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Chapter Six
(Nuclear Reactions)

A large fraction of our knowledge on the properties of nuclei is derived from nuclear reactions. When an incoming particle is scattered off a target nucleus, the outcome depends on a combination of three factors: the reaction mechanism, interaction between the projectile and the target, and the internal structure of the nuclei involved. There are two main categories of nuclear reactions. In the first category, the initial reactant X is a single atom or nucleus that spontaneously changes by emitting one or more particles, i.e.

$X \rightarrow b + Y$

Such a reaction is called radioactive decay. As we have seen from the Chart of the Nuclides, the vast majority of known nuclides are radioactive.

In the second broad category of nuclear reactions are binary reactions in which two nuclear particles (nucleons, nuclei or photons) interact to form different nuclear particles.

For bombarding energies below 100 MeV, nuclear reactions usually produce two products, i.e. they are of the type

$a + X \rightarrow b + Y$

Where a = bombarding particle

X = target (at rest in the lab. system)

b = light reaction product

Y = heavy reaction product

To shorten the notation a reaction of the type above is designated by:

X(a,b)Y

Commonly, one reaction product is light and the other heavy because of the binding energies of the nuclei involved. In some cases b and Y have comparable masses (spallation reaction or fission), or are identical. If b is a gamma ray, we speak of a capture reaction in which Y is the compound nucleus.

In most cases in which more than two products appear, it is possible to describe the process as a rapid sequence of two-product reactions

$a+X \rightarrow b_1+Y_1$

$Y_1 \rightarrow b_2+Y_2$

$Y_2 \rightarrow b_3+Y_3$

For example, see the reaction: $^{4}He + {}^{14}N \rightarrow {}^{1}H + {}^{17}O$

Note that the number of neutrons and protons is conserved. Presently the number of known reactions is in the thousands.

There are two frames to classify the nuclear reactions, consider an elastic collision with a nucleus of mass M. In the lab frame, the nucleus is initially at rest and the particle has energy E_o and momentum mv_o. After the scattering, energy of the particle is E_1, speed v_1 at an angle ϕ with v_o, while the nucleus recoil gives a momentum MV at an angle ψ.

The collision is better analyzed in the center of mass frame, where the condition of elastic scattering implies that the relative velocities only change their direction but not their magnitude. The center of mass velocity is defined as

$$\vec{v}_{C.M.} = \frac{m\vec{v}_o + M\vec{V}_o}{m+M} = \frac{m}{m+M}\vec{v}_o$$

Relative velocities in the center of mass frame are defined as

$$\vec{u} = \vec{v} - \vec{v}_{C.M.}$$

where we defined ϑ as the scattering angle in the center of mass frame, see the figures below:

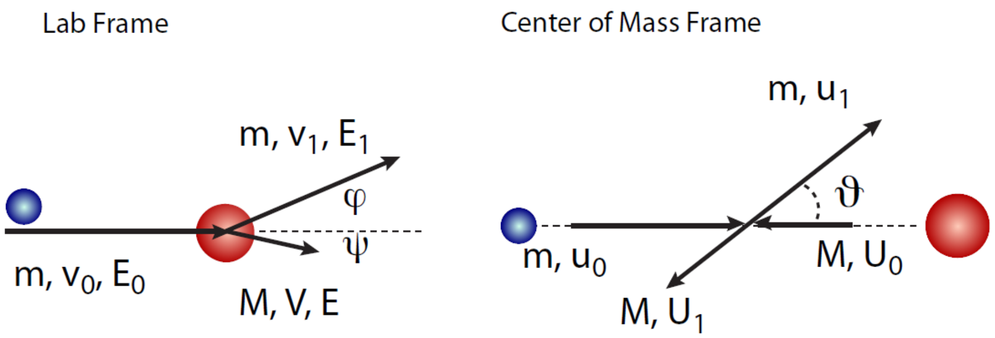

Figure (6-1): Neutron scattering from a nucleus. In left, laboratory frame, in right, center of mass frame.

Types of Nuclear Reactions:

Depending on the circumstances, it is convenient to classify nuclear reactions by the type of bombarding particle, bombarding energy, target, or reaction product. In the first case we distinguish:

Charged-particle reactions, produced by p, d, α, ^{12}C, ^{16}O ...

(p = proton, d = deuteron, α = alpha particle; the last two reactions are called heavy-ion reactions)

Neutron reactions

Photonuclear reactions, produced by gamma rays

Electron-induced reactions

If the bombarding energy is specified we speak informally of

Thermal energies ≈ 1/40 eV

Epithermal energies ≈ 1 eV

Slow-neutron energies ≈ 1 KeV

Fast-neutron energies ≈ 0.1 → 10 MeV

Low-energy charged particles ≈ 0.1 → 10 MeV

High energies ≈ 10 → 100 MeV

Targets are often called

Light nuclei, if $A \leq 40$

Medium-weight nuclei, if $40 < A < 150$

Heavy nuclei, if A ≥ 150

If the light reaction product is identical to the incident particle and has identical energy (in the c.m. system), the reaction is called elastic scattering. If only the energy is different, inelastic scattering occurs. If only gamma rays are emitted, we speak of a capture reaction. If the product nuclei have comparable masses, the reaction is called spallation or fission.

As an illustration, we give the following examples in the shorthand notation

$^{14}N(p,p)^{14}N$ proton elastic scattering

$^{14}N(p,p)^{14}N^*$ proton inelastic scattering

$^{14}N(p,\alpha)^{11}C$ or $^{11}C^*$ (p,α) reaction

$^{14}N(p,\gamma)^{15}O$ or $^{15}O^*$ proton-capture reaction

$^{14}N(\gamma,p)^{13}C$ or $^{13}C^*$ photonuclear reaction

$^{14}N(n,{}^{6}Li)^{9}Be$ or $^{9}Be^*$ spallation reaction

$^{9}Be({}^{6}Li,n)^{14}N$ or $^{14}N^*$ heavy-ion reaction

Conservation laws:

These may be listed as follows.

(i) Conservation of linear momentum, $\sum\vec{P}_i = \sum\vec{P}_f$

(ii) Conservation of total angular momentum i.e.

$$\sum \vec{J}_i + \vec{J}_{rel.i} = \sum \vec{J}_f + \vec{J}_{rel.f}$$

Where $\vec{J}_i, \vec{J}_f$ denote the angular momenta in the initial and final nuclei and $\vec{J}_{rel.i}, \vec{J}_{rel.f}$ denote the relative angular momenta in the entrance (X,a) and final (Y,b) channels.

(iii) Conservation of proton (charge) and neutron number is not a strict conservation law. Under general conditions, one has conservation of charge and conservation of nucleon or baryon (strongly interacting particles) number.

(iv) Conservation of parity, π, such that

$$\pi_X.\pi_a.\pi_{(X,a)} = \pi_Y.\pi_b.\pi_{(Y,b)}$$

Where the parities of the initial and final nuclei and projectiles (incoming, outgoing) are considered.

(v) Conservation of total energy, which becomes

$$T_X + m_X.c^2 + T_a + m_a.c^2 = T_Y + m_Y.c^2 + T_b + m_b.c^2$$

With T the kinetic energy and m.c^2 the mass energy, in the non-relativistic situation, the kinetic energy $T = \frac{1}{2}mv^2$. One defines the Q-value of a given reaction as

$$Q = \sum m_i c^2 - \sum m_f c^2 = (m_X + m_a - m_Y - m_b)c^2$$

which can be rewritten using the kinetic energies as:

$$Q = \sum T_f - \sum T_i = T_Y + T_b - T_X - T_a$$

Examples of Binary Nuclear Reactions:

For every nuclear reaction, we can write a reaction equation. These reaction equations must be balanced, just as chemical reactions must be. Charge (the number of protons) and mass number (the number of nucleons) must be conserved. The number of protons and the number of neutrons must be the same before and after the reaction. We shall illustrate this with some examples of typical nuclear reactions.

1- (α,p) reaction: The first nuclear reaction was reported by Rutherford. He bombarded nitrogen in air with alpha particles (helium nuclei) and observed the production of protons (hydrogen nuclei),

   $^{14}N + {}^{4}He \rightarrow {}^{17}O + {}^{1}H$ or $^{14}N(\alpha,p)^{17}O$

   The product of this reaction is ^{17}O and there are nine protons and nine neutrons on both sides of the equation, so the equation is balanced.

2- (α,n) reaction: In 1932, Chadwick discovered the neutron by bombarding beryllium with alpha particles to produce neutrons from the reaction

   $^{4}He + {}^{5}Be \rightarrow {}^{8}C + n$ or $^{5}Be(\alpha,n)^{8}C$.

3- (γ,n) reaction: Energetic photons (gamma rays) can also interact with a nucleus. For example neutrons can be produced by irradiating deuterium with sufficiently energetic photons according to the reaction

$^{2}H + \gamma \rightarrow {}^{1}H + n$ or $^{2}H(\gamma,n)^{1}H$

4- (p,γ) reaction: Protons can cause nuclear reactions such as the radiative capture of a proton by ^{7}Li, namely

$^{7}Li + {}^{1}H \rightarrow {}^{8}Be + \gamma$ or $^{7}Li(p,\gamma)^{8}Be^{*}$

The product nucleus ^{8}Be is not bound and breaks up (radioactively decays) almost immediately into two alpha particles.

5- (γ,αn) reaction: As an example of a reaction in which more than two products are produced, a high-energy photon can cause ^{17}O to split into ^{12}C, an α particle and a neutron through the reaction

$^{17}O + \gamma \rightarrow {}^{12}C + \alpha + n$

6- (n,p) reaction: Fast neutrons can cause a variety of nuclear reactions. For example, in a reactor core, fast neutrons can interact with ^{16}O to produce ^{16}N, which radioactively decays (half-life of 7.12s) with the emission of a 6.13MeV (69%) or a 7.11MeV (5%) photon. The radionuclide ^{16}N is produced by the reaction

$^{16}O + n \rightarrow {}^{16}N + {}^{1}H$ or $^{16}O(n,p)^{16}N^{*}$.

7- Proton-Proton Cycle

$$p + p \rightarrow d + e^{+} + \nu \Rightarrow p + d \longrightarrow {}^{3}_{2}He + \gamma \Rightarrow {}^{3}_{2}He + {}^{3}_{2}He \longrightarrow {}^{4}_{2}He + p + p$$

i.e. the result of three interactions: $4p \longrightarrow {}^{4}_{2}He + 2e^{+} + 2\nu + \gamma$, Q=24.7MeV

8- Carbon-Nitrogen Cycle

$$p + {}^{12}_{6}C \rightarrow {}^{13}_{7}N + \gamma \Rightarrow {}^{13}_{7}N \longrightarrow {}^{13}_{6}C + e^{+} + \nu \Rightarrow p + {}^{13}_{6}C \longrightarrow {}^{14}_{7}N + \gamma$$

$$\Rightarrow p + {}^{14}_{7}N \longrightarrow {}^{15}_{6}C + \gamma \Rightarrow {}^{15}_{6}C \longrightarrow {}^{15}_{7}N + e^{+} + \nu \Rightarrow p + {}^{15}_{7}N \longrightarrow {}^{12}_{6}C + {}^{4}_{2}He$$

By reduce the Carbon and Nitrogen, Which worked as catalysts we get:

$4p \longrightarrow {}^{4}_{2}He + 2e^{+} + 2\nu + \gamma$, Q=26.7MeV

9- Deuterium-Deuterium and Deuterium-Tritium interactions:

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{1}H + ^{1}_{1}H + 4.03MeV$$

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + n + 17.6MeV$$

Definitions:

1- ***Stopping power***: A more important quantity is the average energy loss of the particle per unit path length, which is called the stopping power.

2- ***Range:*** The range is more precisely defined as the distance a particle travels before coming to rest.

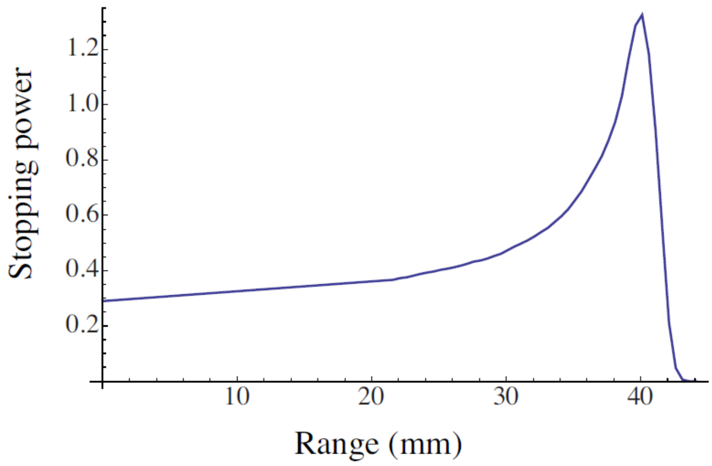

Figure (6-2): Bragg curve for protons (distance in mm)

3- ***Cross Section***:

Classically, the cross section is the area on which a colliding projectile can impact. Thus for example the cross section of a spherical target of radius r is just given by πr^2. The cross section has then units of an area. Let's consider for example a nucleus with mass number A. The radius of the nucleus is then $R=R_oA^{1/3}=1.2A^{1/3}$fm and the classical cross section would be $\sigma = \pi R_o^2 A^{2/3} \approx 5A^{2/3}fm^2$. For a typical heavy nucleus, such as gold, A = 197, we have $\sigma \approx 100fm^2=1barn$ (symbol b, $1b = 10^{-28}m^2= 10^{-24}cm^2= 100fm^2$. When scattering a particle off a target however, what becomes important is not the head-on collision (as between balls) but the interaction between the particle and the target (e.g.

Coulomb, nuclear interaction, weak interaction etc.). For macroscopic objects the details of these interactions are lumped together and hidden. For single particles this is not the case, and for example we can as well have a collision even if the distance between projectile and target is larger than the target radius. Thus the cross section takes on a different meaning and it is now defined as the effective area or more precisely as a measure of the probability of a collision. Even in the classical analogy, it is easy to see why the cross section has this statistical meaning, since in a collision there is a certain (probabilistic) distribution of the impact distance. The cross section also describes the probability of a given (nuclear) reaction to occur, are action that can be generally written as:

$a+X \rightarrow Y+b$ or $X(a,b)Y$

where X is an heavy target and (a) a small projectile (such as a neutron, proton, alpha...) while Y and b are the reaction products (again with b being nucleons or light nucleus, or in some cases a gamma ray). Then let I_a be the current of incoming particles, hitting on an heavy (hence stationary) target. The heavy product Y will also be almost stationary and only (b) will escape the material and be measured. Thus we will observe the (b) products arriving at a detector at a rate R_b. If there are (n) target nuclei per unit area, the cross section can then be written as:

$$\sigma = \frac{R_b}{I_a n}$$

This quantity does not always agree with the estimated cross section based on the nucleus radius. For example, proton scattering x-section can be higher than neutrons, because of the Coulomb interaction. Neutrinos x-section then will be even smaller, because they only interact via the weak interaction.