Foundation of Mathematics

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Course Outline
First Semester

Course Title: Foundation of Mathematics (1)

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Stage: The First

Contents

Chapter 1	Logic Theory	Logic, Truth Table, Tautology, Contradiction, Contingency, Rules of Proof , Logical Implication, Canonical Form, Conjunctive Normal Form, Quantifiers, Logical Reasoning, Mathematical Proof.
Chapter 2	Sets	Definitions, Equality of Sets, Set Laws
Chapter 3	Relations on Set	
Chapter 4	Algebra of Mappings	Mappings, Types of Mappings, Composite Mapping and Inverse.

References

1-Fundamental Concepts of Modern Mathematics. Max D. Larsen. 1970.
2-Introduction to Mathematical Logic, 4th edition. Elliott Mendelson.1997.
3-اسس الرياضيات, الجزء الاول. تاليف د. هادي جابر مصطفى, رياض شاكر نعوم و نادر جورج منصور. 1980.

4- A Mathematical Introduction to Logic, 2nd edition. Herbert B. Enderton. 2001.

THE GREEK ALPHABET

letter	name	capital
α	Alpha	Α
β	Beta	Β
γ	Gamma	Γ
δ	Delta	Δ
ε	Epsilon	Ε
ζ	Zeta	Ζ
η	Eta	Η
θ	Theta	Θ
ι	Iota	Ι
κ	Kappa	Κ
λ	Lambda	Λ
μ	Mu	Μ
ν	Nu	Ν
ξ	Xi	Ξ
ο	Omicron	Ο
π	Pi	Π
ρ	Rho	Ρ
σ ς	Sigma	Σ
τ	Tau	Τ
υ	Upsilon	Υ
φ	Phi	Φ
χ	Chi	Χ
ψ	Psi	Ψ
ω	Omega	Ω

Chapter One

Logic Theory

1.1. Logic

Definition 1.1.1

(i) Logic is the theory of systematic reasoning and symbolic logic is the formal theory of logic.

(ii) A **logical proposition (statement or formula)** is a declarative sentence that is either true (denoted either T or 1) or false (denoted either F or 0) but not both.

Notation: Variables are used to represent logical propositions. The most common variables used are p, q, and r.

Example 1.1.2.

$x + 2 = 2x$ when $x = -2$.

All cars are brown.

$2 \times 2 = 5$.

Here are some sentences that are not logical propositions (**paradox**).

Look out! **(Exclamatory)**

How far is it to the next town? **(Interrogative)**

$x + 2 = 2x$.

"Do you want to go to the movies?" **(Interrogative)**

"Clean up your room." **(Imperative)**

1.2. Truth Table

1.2.1. What is a Truth Table?

(i) A truth table is a tool that helps you analyze statements or arguments (defined later) in order to verify whether or not they are logical, or true.

(ii) A truth table of a logical proposition shows the condition under which the logical proposition is true and those under which it is false.

There are six basic operations called **connectives** that you will utilize when creating a truth table. These operations are given below.

English Name	Math Name	Symbol
"and"	Conjunction	$\wedge$
"or"	Disjunction	$\vee$
"Exclusive"= "or but not both"	xor	$\underline{\vee}$
"if … then"	Implication	$\rightarrow$
"if and only if "	equivalence	$\leftrightarrow$
"not"	Negation	$\sim$

Definition 1.2.2. (Compound Statement)

If two or more logical propositions compound by connectives called compound proposition (statement).

The rules for these connectives (operations) are as follows:

AND ($\wedge$) (conjunction): these statements are true only when both p and q are

AND ∧ (Conjunction)		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

OR ($\vee$) (disjunction): these statements are false only when both p and q are false.

OR ∨ (Disjunction)		
p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive (⊻) one of p or q (read p or else q)

⊻ (Exclusive)		
p	q	p ⊻ q
T	T	F
T	F	T
F	T	T
F	F	F

If → Then Statements – These statements are false only when p is true and q is false (because anything can follow from a false premise).

Equivalent Forms of (**p → q)** read as:

If p then q”:
p implies q
p is a sufficient condition for q
q if p
q whenever p
q is a necessary condition for p.

If → Then		
p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Here, p called **hypothesis (antecedent)** and q called **consequent (conclusion)**.

Note that the statements **p** $\rightarrow$ **q** and **q** $\rightarrow$ **p** are different.

If and only If Statements – These statements are true only when both p and q have the same truth (logical) values.

If ↔ Then		
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

NOT ~ (negation) The "not" is simply the opposite or complement of its original value.

NOT ~ (negation)	
P	~p
T	F
F	T

Note that, the negation is meaningful when used with only one logical proposition. This is not true of the other connectives.

Examples 1.2.3. Write the following statements symbolically, and then make a truth table for the statements.

(i) If I go to the mall or go to the stadium, then I will not go to the gym.

(ii) If the fish is cooked, then dinner is ready and I am hungry.

Solution.

(i) Suppose we set

p = I go to the mall

q = I go to the stadium

r = I will go to the gym

The proposition can then be expressed as "If p or q, then not r," or $(p \vee q) \rightarrow \sim r$.

p	q	r	p ∨ q	~r	(p ∨ q) →~r
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	F	T	T

(ii) Suppose we set

f = the fish is cooked.
r = dinner is ready.
h = I am hungry.

(a) f → (r ∧ h)

(b) (f → r) ∧ h

f	r	h	r ∧ h	f→(r ∧ h)	f → r	(f→r) ∧ h
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	F	T	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

Exercise 1.2,4.
Build a truth table for p → (q → r) and (p ∧ q) → r.