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الجامعة المستنصرية
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التداخل والحيود

Interference & Diffraction

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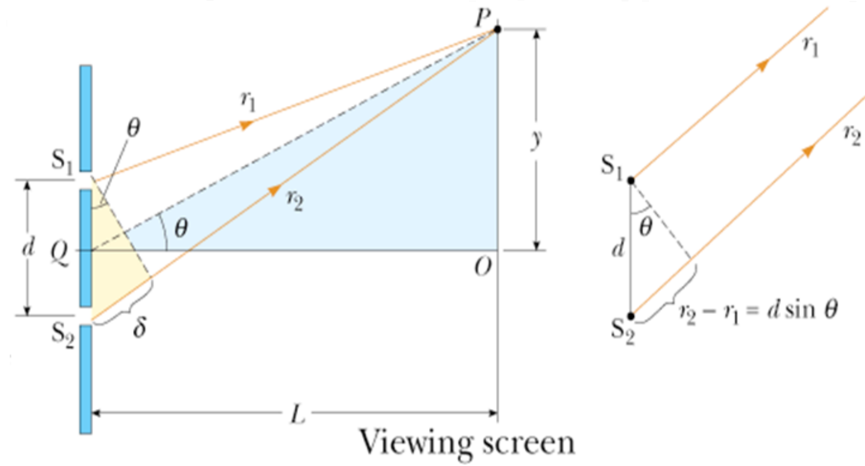
Important Laws of interference

- **Young's Experiment :**

$$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$
(maxima—bright fringes),

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$
(minima—dark fringes),



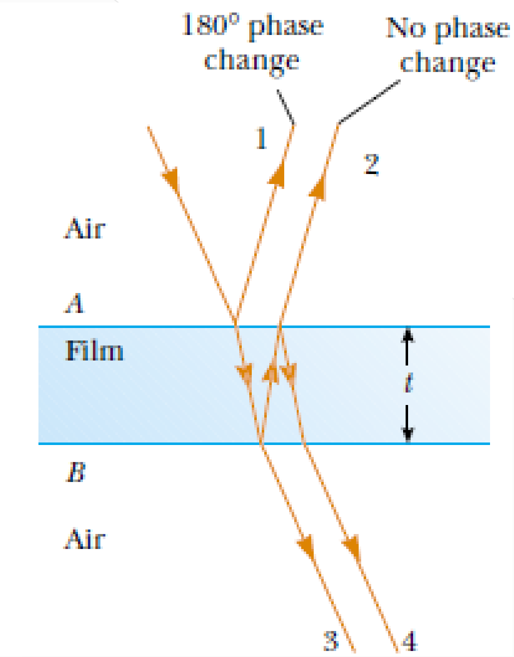
- Intensity in Two-Slit Interference :

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi, \qquad \text{where } \phi = \frac{2\pi d}{\lambda}\sin\theta.$$

- $I = I_1 + I_2\sqrt{I_1 I_2}$ (constructive interference)
- $I = I_1 - I_2\sqrt{I_1 I_2}$ (destructive interference)

Thin-Film Interference

- constructive interference: $$2nt = (m + \tfrac{1}{2})\lambda \qquad (m = 0, 1, 2, \ldots)$$
- destructive interference: $$2nt = m\lambda \qquad (m = 0, 1, 2, \ldots)$$



Newton's rings

- constructive interference: $\frac{1}{2}+\frac{2d}{\lambda}=m \qquad m=1,2,3,\ldots$

R^2 = m λ r

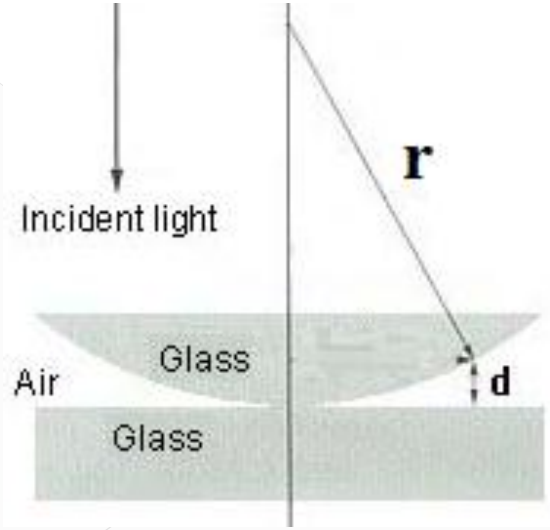
Where:

R= the radius of the ring ; m = order of the ring ; λ = wavelength of light .

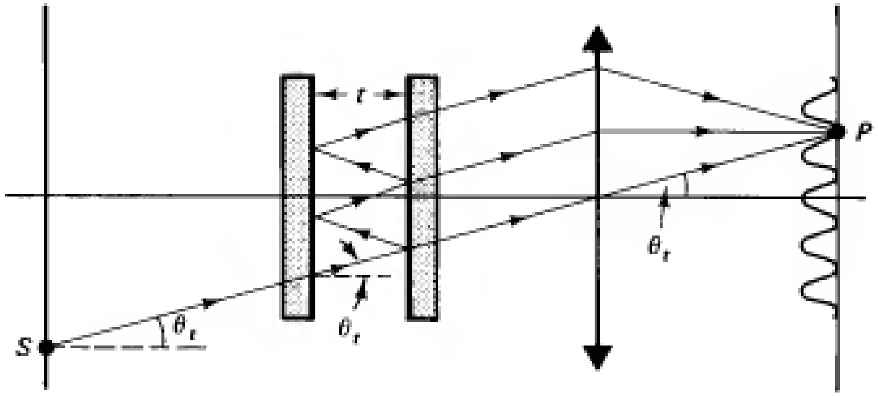
r = the radius of curvature of the convex lens.

- destructive interference: $\frac{1}{2}+\frac{2d}{\lambda}=m+\frac{1}{2} \qquad m=0,1,2,\ldots$

$R^2 = (m + \frac{1}{2})\,\lambda r$



Fabry-Perot interferometer



$\delta = 2\pi\, n\, d \cos\theta / \lambda$ (phase difference)

The transmitted intensity:

$$T = \frac{1}{1 + F\sin^2(\delta/2)}$$

Where F is the coefficient of finesse

$$F \equiv \frac{4r^2}{(1 - r^2)^2}$$

The phase difference between two fringe maxima is $\Delta\delta$ or FWHM :

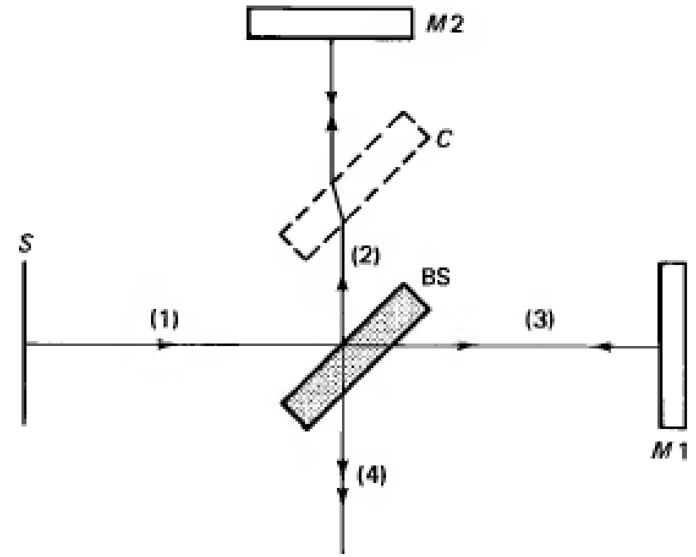
$$\text{FWHM} = \frac{4}{\sqrt{F}}$$

The resolving power $\Re$ is :

$$\Re = = \frac{\lambda}{\Delta\lambda} = \frac{m\pi\sqrt{F}}{2}$$

The finesse ζ is : $\zeta = \dfrac{\pi\sqrt{F}}{2}$

Michelson interferometer



$2d\cos\theta = m\lambda$

The optical path difference Δ is :

$\Delta = 2d\cos\theta$

Intensity of the fringe is :

$I = 4I_o\cos^2\left(\frac{\delta}{2}\right)$

The phase difference δ is:

$\delta = \left(\frac{2\pi}{\lambda}\right)\Delta$

The order of center dark fringe is:

$m = \frac{2d}{\lambda}$

For a mirror translation Δd, the number Δm of fringes passes at or near the center of the pattern of the interference is:

$\Delta m = \frac{2\Delta d}{\lambda}$

As the moveable mirror is displaced by $\lambda/2$, each fringe will move to the position previously occupied by an adjacent fringe. The number of fringes *N*, that have moved past a reference point to determine the distance traveled by the mirror Δd, that is,

$\Delta d = N\left(\frac{\lambda}{2}\right)$

Diffraction

The Fraunhofer diffraction pattern produced by a single slit of width a on a distant screen consists of a central bright fringe and alternating bright and dark fringes of much lower intensities. The angles θ_{dark} at which the diffraction pattern has zero intensity, corresponding to destructive interference, are given by :

$$\sin \theta_{dark} = m \lambda / d \quad ; \quad m = 1, 2, 3, \ldots.$$

Rayleigh's criterion: if two objects are separated by less than the minimum angle, they cannot be distinguished, The limiting angle of resolution for a slit of width a is $\theta_{min} = \lambda/d$, and the limiting angle of resolution for a circular aperture of diameter D is :

$$\theta_{min} = 1.22 \ \lambda / d$$

A diffraction grating is a large number of small slits. Principal maxima:

$$d \sin \theta = m \lambda \qquad\qquad m = 0,1,2,3, \ldots\ldots$$

1) What will happen if the monochromatic light used in Young's experiment is replaced by white light?

Solution:

Only the central fringe is white, all the other fringes are colored.

2) What will happen if one of the slits in Young's experiment is covered with cellophane paper which absorbs a fraction of the intensity of light from the slit?

Solution:

The bright fringes will become less bright and the dark fringes will not be completely dark.

3) How is the interference pattern affected if the Young's experiment was performed in still water than air?

Solution:

Fringes will be narrower. (width of the fringe $\propto \lambda$, and λ will decrease in water)

4) Two coherent sources of intensity ratio 100:1 interfere. What is the ratio of the intensity between the maxima and minima in the interference pattern?

Solution:

I $= I_1 + I_2\sqrt{I_1 I_2}$ = 100+1+2$\sqrt{100}$ = 121 (maxima or constructive interference)

I $= I_1 - I_2\sqrt{I_1 I_2}$ = 100+1-2$\sqrt{100}$ = 81 (minima or destructive interference)

$$\frac{121}{81} = \frac{3}{2} = 1.5$$

5) What the is the effect on the interference fringes in Young's experiment if the width of the source slit is increased?

Solution:

The fringes become less distinct.

6) What the is the effect on the interference fringes in Young's experiment if the widths of the two slits are increased?

Solution:

The bright fringes are no longer equally bright and equally spaced.

7) In Young's experiment the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the 4^{th} bright fringe and the central bright fringe is 1.2 cm. what is the wavelength of light used in the experiment?

Solution:

$x = m \lambda L / d \Longrightarrow \lambda = x d / m L$

x = width of the fringe; m = order of the fringe; L= distance of the screen;

d = distance between the slits; λ = the wavelength of light.

$\lambda = (1.2)(0.028) / (4)(140)$

$\lambda = 0.00006$ cm

$\lambda = 600$ nm

8) A series of rings formed in Newton's rings experiment with sodium light was viewed by reflection. The diameter of the m^{th} dark ring was found to be 0.28 cm and that of the $(m + 10)^{th}$ 0.68 cm. If the wavelength of sodium light is 589 nm, calculate the radius of curvature of the lens surface.

Solution:

R = D/2 = 0.14 cm for the dark ring

R = D/2 = 0.34 cm for the bright ring

$R^2 = m\,\lambda\,r$ for the dark ring

$R^2 = (m + \frac{1}{2})\,\lambda r$ for the bright ring

$$r = \frac{(0.14)^2}{(m)(589\times10^{-7})} = \frac{(0.34)^2}{(m+10)(589\times10^{-7})}$$

$\therefore$ m = 2

$$r = \frac{(0.14)^2}{(2)(589\times10^{-7})} = 166 \text{ cm} = 1.66 \text{ m}$$

9) Calculate the minimum thickness of a soap-bubble film (n=1.33 for water) that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is λ = 600 nm . What if the film is twice as thick? Does this situation produce constructive interference?

Solution:

The minimum film thickness for constructive interference in the reflected light corresponds to $m = 0$. This gives $2nt = \lambda/2$, or

$$t = \frac{\lambda}{4n} = \frac{600}{4(1.33)} = 133 \text{ nm}.$$

For the thicknesses at which constructive interference will occur :

$$t = (m+\frac{1}{2})\frac{\lambda}{2n} = (2m+1)\frac{\lambda}{4n} \qquad (m= 0, 1, 2, 3, \ldots.)$$

The allowed values of m show that constructive interference will occur for **odd** multiples of the thickness corresponding to m = 0, t = 113 nm. Thus, constructive interference will **not** occur for a film that is twice.

10) Light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective has a diameter of 0.900 cm, (a) what is the limiting angle of resolution? If it were possible to use visible light of any wavelength, (b)what would be the maximum limit of resolution for this microscope?

Solution:

(a) The limiting angle of resolution is:

$$\theta_{min} = 1.22\, \lambda / D = 1.22 \times \frac{589 \times 10^{-9}}{0.9 \times 10^{-2}} = 7.98 \times 10^{-5} \text{ rad.}$$

This means that any two points on the object subtending an angle smaller than this at the objective cannot be distinguished in the image.

(b) To obtain the smallest limiting angle, we have to use the shortest wavelength available in the visible spectrum. Violet light (400 nm) gives a limiting angle of resolution of

$$\theta_{min} = 1.22\, \lambda / D = 1.22 \times \frac{400 \times 10^{-9}}{0.9 \times 10^{-2}} = 5.42 \times 10^{-5} \text{ rad.}$$

11) Monochromatic light from a helium–neon laser (λ = 632.8 nm) is incident normally on a diffraction grating containing 6 000 grooves per centimeter. Find the angles at which the first- and second-order maxima are observed.

Solution:

First, we must calculate the slit separation, which is equal to the inverse of the number of grooves per centimeter:

$$d = \frac{1}{6000} \text{ cm} = 1.667 \times 10^{-4} \text{ cm} = 1667 \text{ nm}$$

For the first-order maximum (m = 1), we obtain:

$$\sin\theta_1 = \lambda / d = \frac{632.8}{1667} = 0.379$$

$$\theta_1 = 22.31°$$

For the second-order maximum (m =2), we find

$$\sin\theta_2 = 2\lambda / d = \frac{2(632.8)}{1667} = 0.759$$

$$\theta_2 = 49.39°$$

12) A Fabry-perot interferometer has a 1 cm spacing and reflection coefficient of r = 0.95. For a wavelength around 500 nm, determine its maximum order of interference, its coefficient of finesse, its minimum resolvable wavelength, and its resolving power.

Solution:

$$m_{max} = \frac{2t}{\lambda} = \frac{2(1\times10^{-2})}{500\times10^{-9}} = 40{,}000$$

$$F = \frac{4r^2}{(1-r)^2} = \frac{4(0.95)^2}{(1-0.95)^2} = 380$$

$$(\Delta\lambda)_{min} = \frac{2\lambda}{m\pi\sqrt{F}} = \frac{2(5000\text{Å})}{(40000)\pi\sqrt{380}} = 0.004\ \text{Å}$$

$$\Re = \frac{\lambda}{\Delta\lambda} = \frac{5000}{0.004} = 1.2\times\ 10^6$$

13) A Fabry-perot interferometer have an amplitude reflectance coefficient of r = 0.8944, find its coefficient of finesse, its half-width, and its finesse.

Solution:

$$F = \frac{4r^2}{(1-r)^2} = \frac{4(0.8944)^2}{(1-0.8944)^2} = 80$$

$$\text{FWHM} = \frac{4}{\sqrt{F}} = \frac{4}{\sqrt{80}} = 0.448$$

$$\zeta = \frac{\pi\sqrt{F}}{2} = \frac{\pi\sqrt{80}}{2} = 14$$

14) One of the mirrors of a Michelson Interferometer is moved. and 1000 fringe-pairs shift past the hairline in a viewing telescope during the process. If the device is illuminated with 500-nm light, how far was the mirror moved?

Solution:

$$\Delta\, \mathrm{m} = \frac{2\Delta d}{\lambda}$$

$$\Delta\, \mathrm{d} = \frac{\Delta m\, \lambda}{2} = \frac{1000\ (500\ nm)}{2} = 25 \times 10^{4}\ \mathrm{nm} = 2.5 \times 10^{-4}\ \mathrm{m}$$

15) A Michelson Interferometer is illuminated with monochromatic light. One of its mirrors is then moved 2.53×10^{-5} m, and it is observed that 92 fringe-pairs, bright and dark, pass by in the process. Determine the wavelength of the incident beam.

Solution:

A motion of $\frac{\lambda}{2}$ causes a single fringe pair to shift past, hence :

$$\Delta d = \mathrm{N}\left(\frac{\lambda}{2}\right)$$

$$\lambda = \frac{2\,\Delta d}{N}$$

$$\frac{92\ \lambda}{2} = 2.53\times 10^{-5}\ \mathrm{m}$$

$$\lambda = \frac{2\,\Delta d}{N} = 550\ \mathrm{nm}$$

16) A glass camera lens with an index of 1.55 is to be coated with a cryolite film (n = 1.30) to decrease the reflection of normally incident green light (λ = 500 nm). What thickness should be deposited on the lens?

Solution:

$$t = \frac{\lambda}{4n} = \frac{5.5 \times 10^{-7}}{4(1.55)} = 8.87 \times 10^{-6}\ \text{m}.$$

17) Solar cell devices that generate electricity when exposed to sunlight are often coated with a transparent, thin film of a low refractive index material to minimize reflective losses from the surface. Suppose that a silicon solar cell (*n* = 3.5) is coated for this purpose. The minimum film thickness that produces the least reflection at a wavelength of 550 nm, is 94.8 nm. Determine the refractive index of the thin film material.

Solution:

$$t = \frac{\lambda}{4n}$$

$$n = \frac{\lambda}{4t} = \frac{550}{4(94.8)} = 1.45$$ (this material is SiO)

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