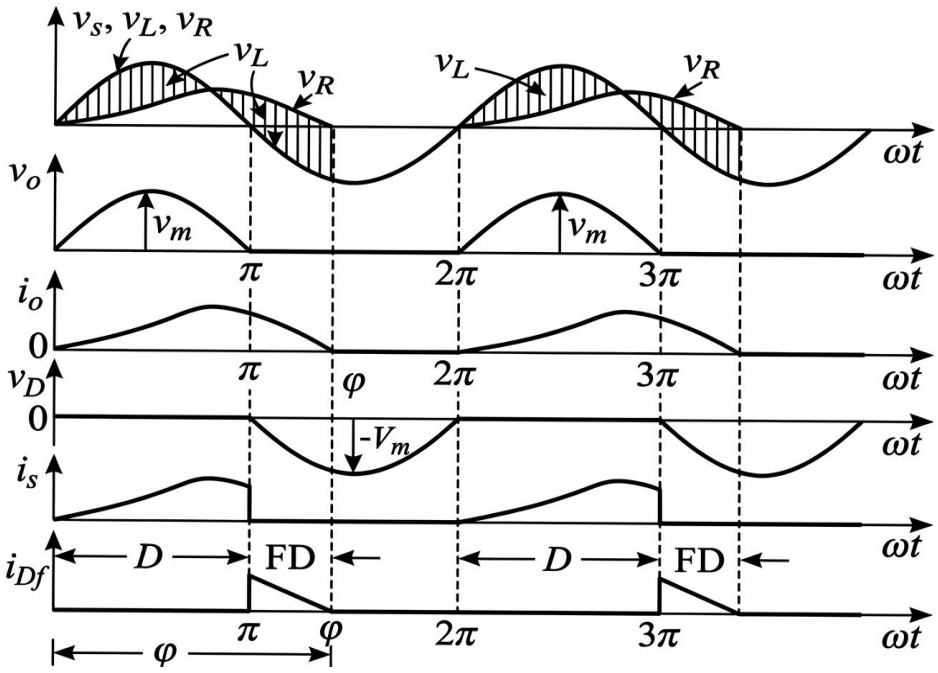


Figure 7: Half-wave rectifier with free-wheeling diode and an R-L load waveforms.

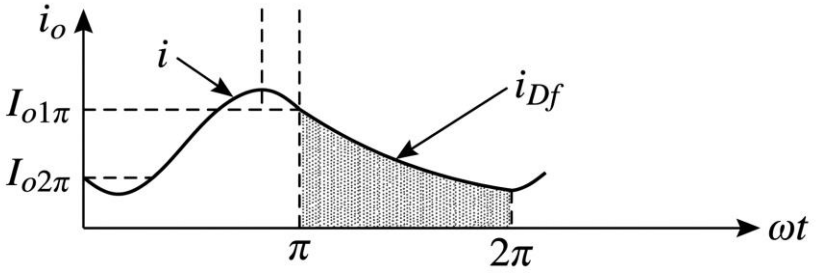


Figure 7: Half-wave rectifier with free-wheeling diode and inductive load output current.

The output voltage is the positive half of the sinusoidal input voltage. The mean output voltage is given by:

$$V_{\text{mean}} = \frac{V_{\text{m}}}{\pi} \tag{6}$$

The rms value of the load circuit voltage v_0 is given by:

$$V_{\text{rms}} = \sqrt{\left\{\left(\frac{1}{2\pi}\right)\int_0^{\pi}(V_{\text{m}}\sin\omega t)^2 d(\omega t)\right\}} = \frac{V_{\text{m}}}{2} \tag{7}$$

The ripple voltage is defined as

$$V_{RI} = \sqrt{{V_{\text{rms}}}^2 - {V_{\text{mean}}}^2} = 0.3855\, V_{\text{m}} \tag{8}$$

Hence the ripple factor is defined as

$$K_{\text{v}} = \frac{V_{RI}}{V_{\text{mean}}} = \frac{(0.3855\, V_{\text{m}})}{V_m/\pi} = 1.2 \tag{9}$$

After a large number of supply cycles, steady-state load conditions are established, and the load current is given by

$$i^0 = \left(\frac{V_{\text{m}}}{Z}\right)\sin(\omega t - \theta) + \left(I_{o2\pi} + \left(\frac{V_{\text{m}}}{Z}\right)\sin\theta\right)e^{\left(\frac{-\omega t}{\tan\theta}\right)} \tag{10}$$

for $0 \le \omega t \le \pi$

$$I_{o2\pi} = \left(\frac{V_{\text{m}}}{Z}\right)\sin\theta \cdot \frac{\left[1 + e^{\left(\frac{-\pi}{\tan\theta}\right)}\right]}{\left[e^{\left(\frac{\pi}{\tan\theta}\right)} - e^{\left(\frac{-\pi}{\tan\theta}\right)}\right]} \tag{11}$$

where $Z = \sqrt{(R^2 + (\omega L)^2)}$ and $\tan\theta = \frac{\omega L}{R}$

The diode current and hence load current is given by

$$i^0 = i_{Df} = I_{o1\pi}\, e^{\left\{-\frac{(\omega t - \pi)}{\tan\theta}\right\}} \qquad (12)$$

$$\pi \leq \omega t \leq 2\pi$$

for

$$I_{o1\pi} = I_{02\pi}\, e^{\left(\frac{\pi}{\tan\theta}\right)}$$

In Fig. 6 it will be seen that although the load current is continuous, the supply current is discontinuous and therefore has a very high harmonic content.

Example 2: In the circuit of Figure 6 the source voltage is $240\sqrt{2}\, sin(2\pi 50t)$ V, $R = 10\,\Omega\, and\, L = 50\, m$H. calculate **(a)** the mean value of the load current (I_{mean}). **(b)** the current boundary conditions, namely $I_{01}\pi$ and $I_{02}\pi$.

Solution:

$$(a)\; V_{mean} = \frac{V_{\mathrm{m}}}{\pi} = \frac{(\sqrt{2}\times 240)}{\pi}$$

hence the mean load current

$$I_{mean} = \frac{V_{mean}}{R} = \frac{(\sqrt{2}\times\ 240)}{(\pi\ \times\ 10)} = 10.8\ \mathrm{A}$$

(b) The load impedance is characterized by

$$Z = \sqrt{(R^2 + (\omega L)^2)} = \sqrt{(10^2 + (2\pi \times 50 \times 50 \times 10^{-3})^2} = 18.62\,\Omega$$

$$and\ \theta = tan^{-1}\left(\frac{\omega L}{R}\right) = 57.51°$$

$$or\ \tan\theta = \frac{\omega L}{R} = 1.57$$

$$I_{o2\pi} = \left(\frac{\sqrt{2}\times\ 240}{18.62}\right)\sin(57.51°)\cdot\frac{\left[1\ +\ e^{\left(\frac{-\pi}{1.57}\right)}\right]}{\left[e^{\left(\frac{\pi}{1.57}\right)} - e^{\left(\frac{-\pi}{1.57}\right)}\right]}$$

$$I_{o2\pi} = 15.375 \times \left(\frac{1.13519}{7.26135}\right) = 2.4\ \mathrm{A}$$

$$I_{o1\pi} = I_{o2\pi}\, e^{\left(\frac{\pi}{\tan\theta}\right)} = I_{o2\pi}\, e^{\left(\frac{\pi}{1.57}\right)} = 17.73\ \mathrm{A}$$