بسم الله الرحمن الرحيم

Textbooks.

The recommended refrences text for the course are:

1- J. R. Holton : An Introduction to dynamic Meteorology 3rd Edition (1992) by Academic press. Note that is a 4th addition available, dated 2004.

2- A.E. Gill : Atmosphere - Ocean Dynamics (1982) by Academic press.

3- WMO Report : compendium of meteorology Vol. 1 part 1 Dynamic Meteorology No: 364.

4- J. T. Houghton : The Physics of Atmospheres 2nd Edition (1986) by Cambridge Univ. press.

5- Haltiner & Martin: Dynamical & physical Meteorology (1957).

Review of Vectors.

Some physical quantities such as Temperature, time and mass, can be described by a single number. Because these quantities are describable by giving only a magnitude, they are called "scalars".

on the other hand physical quantities such as wind velocity, displacement, force and acceleration require both a magnitude and a direction to completely describe them. such quantities are called "Vectors".

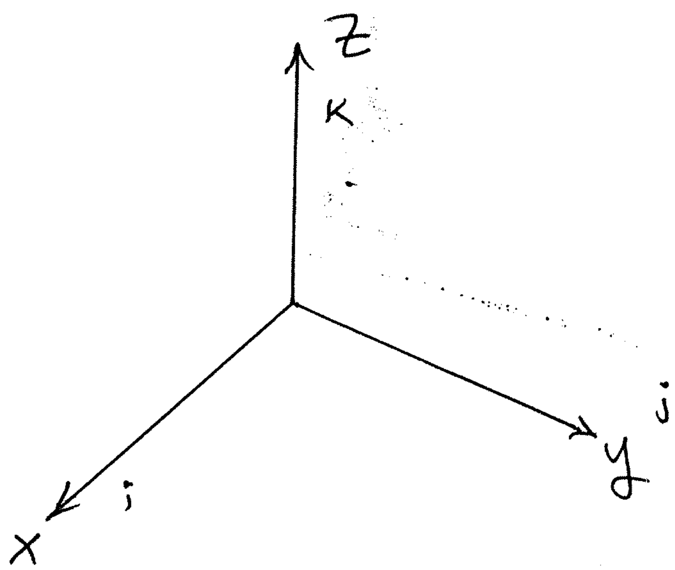
* Vectors have both a magnitude and a direction.
* Vectors are denoted by placing an arrow over the top ($\vec{A}$).
* The magnitude of a vector is denoted either by A or $|\vec{A}|$.

* An Vector can be written in terms of components along the coordinate system axes as:

$$\vec{A} = A_x i + A_y j + A_z k .$$

i, j, K are unit vectors along the x, y and z respectively, the magnitude of unit vectors is one.

In meteorology we use the following coordinate system:



The X-coordinate increases eastward.
The y-coordinate increases northward.
The Z-coordinate increases upward.

the Velocity along each coordinate direction are defined as:

$u = \frac{dx}{dt}$; u is the speed in eastward direction (Zonal wind).

$v = \frac{dy}{dt}$; v is the speed in northward direction (meridional wind).

$w = \frac{dz}{dt}$; w is the speed in upward direction (Vertical wind).

Vector Decomposition:

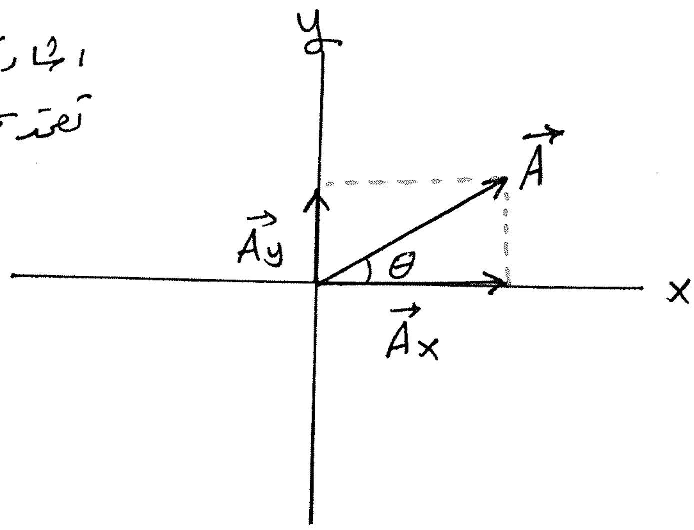
We can decompose a vector into component vectors along each coordinate axis.

A vector $\vec{A}$ can be decomposed into the vectors.

اشارة المركبة A_x و A_y

تعتمد على الزاوية θ

A_x − A_y +	A_x + A_y +
A_x − A_y −	A_x + A_y −



$$\cos\theta = \frac{A_x}{A} \Longrightarrow A_x = A\cos\theta.$$

$$\sin\theta = \frac{A_y}{A} \Longrightarrow A_y = A\sin\theta.$$

* The magnitude of $\vec{A}$ is:

$$A = \sqrt{(A_x)^2 + (A_y)^2}$$

* The direction of vector $\vec{A}$ is:

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

Atmospheric Dynamic: the branch of meteorology dealing with study of the causes and nature of motion of the atmosphere on all scales.

The motion in the atmosphere is governed by a set of equations, Known as the Navier-stokes equations. These equations, solved numerically by Computers, are used to produce our weather forecasts.

The scales of atmospheric motion are:

	Scale	Time	Distance	Example
1-	planetary scale	weeks, or longer	1000 to 40000 km	trade wind.
2-	Synoptic Scale	Days to weeks	100 to 5000 Km	cyclones
3-	Meso scale	Minutes to hours	1 to 100 Km	Tornado
4-	Microscale	second to minutes	$<$ 1 Km	Turbulence

in order to describe the dynamical behavior of the atmosphere, we treat it as a fluid. The circulation of planet's atmosphere is governed by four basic principles:

1- Newton's law of motion. "second law).
2- conservation of energy. "first law of thermodynamics."
3- conservation of mass. "equation of continuity"
4- the equation of state.

The basics of the equation of motion for the atmosphere is Newton's second law.

$$ma = \sum_{i=1}^{n} F_i$$

Newton's second law is applicable in a so-called inertial system, is a system which is remains fixed relative to the stars.
Velocity and acceleration measured in the inertial system are called the absolute velocity V_o and absolute acceleration a_o

We must modify Newton's second law in such away that it applies to a system which is fixed relative to the rotating earth.

$$a = \sum_{i=1}^{n} \frac{F_i}{m}$$

$$\left(\frac{d\vec{V_a}}{dt} \right)_{\text{fix. sys.}} = \sum_{i=1}^{n} \vec{f_i}$$

where $\vec{f_i}$ is the force per unit mass.

The Total Derivative

motorological variables such as $P, T, \vec{V}$, can vary both in space and time, function (x, y, z, t).

Using Taylor series expansion for Temperature.

$$\Rightarrow dT = \frac{\partial T}{\partial t} dt + \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz .$$

Dividing by dt

$$\frac{dT}{dt} = \frac{\partial T}{\partial t}\frac{dt}{dt} + \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} + \frac{\partial T}{\partial z}\frac{dz}{dt}$$

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

where $\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$ and $\frac{dz}{dt} = w$.

which can also be written as

$$\boxed{\frac{dT}{dt} = \frac{\partial T}{\partial t} + \vec{V}\cdot\vec{\nabla} T}$$

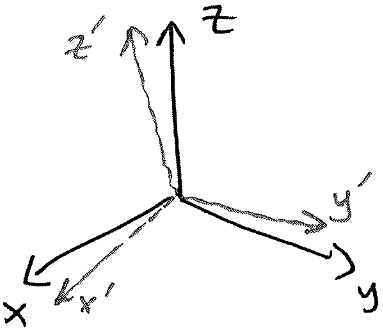
This shows that, the partial derivative is not equal to the total derivative.

Relation between fixed and Rotating system.

$\vec{A}$ is a vector in a fixed system

$$\vec{A} = A_x i + A_y j + A_z K \quad \text{(fixed system)}.$$

$$\vec{A}^* = A_x' i' + A_y' j' + A_z' K' \quad \text{(Rotated system)}.$$



$$\vec{A} = A_x i + A_y j + A_z K = A_x' i' + A_y' j' + A_z' K'.$$

$$\frac{d\vec{A}}{dt} = \frac{dA_x}{dt} i + \frac{dA_y}{dt} j + \frac{dA_z}{dt} K = \frac{dA_x'}{dt} i' + \frac{dA_y'}{dt} j' + \frac{dA_z'}{dt} K + A_x' \frac{di'}{dt} + A_y' \frac{dj'}{dt} + A_z' \frac{dK'}{dt}.$$

for any vector

$$\frac{d}{dt}(\text{any vector}) = \vec{\Omega} \times (\text{the vector})$$

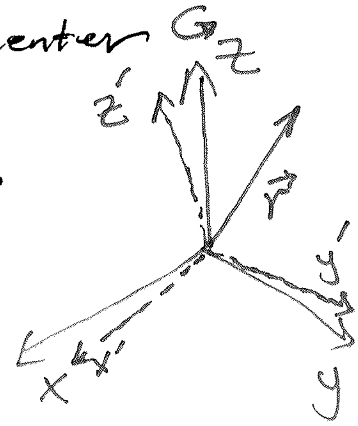
$$\frac{di'}{dt} = \vec{\Omega} \times i', \quad \frac{dj'}{dt} = \vec{\Omega} \times j', \quad \frac{dK'}{dt} = \vec{\Omega} \times K'$$

Thus
$$\frac{dA_x}{dt} i + \frac{dA_y}{dt} j + \frac{dA_z}{dt} K = \frac{dA_x'}{dt} i' + \frac{dA_y'}{dt} j' + \frac{dA_z'}{dt} K + A_x'(\vec{\Omega} \times i') + A_y'(\vec{\Omega} \times j') + A_z'(\vec{\Omega} \times K')$$

$$\left(\frac{d\vec{A}}{dt}\right)_{\text{fix. sys}} = \left(\frac{d\vec{A}}{dt}\right)_{\text{rota sys}} + \vec{\Omega} \times \vec{A}$$

Equation of motion in the Absolute and Rotating Coordinates.

let $\vec{r}$ is position vector from origin (center of earth)



Recall that : $\left(\frac{d\vec{A}}{dt}\right)_{\substack{fix\\ sys}} = \left(\frac{d\vec{A}}{dt}\right)_{\substack{rota.\\ syst}} + \vec{\Omega} \times \vec{A}$

$$\left(\frac{d\vec{r}}{dt}\right)_{\substack{fix.\\ sys.}} = \left(\frac{d\vec{r}}{dt}\right)_{\substack{rota.\\ sys.}} + \vec{\Omega} \times \vec{r}$$

$$\boxed{\vec{V}_a = \vec{V}_r + \vec{\Omega} \times \vec{r}}$$

$$\left(\frac{d\vec{V}_a}{dt}\right)_{\substack{fix\\ sys}} = \left(\frac{dV_a}{dt}\right)_{\substack{rota\\ sys}} + \vec{\Omega} \times \vec{V}_a$$

$$\left(\frac{d\vec{V}_a}{dt}\right)_{\substack{fix\\ sys}} = \frac{d}{dt}\left[\vec{V}_r + \vec{\Omega} \times \vec{r}\right]_{\substack{rota\\ sys}} + \vec{\Omega} \times \left[\vec{V}_r + \vec{\Omega} \times \vec{r}\right]$$

$$\left(\frac{d\vec{V}_a}{dt}\right)_{\substack{fix\\ sys}} = \left(\frac{d\vec{V}_r}{dt}\right)_{\substack{rota\\ sys}} + \frac{d}{dt}\left(\vec{\Omega} \times \vec{r}\right)_{\substack{rota\\ sys}} + \vec{\Omega} \times \left[\vec{V}_r + \vec{\Omega} \times \vec{r}\right]$$

$$\left(\frac{dV_a}{dt}\right)_{\substack{fix\\ sys}} = \left(\frac{d\vec{V}_r}{dt}\right)_{\substack{rota\\ sys}} + \frac{d}{dt}\left(\vec{\Omega} \times \vec{r}\right) + \vec{\Omega} \times \vec{V}_r + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right)$$

$$\therefore \underset{\substack{fix\\sys}}{\left(\frac{d\vec{V_a}}{dt}\right)} = \underset{\substack{rota\\sys}}{\left(\frac{d\vec{V_r}}{dt}\right)} + 2(\vec{\Omega} \times \vec{V_r}) - \vec{\Omega}^2 \vec{r}$$

Apply Newton's second law.

$$\left(\frac{d\vec{V_a}}{dt}\right) = \left(\frac{d\vec{V_r}}{dt}\right) + 2(\vec{\Omega} \times \vec{V_r}) - \vec{\Omega}^2 \vec{r} = \sum f_i$$

$$\boxed{\therefore \underbrace{\frac{d\vec{V_a}}{dt}}_{\text{Abs. Accel}} = \underbrace{\frac{d\vec{V}}{dt}}_{\text{Rel. Accel}} + \underbrace{2(\vec{\Omega} \times \vec{V})}_{\text{Coriolis Accel}} - \underbrace{\vec{\Omega}^2 \vec{R}}_{\text{Centripetal Accel}} = \underbrace{\sum f_i}_{\text{Net real forces}}}$$

The forces on the right hand side of equation are:

1- Gravity $\vec{g}$ 2- PGF $-\frac{1}{\rho}\vec{\nabla} P$ 3- Friction $f_{friction}$.

$$\frac{d\vec{V}}{dt} + 2(\vec{\Omega} \times \vec{V}) - \vec{\Omega}^2 \vec{R} = \vec{g} - \frac{1}{\rho}\vec{\nabla} P + \vec{f}$$

$$\boxed{\underbrace{\frac{d\vec{V}}{dt}}_{\text{Rela. accel.}} = \underbrace{-\frac{1}{\rho}\nabla \vec{P} + \vec{g} + \vec{f}}_{\text{Real force}} \underbrace{- 2(\vec{\Omega} \times \vec{V}) + \vec{\Omega}^2 \vec{R}}_{\text{apparent forces due to earth rotation}}}$$

This is THE EQUATION OF MOTION used in meteorology.
it is the back bone of atmosphere science.

$$\left(\frac{d\vec{V_a}}{dt}\right)_{\substack{\text{fix}\\ \text{sys}}} = \left(\frac{d\vec{V_r}}{dt}\right)_{\substack{\text{rota}\\ \text{sys}}} + \left(\frac{d\Omega}{dt}\times r\right)_{\substack{\text{rota}\\ \text{sys}}} + \left(\vec{\Omega}\times\frac{d\vec{r}}{dt}\right)_{\substack{\text{rota}\\ \text{sys}}} + \vec{\Omega}\times\vec{V_r} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

$$\left(\frac{d\vec{\Omega}}{dt}\times\vec{r}\right)_{\substack{\text{rota}\\ \text{sys}}} = \text{Zero} \quad \text{--- Show that.}$$

Thus :

$$\left(\frac{d\vec{V_a}}{dt}\right)_{\substack{\text{fix}\\ \text{sys}}} = \left(\frac{d\vec{V_r}}{dt}\right)_{\substack{\text{rota}\\ \text{sys}}} + 0 + \left(\vec{\Omega}\times\frac{d\vec{r}}{dt}\right) + \vec{\Omega}\times\vec{V_r} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

Recall that $\vec{V_r} = \dfrac{d\vec{r}}{dt}$

$$\left(\frac{d\vec{V_a}}{dt}\right)_{\substack{\text{fix}\\ \text{sys}}} = \left(\frac{d\vec{V_r}}{dt}\right)_{\substack{\text{rota}\\ \text{sys}}} + 2\left(\vec{\Omega}\times\vec{V_r}\right) + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

Recall that $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$

Thus $\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = (\vec{\Omega}\cdot\vec{r})\vec{\Omega} - (\vec{\Omega}\cdot\vec{\Omega})\vec{r}$

where $\vec{\Omega}\perp\vec{r} \Longrightarrow \boxed{\vec{\Omega}\cdot\vec{r} = 0}$ show that

$$\therefore \vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = 0 - (\vec{\Omega}\cdot\vec{\Omega})\vec{r}$$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = -\vec{\Omega}^2\vec{r}$$