

## The Newtonian Lens Equation

We have been using the “Gaussian Lens Formula;

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

An alternate lens formula is known as the Newtonian Lens Formula;

$$x_1 x_2 = f^2$$

Which can be verified by substituting;

$$p = f + x_1$$

$$q = f + x_2$$

Into the Gaussian Lens Formula, Here,  $x_1$  and  $x_2$  are the distances to the object and image respectively from the focal points. ( $f$  is negative for a diverging lens)

$$x_1 = p - f$$

$$x_2 = q - f$$

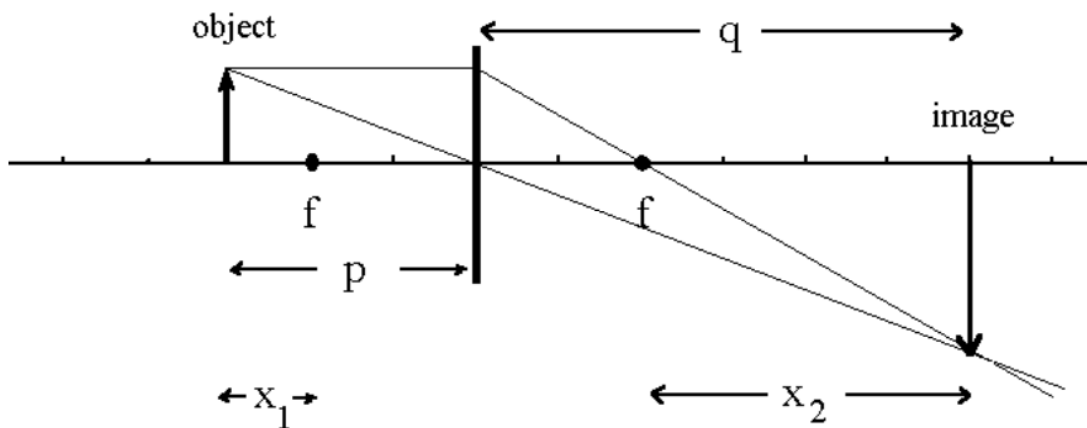


Figure 18. Gaussian & Newtonian Lens Equations Relation

**Example1:**

An object is 15 cm from a converging lens which has a focal length of 10 cm, where is the image?

$$x_1 = p - f = 15 - 10 = 5 \text{ cm}$$

$$x_2 = \frac{f^2}{x_1} = \frac{100}{5} = 20 \text{ cm}$$

So the image is  $(10+20) \text{ cm} = 30 \text{ cm}$  to the right of the lens.

**Example2:**

An object is 15 cm from a diverging lens which has a focal length of -10 cm, where is the image?

$$x_1 = p - f = 15 - (-10) = 25 \text{ cm}$$

$$x_2 = \frac{f^2}{x_1} = \frac{100}{25} = 4 \text{ cm}$$

So the image is  $((-10) + 4) \text{ cm} = -6 \text{ cm}$  to right of lens, which is 6 cm to left of lens

**Lens maker's Equation**

The lens maker's equation is another formula used for giving a relationship between the (focal length, refractive index and radii of curvature of the two spheres) used in the lens. Each face of a lens is part of a sphere where the thickness of the lens must be considered.

For a thin lens,

$$p = \frac{1}{f} = (n_{Lens} - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

$P$  is the power of the lens,

$f$  is the focal length of the lens,

$n$  is the refractive index of the lens material,

$R_j$  is the radius of curvature of the face closest to the object

$R_2$  is the radius of curvature of other face

For a Convex (converging) Lens	For a Concave (diverging) Lens
$R_1 = \text{positive}$	$R_1 = \text{negative}$
$R_2 = \text{negative}$	$R_2 = \text{positive}$
$f = \text{positive}$	$f = \text{negative}$

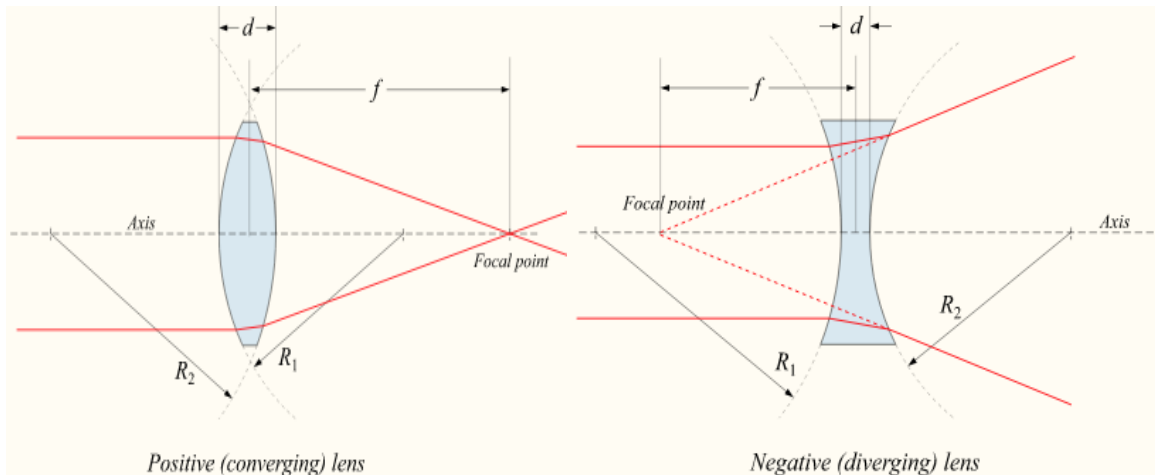


Figure 19. Thickness of the lens effect

**Example:**

The refractive index of a lens is given as 1.5, two curved surfaces have radius of curvature 10cm and -15cm respectively. Calculate the focal length of the given lens?

$n=1.50$ ,  $R_1=10$  cm and  $R_2 = -15$  cm

Lens maker's formula is given as,

$$\frac{1}{f} = (n_{Lens} - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{f} = (1.5 - 1) \left[ \frac{1}{10} + \frac{1}{15} \right]$$

$$= 0.5 \times (0.1 + 0.0666) = 0.5 \times 0.1666 = 0.0833$$

$$f = \frac{1}{0.0833} = 12 \text{ cm}$$