Chapter (3)

TURBULENCE KINETIC ENERGY

3.1 The TKE budget Derivation :

The definition of TKE presented is TKE/m= e = 0.5 ($u'^2 + v'^2 + w'^2$) . we recognize immediately that TKE/m is nothing more than the summed velocity variances divided by two .

If we choose a coordinate system aligned with the mean wind , assume **horizontal homogeneity** , and **neglect subsidence** , then a special from of the TKE budget equation can be written :

$$\frac{\partial \bar{e}}{\partial t} = \frac{g}{\theta}\,\overline{(w'\theta')} - \overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \frac{\partial(\overline{\overline{w'e}})}{\partial z} - \frac{1}{\rho}\frac{\partial(\overline{w'p'})}{\partial z} - \varepsilon \;\ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

$\frac{\partial \bar{e}}{\partial t}$: Represent local storage or tendency of TKE .

$\frac{g}{\theta}\,\overline{(w'\theta')}$: is the buoyant production or consumption term . it is a production or loss term depending on whether the heat flux $\overline{(w'\theta')}$ is positive (during daytime over land) or negative (at night over land) .

$-\overline{u'w'}\frac{\partial \bar{u}}{\partial z}$: is a Mechanical or shear productionless term . the momentum flux $\overline{u'w'}$ is usually of opposite sign from the mean wind shear , because the momentum of wind is usually lost downward to the ground . thus , in this term is positive contribution to TKE when multiplied by a negative sign .

$-\frac{\partial(\overline{w'e})}{\partial z}$: Represents the turbulent transport of TKE . it describes how TKE is moved around by the turbulent eddies w' .

$-\frac{1}{\rho}\frac{\partial(\overline{w'p'})}{\partial z}$: is a pressure correlation term that describes how TKE is redistributed by pressure perturbations . it is often associated with oscillations in the air (buoyancy or gravity waves).

ε : represents the viscous dissipation of TKE , i.e. the conversion of TKE it to heat .

3.2 TKE BUDGET TERMS :

3.2.1 Buoyant Term :

Production : fig 4.1 shows the variation of a number of TKE budget terms with height within a fair – weather convective ML . the most important part of the buoyancy term is the flux of virtual potential temperature, $\overline{(w'\theta'_v)}$. this flux is positive and decreases roughly linearly with height within the bottom 2/3 of the convective ML .

In convection boundary layer capped with actively growing cumulus clouds , the positive buoyancy within the cloud can contribute to the production of TKE (see fig. 4.2) . between this cloud layer contribution and the contribution near the bottom of the subcloud layer , there may be a region near cloud base where the air is statically stable and the buoyancy term is therefore negative .

Consumpation : In statically stable conditions, an air parcel displaced vertically by turbulence would experience a buoyancy force pushing it back towards it starting height . Static stability thereby tends to suppress , or consume , TKE , and is associated with negative values of term buoyancy . such conditions are present in the SBL at night over land , or anytime the surface is colder than the overlying air. an example of the decay of turbulence in negatively buoyant conditions just after sunset is shown in the budget profiles of fig 3.3 .

This same type of consumption can occur at the top of a ML , where warmer air entrained downward by turbulence opposes the descent because of its buoyancy . this is related to the negative values of the buoyancy term near the top of the ML in fig 3.1 .

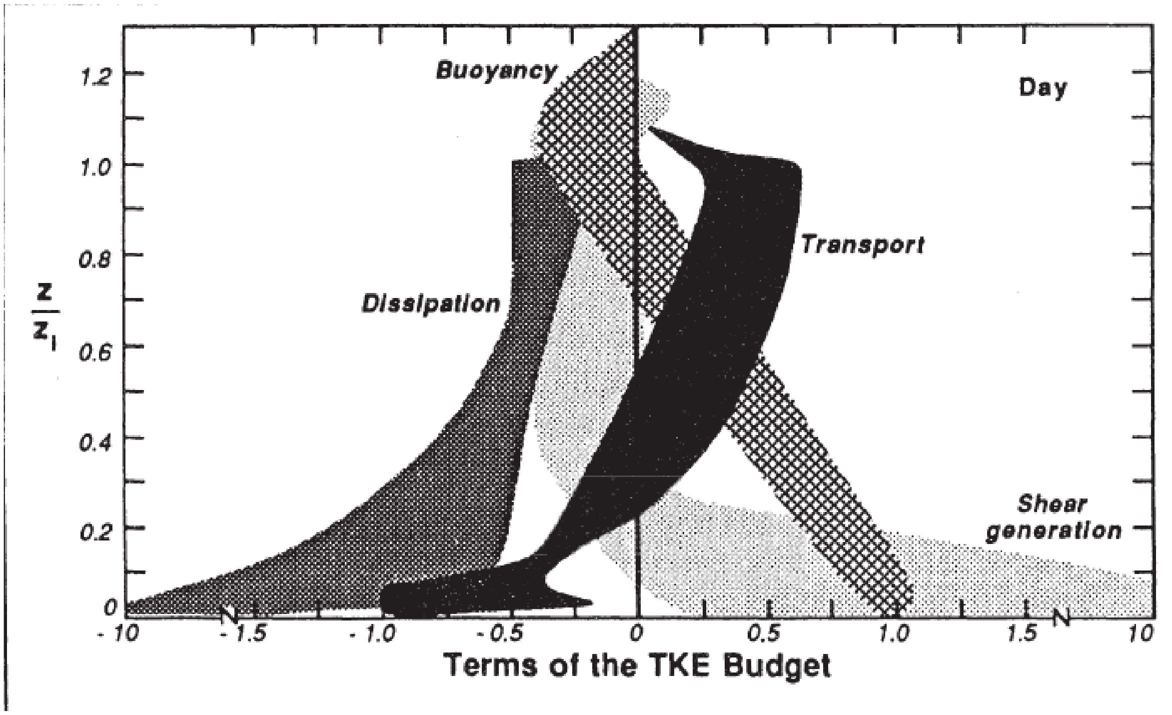


Figure (3.1) : normalized terms in turbulent kinetic energy equation , shaded area indicate ranges of values . all terms are divided by w_*^2/z_i

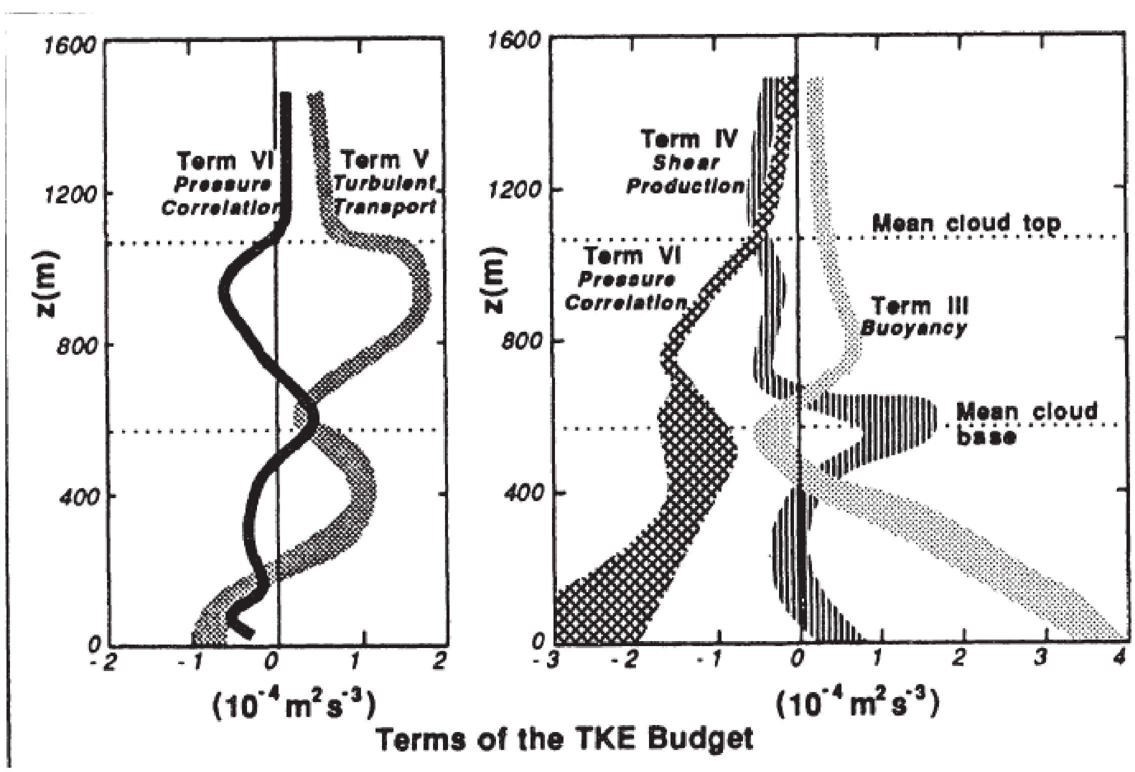


Figure (3.2) : range of terms in the turbulent kinetic energy budget for a cloud topped tropical boundary layer . transport term is split in to the pressure correlation (PC) and turbulent transport (T) parts .

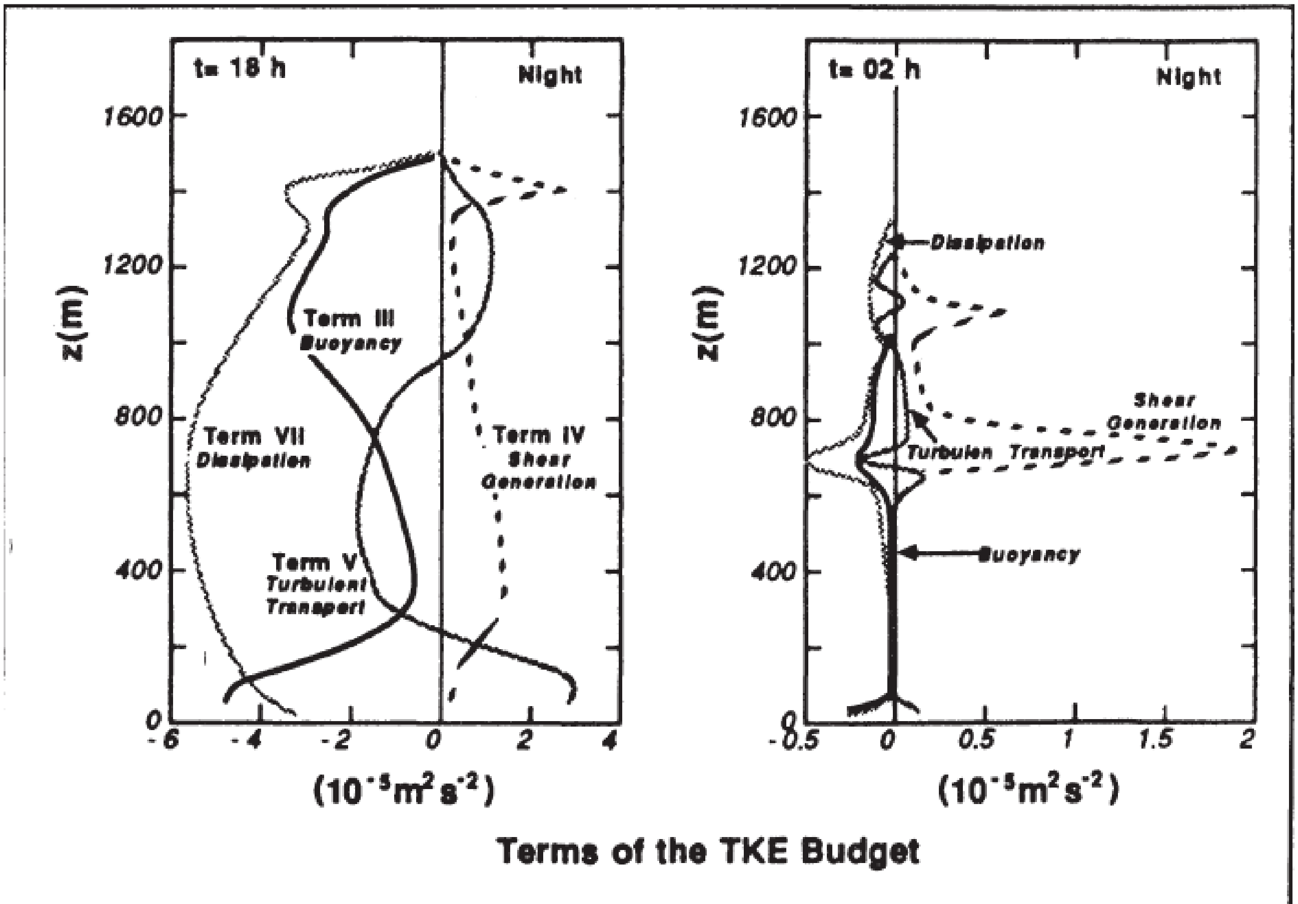


Figure (3.3) : modeled turbulent kinetic energy budget at t=18h , and t=02 h during night 33-34 .

3.2.2 Mechanical (shear) production :

When there is a turbulent momentum flux in the presence of a mean wind shear , the interaction between the two tends to generate more turbulence . even through a negative sign precedes term shear , the momentum flux is usually of opposite sign from the mean shear , resulting in production , not loss , of turbulence .

Fig. 3.1 shows case studies of the contribution of shear production to the TKE budget for convective situations. The greatest wind shear magnitude occurs at the surface . not surprisingly , the maximum shear production rate also occurs there . the wind speed frequently varies little with height in the ML above the surface layer , resulting in near zero shear and near zero shear production of turbulence . shear production is often associated with the surface layer because of its limited vertical extent .

A smaller maximum of shear production sometimes occurs at the top of the ML because of the wind shear across the entrainment zone . in that region , the subgeostrophic winds of the ML recover to their geostrophic values above the ML .

The relative contributions of the buoyancy and shear terms can be used to classify the nature of convection (see fig 3.4) **free convection** scaling is valid when the buoyancy term is much larger than the mechanical scaling is valid when the buoyancy term is much larger than the mechanical term , **forced convection** scaling is valid when the opposite is true .

Magnitudes of the shear production term in the surface layer are obviously greatest on a windy day , and are small on a calm day . in synoptic –scale cyclones the strong winds and overcast skies suggest that forced convection is applicable . on many days , turbulence is neither in state of free nor forced convection because both the shear and buoyancy terms are contributing to the production of turbulence .

At night over land , or anytime the ground is colder than the air , the shear term is often the only term that generate turbulence .

The greatest shears are associated with the change of u and v components of mean wind with height . except in thunderstorms, shear of w negligible in the BL .

Both the buoyant and shear production terms can generate anisotropic turbulence . the difference is that shear generation produces turbulence primarily in the horizontal directions , while buoyant generation produces it primarily in the vertical .

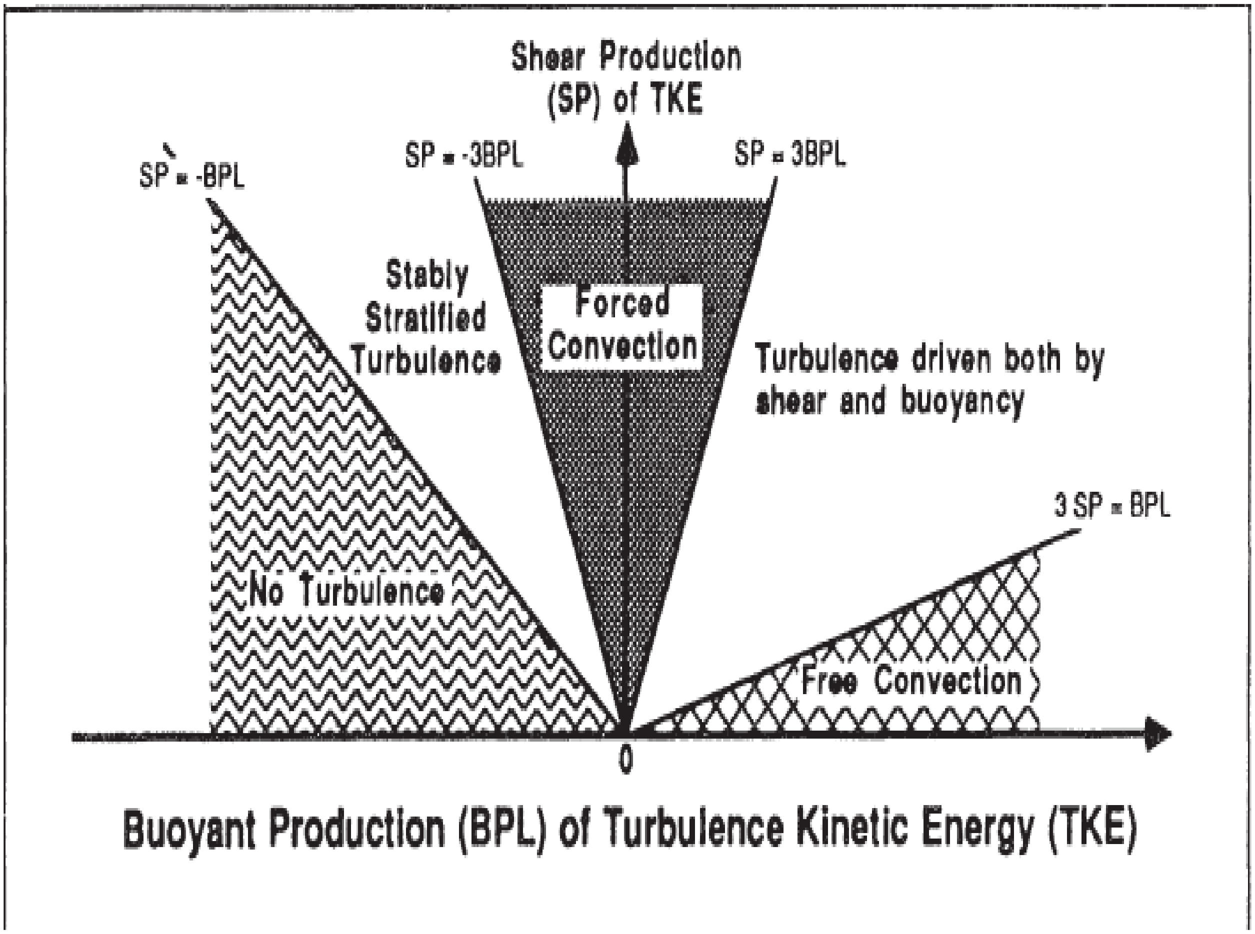


Figure (3.4) : regimes of free and forced convection

3.2.3 Turbulent Transport :

The quantity $\overline{w'e}$ represents the vertical turbulent flux of TKE . the change in flux with height is more important than the magnitude of flux . term transport v is a flux divergence term , if there is more flux in to a layer than leaves then the magnitude of TKE increases . term transport act as either production or loss , depending on whether there is a flux convergence or divergence when integrated over the depth of the ML , however , term transport becomes identically zero , assuming as bottom and top boundary conditions that the earth is not turbulent , and that there is negligible turbulence above the top of the ML . overall , this term is neither creates nor destroys TKE , it just moves or redistributes TKE from one location in the BL to another .

3.2.4 Pressure correlation :

Static pressure fluctuations are exceedingly difficult to measure in the atmosphere . the magnitudes of these fluctuations are very small , being on the order of 0.005kpa (0.05mb) in the convective surface layer to 0.001 kpa(0.01 mb) or less in the ML . pressure sensors with sufficient sensitivity to measure these static pressure fluctuations are contaminated by the large dynamic pressure fluctuations associated with turbulent and mean motions . as a result, correlations such as $\overline{w'p'}$ calculated from experimental data often contain more noise than signal .

3.2.5 Disspation :

Daytime dissipation rates are often largest near the surface , and then become relatively constant with height in mixed layer . above the ML top , the dissipation rate rapidly decreases to near zero . at night , both TKE and dissipation rate decrease very rapidly with height .

PROPLEMS:

1- at 1/3 from mixing layer height , practical atmospheric condition was : $\overline{u'w'}$= -0.03 m2/s , $\overline{w'\theta'}$ =0.1 k.m/s , du/dz= 0.03s^{-1} , θ_v = 293k (assume we neglected pressure correlation term vertical transport of turbulent) , find the dissipation term ε , where the rate of turbulent producted is 0.002746 m^2/s^3 at this height . (10 marks)

2- What is the flux , and how we can attain kinematic flux from flux ? use the units of heat flux ($\frac{j}{m^2\ s}$) , moisture ($\frac{kg_{wter}}{m^2\ s}$) , momentum ($\frac{kg\ ms^{-1}}{m^2\ s}$) , to drive the kinematics flux units for this variable

3- write the final mode of the turbulent kinentic energy bughet equation , and state each terms . is the turbulent kinetic energy conserved quantity and why ?

PROBLEM

Q1) At a height of z=300m in a 1000m thick mixed layer the following condition were Observed : $\frac{\partial u}{\partial z} = 0.01s^{-1}$, $\overline{\theta_v} = 25^0C$,$\overline{W'\theta'} = 0.15\ K\frac{m}{s}$, and $\overline{u'w'} = -0.03\ m^2s^{-2}$, also , the surface virtual heat is 0.24 K. m/s . if the pressure and turbulent transports are neglected then :

(a) What dissipation rate is required to maintain a locally steady state at z= 300m .

(b) What are the values of the normalized TKE terms ?

The answer :

a- Since no information was given about the v- component of velocity or stress , let 's assume that the x-axis has been chosen to be aligned with the mean wind . and we know that term local storage must be zero for steady state , and term disspation and turbulent transports are zero as specified in the statement of the problem . thus , the remaining terms can be manipulated to solve for ε .

$$\varepsilon = \frac{g}{\bar{\theta}}\ \overline{w'\theta_v'} - \ \overline{u'w'}\frac{\partial \bar{u}}{\partial z} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

Plugging in the values given above yields :

$$\varepsilon = \left\{\left(\frac{9.8\ m/s^2}{[(273.15+25]K}\right)\right\}.(\ 0.15\ k\ ms^{-1}\ ) - (-0.03\ m^2s^{-2}\ ).(\ 0.01\ s^{-1}\ )$$

$$\varepsilon = 4.93x\ 10^{-3} + 3x\ 10^{-3}\ \left(\frac{m^2}{s^3}\right)$$

$$\varepsilon = 5.23\ x\ 10^{-3}\ \ (m^2\ s^{-3})$$

b- To normalize the equations we first use $\frac{w_*}{z_i} = \frac{g}{\theta}\ (\overline{w'\theta'})$, which for our case equals 7.89x 10^{-3} (m^2s^{-3}) . dividing our terms by this value

$$0 = 0.625 + 0.038 - 0 - 0 - 0.663$$

Discussion :

This buoyant production term is about an order of magnitude larger than the mechanical production term , meaning that the turbulence is in a state of free convection . in regions of strong turbulent production , the transport term usually removes some of the TKE and deposits it where there is a net loss of TKE , such as in the entrainment zone . thus , we might expect that the local dissipation rate at z= 300m is smaller than the value calculated above .

Q2) why doesn't turbulent energy cascade from small to large eddies (or wavelengths) in the boundary layer ?

Q3) refer to the TKE equation . which terms , if any , represent the production of turbulence during a day when there are light winds and strong solar heating of the boundary layer ?

Q4) given the following TKE equation :

$$\frac{\partial e}{\partial t} + u_j \frac{\partial e}{\partial x_j} = -\ \overline{u'w'}\frac{\partial u}{\partial z} + \frac{g}{\theta}\ (\overline{w'\theta'}) - \frac{\partial(\overline{w'e})}{\partial z} - \frac{1}{\bar{\rho}}\ \frac{\partial(\overline{w'p'})}{\partial z} + \varepsilon \ \ldots\ldots\ldots\ldots$$

a. Which terms are always loss terms ?
b. Which terms neither create nor destroy TKE ?
c. Which terms can be either production or loss ?
d. Which terms are due to molecular effects ?
e. Which production terms are largest on a cloudy , windy day ?
f. Which production terms are largest on a calm sunny day over land ?
g. Which terms tend to make turbulence more homogeneous ?
h. Which terms tend to make turbulence less isotropic ?
i. Which terms describe the stationarity of the turbulence ?
j. Which terms describe the kinetic energy lost from the mean wind ?

Q5) fill in the table based on the regions A-H labeled on the attached diagram .

Property	Lapse rate	Heat flux	Static stabilit	Turbulent ?	name
Choices	Subadiab.	up	Stable	yes	Noct. inversion
	Adiabatic	zero	neutral	unknown	Cloud layer
	Superad.	down	unstable	no	Mixed layer

				sporadic	Entrainment zone
					Capping inversion
					Free atmosphere
Region					
A					Surface
B					
C	Subadiab.				
D					
E					Residual
F		zero			
G				unknown	
H			stable		

Q6) **a**- rewrite the conservation equation for mean kinetic energy in terms of the geostrophic wind ?

b-suppose that $\overline{u'w'}$ = -0.05 $m^2\ s^{-2}$ and $\frac{\partial u}{\partial z}\ 5\ s^{-1}\ and\ \bar{v} = 0$ within the surface layer . if there are no pressure gradients , then what is the value of the rate of change of mean kinetic energy , and what does it mean concerning the change in mean wind speed during a 1 minute period ?

Q7) on the planet krypton suppose that turbulent motions are affected by a strange from of viscosity that dissipates only the vertical motions . how would the tke be affected ?

Q8) define the following types of convection . under what weather conditions is each type of convection most likely ? what term in the equation is small under each condition ? a- free convection b- forced convection

Q9) given the term : uj= d (½ v2)/dxj , which represent the advection of total horizontal v component of kinetic energy . expand the variables uj and v in to mean and turbulent parts , Reynolds average , and simplify as possible .

Q10) given the following sounding in the moring boundary layer . determine whether each layer is stable or unstable (in both the static and dynamic sense) , and state if the flow is turbulent . indicate your results in the table to the right of the figures .

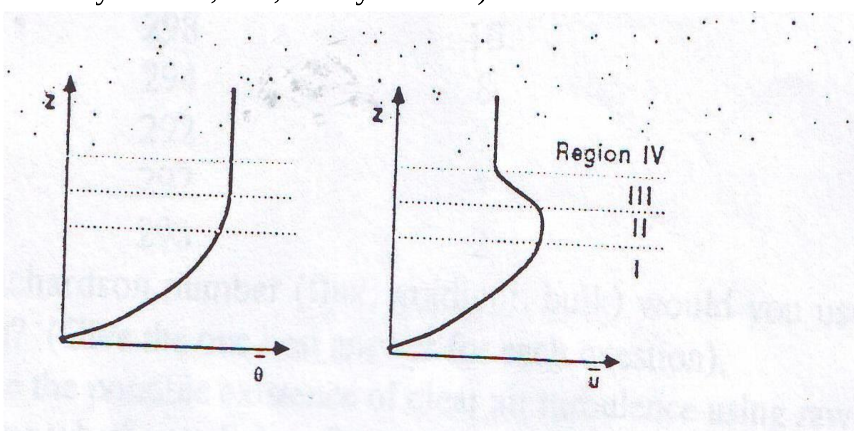
Q5) define the following types of convection . under what weather conditions is each type of convection most likely ? what term in the equation is small under each condition ? a- free convection b- forced convection

Q6) if the tke at 10m is at steady state , and if $\varepsilon = 0.01\, m2s - 3$, then is the transport term supplying or removing tke from the air at z= 10m ?

Q14) given the tke equation with terms labeled a to e belogw :

$$\underset{A}{\frac{\partial y}{\partial x}} = \underset{B}{-uw\frac{\partial y}{\partial x}} + \underset{C}{\frac{g}{\theta}\, w'\theta} - \underset{D}{\frac{\partial y}{\partial x}\, w'\left(\frac{p'}{\rho} + e\right)} - \underset{E}{\varepsilon} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots$$

And given 4 regions of the stable boundary layer , labeled A, B, C, D, E In figure below , determine the sign (+ , - or near zero) of each term in each region .(assume : that term A is always zero , i.e , steady state .)



Q9) given the following turbulence statistics

Where	Location a		Location b	
when (UTC) STATISTIC	1000	1100	1000	1100
U'3	0.5	0.5	0.7	0.5
V'3	0.25	0.5	0.25	0.25
W'3	0.7	0.5	0.7	0.25

Where and when is the turbulence

a- Stationary b- homogeneous b- isotropic