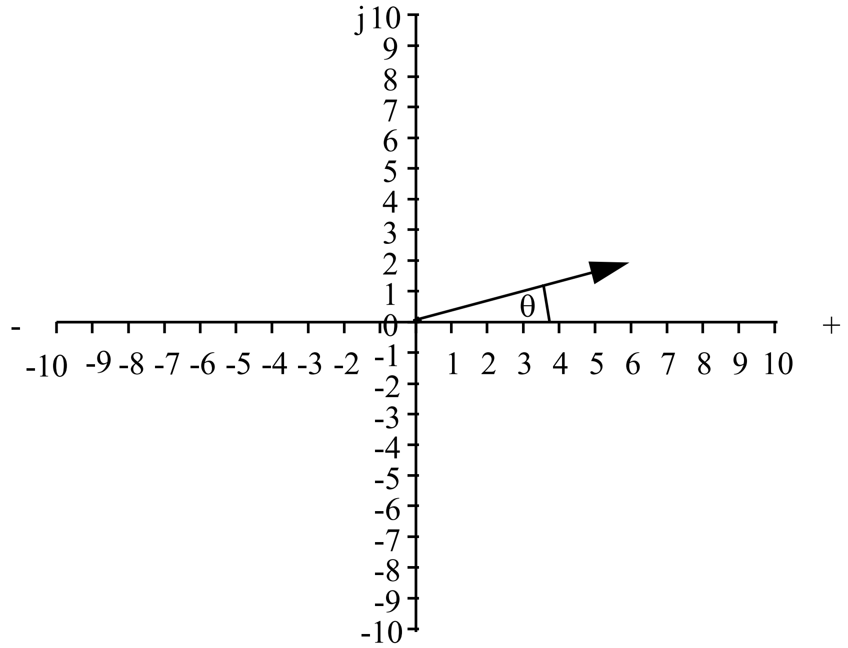
8.1 Complex Numbers

8.1.2 Complex Numbers Problem Class II

1. Determine the modulus and argument of the following complex numbers and hence express them in polar form :-

(i) $6 + j2$ Firstly, represent the complex number on an Argand diagram. This ensures you will get θ correct, particularly, when the complex number contains negative terms.



$$\text{modulus} = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} = 6.325$$

Argument (θ), $\tan\theta = \frac{2}{6} = \frac{1}{3}$

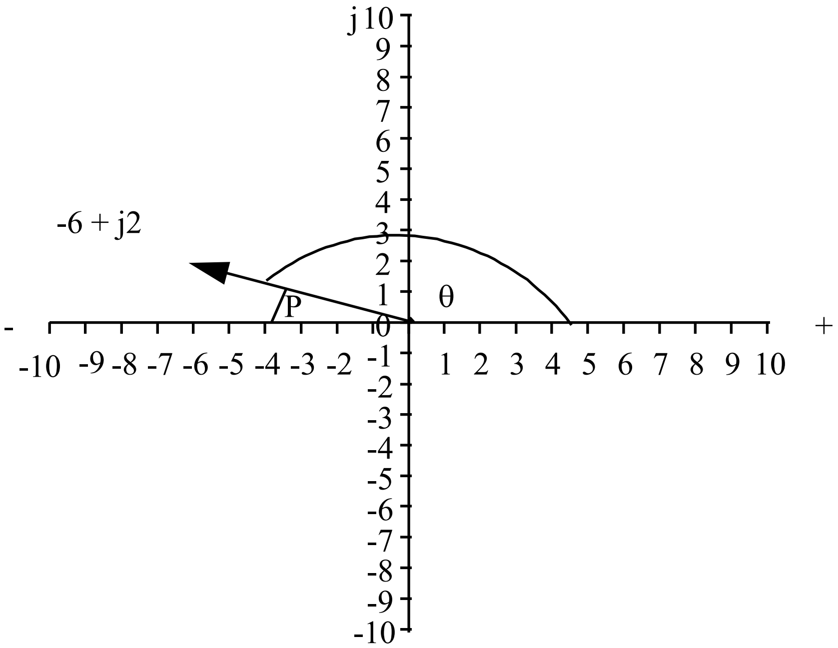
Therefore, $\theta = \tan^{-1}\frac{1}{3} = 18.43°$

Now that the modulus (r) and argument (θ) have been determined the complex number can be written in polar form. Based on the general polar form of a complex number :-

$z = r(\cos\theta + j\sin\theta)$

$6 + j2$ is $6.325(\cos 18.43° + j\sin 18.43°)$ in polar form.

(ii) - 6 + j2



modulus (r) = $\sqrt{6^2 + 2^2} = \sqrt{40} = 6.325$

The argument (θ) = 180° - the angle P

tan P = $\frac{2}{6} = \frac{1}{3}$ Therefore, P = 18.43°

Therefore, θ = 180° - 18.43° = 161.57°

So, - 6 + j2 in polar form is 6.325(cos 161.57° + jsin 161.57°)

(iii) 4 - j5 in polar form is 6.403(cos 308.66° + jsin 308.66°)

(iv) 2 + j6 in polar form is 6.325(cos 71.57° + jsin 71.57°)

(v) the conjugate of (iv) is 2 - j6 and in polar form this is

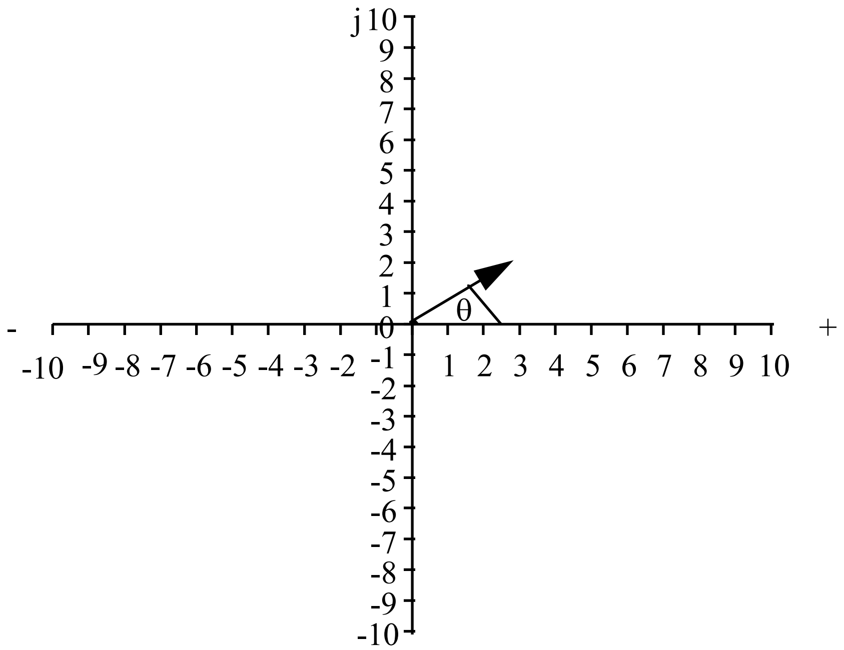
6.325(cos 288.43° + jsin 288.43°)

2. Express the following in exponential form :-

(i) 3 + j2

As with question 1 calculate the modulus ‘r’ and the argument ‘θ’. Then substitute ‘r’ and ‘θ’ into the general expression for the exponential form of a complex number :-

$$z = re^{j\theta}$$



$\theta = \tan^{-1}\dfrac{2}{3} = 33.69°$

$r = \sqrt{3^2 + 2^2} = \sqrt{9+4} = 3.61$

The general exponential form of a complex number is $z = re^{j\theta}$ therefore :-

3 + j2 in exponential form is $3.61e^{j.588}$

(ii) -4 - j3 in exponential form is $5e^{j3.785}$

(iii) 2 + j4 in exponential form is $4.47e^{j1.107}$

3. Calculate the following :-

(i) $6(\cos 25° + j\sin 25°) \times 4(\cos 75° + j\sin 75°)$

Simply add together the angles and multiply the 'r' values

$6(\cos 25° + j\sin 25°) \times 4(\cos 75° + j\sin 75°)$

$= (6 \times 4)[\cos(25° + 75°) + j\sin(25° + 75°)]$

$= 24(\cos 100° + j\sin 100°)$

(ii) $10(\cos 90° + j\sin 90°) \times 3(\cos 50° + j\sin 50°) = 30(\cos 140° + j\sin 140°)$

(iii) $\dfrac{8(\cos 60° + j\sin 60°)}{12(\cos 30° + j\sin 30°)}$

Division of complex numbers in polar form :- divide the 'r' terms and subtract the angles.

$$\frac{8(\cos 60° + j\sin 60°)}{12(\cos 30° + j\sin 30°)} = \frac{8}{12}[\cos(60° - 30°) + j\sin(60° - 30°)]$$

$$= \frac{2}{3}(\cos 30° + j\sin 30°)$$

(iv) $\frac{42(\cos 60° + j\sin 60°)}{21(\cos 45° + j\sin 45°)} = 2(\cos 15° + j\sin 15°)$

(v) $[10(\cos 20° + j\sin 20°]^4$

To solve this use DeMoivres Theorem :-

RULE - $[r(\cos\theta + j\sin\theta)]^n = r^n(\cos n\theta + j\sin n\theta)$

Therefore, $[10(\cos 20° + j\sin 20°]^4 = 10^4[\cos(4 \times 20°) + j\sin(4 \times 20°)]$

$$= 10000(\cos 80° + j\sin 80°)$$

(vi) $[2(\cos 60° + j\sin 60°)]^3 = 8(\cos 180° + j\sin 180°)$

(vii) $[3(\cos 40° + j\sin 40°)]^{\frac{1}{2}} = 3^{\frac{1}{2}}(\cos\frac{40}{2} + j\sin\frac{40}{2})$

$$= \sqrt{3}(\cos 20° + j\sin 20°)$$

$$= 1.732(\cos 20° + j\sin 20°)$$

(viii) $\sqrt{[3(\cos 40° + j\sin 40°)]} = [3(\cos 40° + j\sin 40°)]^{\frac{1}{2}}$

$$= 1.732(\cos 20° + j\sin 20°)$$

(ix) $\sqrt{4(\cos 20° + j\sin 20°)} = 2(\cos 10° + j\sin 10°)$

(x) $\sqrt[3]{3 + j4} = (3 + j4)^{\frac{1}{3}}$

Firstly, the complex number must be converted into polar form :-

$r = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ $\theta = \tan^{-1}\dfrac{4}{3} = 53.13°$

Therefore, 3 + j4 in polar form is 5 (cos 53.13° + jsin 53.13°)

$\left[5(\cos 53.13° + j\sin 53.13°\right]^{\frac{1}{3}} = 1.71\ (\cos 17.71° + j\sin 17.71°)$