Numerical Prediction

This course deals with the numerical Weather and physical refinement incorporated in atmospheric models used for numerical weather prediction.

1. general aspects
2. The prediction problem in the P – system.
3. The equivalent parotropic model.
4. Finite – difference techniques.
5. Quasi – geostrophic baroclinic models.
6. The w – equation.
7. Non – geostrophic models.
8. Solution of the primitive equation.
9. General circulation models.

CHAPTER ONE: GENERAL ASPECTS

1.1 Introduction:

Numerical weather prediction is concerned with the prediction of meteorological parameters by the numerical station of the hydrodynamic equations which govern atmospheric motion. A snormous number of arithmetic and logical operations are required for this task. For this reason, electronic computers are needed to process the date within reasonable time limits.

In this chapter we briefly discuss the automatic date processing of synoptic meteorological data. The prediction problem is then presented and this is followed by historical survey of the development of number weather prediction.

1.2 Historical Survey

Numerical methods were first used in 1922 by L. F. Richardson in an attempt to predict the large – scale motion of the atmosphere from knowledge of its initial stats. In order to solve the hydrodynamic equations which govern the motion of the atmosphere, Richardson replaced partial derivatives by finite differences. In this way he obtained a system of algebraic equations.

Richardson found that the computations were very time-consuming with the facilities available to him. The results were also disappointing in that computed sea-level tendencies were much larger than those that were actually observed. At the time, he believed that this was due to insufficient and inaccurate data. Actually, two other factors were involved.

First, his prognostic equations were such that his computed pressure and wind speed increments were small differences between large numbers. They were therefore oversensitive to initial errors in measurement.

Secondly, a mathematical difficulty arose. His time steps were too large for the spatial increments he used to obtain his finite-differences solutions. Fast gravity waves could traverse the grid distance during the time interval he selected. This led to computational instability.

The term "meteorological noise" was first used by J. G. Charney in 1948 to describe high-speed waves which were meteorologically unimportant. He showed that suitable techniques could be used to "filter out" such waves.

Sound waves were excluded by using the hydrostatic equation – a method that had previously been used by Richardson. **Charney** then eliminated gravity waves by making use of the fact that large-scale motion was approximately geostrophic.

By employing a type of dimensional analysis, he was able to show that the filtering action could be achieved by making the geostrophic approximation. This assumption could not, however, be used with the primitive equations motion. Zero acceleration would have resulted from the balance of the pressure and Coriolis forces.

The difficulty was overcome by using the vorticitye equation and eliminating the divergence by means of the equation of continuity. In spit of the simplifications achieved with these quasi- geostrophic equation, further modeling approximations were found to be necessary.

This led to the development of the equivalent barotropic model by Charney in 1949. In this model the wind maintained a constant direction with height, but its speed could vary in the vertical. It is of ten referred as the barotropic model, to which it becomes equivalent when applied at the so-called level of non-divergence. Experience has indicated that the equivalent barotropic model is a reasonable representation of atmospheric conditions except in sporadic periods of baroclinic instability.

The next development brought forth advections models. The wind direction was permitted to vary with height, and so horizontal advection of potential temperature could occur. However, vertical transport of temperature was still neglected.

The next step was to remove the restriction of more horizontal incorporate vertical motion and these were capable of producing numerical progresses two levels. For this reason, they were known as two level or two-parameter models.

Three, - four and five parameter models then followed. However, these multi-parameter models did not show any marked improvement over the simpler two-parameter type.

By this time, it was realized that some of the simplifications used to determine the prognostic equations necessitated the omission of related terms for the sake o consistency.

The use of the geostrophic approximation was also questioned. It was found that this assumption was often being applied inconsistently. In some cases, it was also felt that its filtering action was too strong. This resulted in the replacement of the geostrophic approximation by the more general assumption of non-divergence implied by a balance equation. Stream function techniques were also introduced into these non-geostrophic models.

Further developments led to the introduction of previously neglected physical effects. These included topography, surface friction and non-adiabatic influences. Many of these refinements have been incorporated into models specifically designed to study the general circulation of the atmosphere.

At the same time, the development of larger and faster computer systems revived interest in the primitive equations. Today, multi-level models based on these equations are being used in many centers, both for meteorological research and for operational forecasting.

Meanwhile, investigations into deficiencies of the finite-difference technique are proceeding. This technique when applied to the primitive equations produces a number of difficulties, which include not only truncation errors but linear and non-linear instability. Spectral methods are being studied in an endeavor to avoid these problems.

Now that computing facilities are being steadily improved and extended more realistic atmospheric models are being developed. The new computer system is capable of processing an increasing amount of meteorological data for these more sophisticated models. It is therefore evident that further advances in the field of numerical weather prediction can be expected in the future.

1.3 The Prediction Problem

The basic equations which are relevant to motion in the atmosphere are as follows:

1) Newton's second law of motion.
2) The first law of thermodynamics (i.e. the thermodynamic energy equation).
3) The law conservation of mass (i.e. the continuity equation).
4) The equation of stats.
5) The conservation equation for the water substances.

For a large of atmospheric motions the atmosphere may be treated as a perfect gas. This assumption will be made during the current studies.
In the case of dry air it is possible to derive a completes system of five scalar equations and five unknowns:

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= -\vec{v}.\nabla u - \frac{1}{\rho}.\frac{\partial p}{\partial x} + fv - ew + \frac{1}{\rho}F_x \\
\frac{\partial v}{\partial t} &= -\vec{v}.\nabla v - \frac{1}{\rho}\frac{\partial p}{\partial y} - fu + \frac{1}{\rho}F_y \\
\frac{\partial w}{\partial t} &= -\vec{v}.\nabla w - \frac{1}{\rho}\frac{\partial p}{\partial z} + eu - g + \frac{1}{\rho}F_z \qquad \text{------ ----------- -----} \quad (1.1) \\
\frac{\partial \rho}{\partial t} &= -\vec{v}.\nabla \rho - \rho\nabla.\vec{v} \\
\frac{\partial p}{\partial t} &= -\vec{v}.\nabla p - \frac{c_p}{c_v p}\nabla.\vec{v} + \frac{R}{c_v}\rho Q
\end{aligned}
$$

These prognostic equations are of the first order with respect to times and can be used to solve general prediction problem in the z-system of coordinates. The five variables (u,v,w,ρ, and ps)
Although T does not appear it can always be derived for the equation of stats if p and ρ are known.
It is assumed that the components of the friction force (Fx, Fy, and Fz) and the diabetic heating Q (dh\dt) are either know functions or expressible in terms of the other variables. Hence, in principles, all future states can be determined by solution of this system.

When moisture is included, some modifications are necessary to the equation of stat and the thermodynamic energy equation. In addition, an equation is needed to express the conservation of the water substances. For the present, we shall however confine our discussion to dry air.

In our studies in dynamic meteorology we discussed the quasi-static prediction problem, which arises if we replace the third equation by the hydrostatic equation. It is then necessary to introduce Richardson's equation if we are to derive the vertical velocity (w).

If we w_R are the vertical velocity necessary to maintain hydrostatic equilibrium at all points and at all times, we have:

$$\frac{\partial w_R}{\partial z} = \frac{c_v}{c_p}\frac{1}{p}\left[-\vec{v}_H.\nabla_H p - \frac{c_v}{c_p}\nabla_H.\vec{v} + \frac{R}{c_v}\rho Q + g\int_z^{\infty}\nabla_H.\rho\vec{v}dz\right] \quad \text{-------- -------} \quad (1.2)$$

If this is integrated from the earth's surface to the height z, we obtain a diagnostic equation for determining w_R in the form:

$$w_R = w_{\mathrm{O}}(x, y, z_o) + \int_{z_o}^{z} A(x, y, z')dz' \quad \text{-------------- -------------------} \quad (1.3)$$

Where: $w_o w(x, y, w_o)$ is the vertical velocity at $z = z_o(x, y)$ which is the height of the topography.

z' = a dummy for integration.

On replacing the third equation of (1.1) by Richardson's equation, we obtain the quasi – static system of equations. These may be used to predict the future stats of the atmosphere. They may be written in the form:

$$\begin{aligned}
\frac{\partial u}{\partial t} &= -\vec{v}.\nabla u - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{1}{\rho}F_x \\
\frac{\partial v}{\partial t} &= -\vec{v}.\nabla v - \frac{1}{\rho}\frac{\partial p}{\partial y} - fu + \frac{1}{\rho}F_y \\
\frac{\partial p}{\partial t} &= -\vec{v}.\nabla p - \frac{c_p}{cv}p\nabla.\vec{v} + \frac{R}{c_v}\rho Q \quad \text{-------- --------- --------} \quad (1.4) \\
\rho &= -\frac{1}{g}\frac{\partial p}{\partial z} \\
w_R &= w_\circ + \int_{z_o}^{z} A(x, y, z')dz'
\end{aligned}$$

The equations (1.4) or any equivalent system with another vertical coordinates are known as the *Primitive equation*.

1.4 Numerical Models

The complexity of atmospheric processes has made it necessary for meteorologists to use model atmospheric. These models are simplified representations of the atmosphere devised to study atmospheric behavior and to produce numerical weather predictions. The simplifications allow attention to be focused on these atmospheric properties which are considered to be of greatest importance. By using physical models it is possible to represent these properties in the mathematical system of equations. For many the earth's atmosphere can be regarded as behaving as a stratified fluid. It can therefore be treated as consisting of a number of layers whose properties and interrelations can be expressed mathematically.

The simplest and most restrictive is the (equivalent) barotropic model. A single pressure surface (usually 500 mb) is considered. At this level it is assumed that the motion is equivalent to a hypothetical barotropic atmosphere in which the surfaces of pressure and density (or specific volume) coincide at all levels. This is a useful approximation in same type of atmospheric flow. More sophisticated baroclinic involves consideration of many isobaric surfaces. This enables vertical variations in winds and temperatures to be represented more realistically.

By considering the distribution of moisture, the effects of condensation and evaporation can be incorporated in numerical prediction models. Other physical processes, such as frication and radiation, can also be introduced to make the models more representatives of the true atmospheres.

The development of numerical weather predication has been closely related to advances in computer technology. As the spead and capacity of computers have increased, so it has been possible to incorporate more physical refinements into the atmospheric models.

1.5 Automatic Data Processing

In many countries synoptic chart analyses and prognoses are produced automatically by high – speed electronic computers. They are called analyses and numerical prognoses respectively.

With the air of auxiliary telecommunications equipment, the output of the teletype system has been made compatible with the input of the computer system. In addition, date from satellites, radars; automatic weather station, etc. can be introduced by mean of a manually interactive automatic control system.

Meteorological data of all types and origin can therefore by recorded almost instantaneously in the information storage (memory) of the computer. They are then processed automatically.

In the case of coded massages, a sequence of stored programmed instruction enables the machine to identify the type and origin of each report. It then decodes the massage and selects the data relevant to its ultimate purpose. These data are then converted to a standard form in a common system of units. Checks for consistency with other meteorological information are also made.

After the initial message recognition and data validation have been completed, the machine prints cut a list of observing stations whose reports are considered suspect. The probable nature of the error is able displayed.

If the analyst confirms that the machine is correct, he may decide either to refer a massage for further checking and modification or to process with the analysis. In the later case, the meteorological data are analyzed by an objective process of interpolation and smoothing. When the numerical analysis has been completed, it is printed out in a selected form, e.g. contours on a surface of constant pressure.

If the numerical analyses for the various isobaric surfaces appear to be consistent with the current synoptic situation, the machine is instructed to proceed with the numerical prognoses.

The numerical weather prediction methods are objective and based on physical theory. Each numerical prognosis involves the solving of a set of equations which express in mathematical form the physical laws governing the behavior of a mixture of gases, such as the atmosphere.

Owing to the complexity of these equations, a 24 hour prognosis may require many millions of elementary arithmetical operations, exclusive of purely logical operations. Nevertheless, these computations can often be completed in friction of an hour in the case, if certain modeling approximations are adopted.

1.6 The Filtering Problem

The general atmospheric equations permit solutions of many types' sound waves, internal and external gravity waves, inertia waves, Rossby waves, etc. However, experience and the understanding of the weather systems indicants these various wave types are of unequal importance in weather predictions.

Synoptic charts show that the motion systems which appear to carry the weather along are rather long waves. These have a wavelength of a few thousand kilometers and move rather slowly from at speeds of the order of (**10** m/sec.). The Rossby waves are of this type and move from west to east at a slow speed if their wavelength is sufficiently short.

Some wave types are not relevant to weather prediction. The sound and gravity waves too fast compared with the motion of the weather systems. The inertia waves in a pure from hardly exist in the atmosphere because they required a vanishing pressure force.

Hence, it is desirable to have a system to equations which describes the Rossby waves accurately. The pressure amplitude of Rossby waves is usually large (often exceeding **20** mb), while the characteristic amplitude of sound waves is small fraction of a millibar. Gravity waves have amplitudes ranging from about a millibar (atmospheric tides).

Considered as components of the atmospheric-pressure pattern, sound and gravity waves can be dispensed with altogether. The meteorological filtering problem is to design a set of equations which eliminate the sound and gravity waves, but possess solutions corresponding to Rossby waves.

The sound waves can be excluded by adopting the assumption of incompressibility. Sound waves can exist only in a medium which is compressible.

However, this assumption is too drastic, because compressibility is important for some of the weather phenomena we wish to consider.

The external gravity wave can be eliminated by adopting a modified lower boundary condition, i.e. $w = 0, at, ... p = p_{\circ}$. This would replace the more correct general condition. Changing the general condition at the lower boundary at w= 0 is therefore an example of a filter approximation. We exclude a physical phenomenon (external gravity waves) because they are of little importance for the phenomenon under investigation, i.e. the prediction of weather systems.

These examples show that the filter problem is a difficult one. It requires considerable meteorological insight to design a set of equations which remove all unimportant phenomena. This has to be done in such a way that the essential features of the motion of atmospheric weather systems are only slight modified.

It is not possible to know if a unique solution to such a problem exists. However, a systematic attack on the problem has been made. This involves the use the so-called scale analysis.

1.7 Quasi-geostrophic Equations

Using a two-level atmospheric model the linearized perturbation equations can be studied. It can be shown that external gravity waves can be eliminated by using the boundary condition $w = 0, at, ... p = p_{\circ}$. The internal gravity waves and inertia waves can then be filtered out of the systems b applying the geostrophic approximation in a selective way.

The geostrophic wind approximation is used to compute the horizontal wind and the vorticitye, but not the velocity divergence. This technique filters out the internal gravity-inertia waves from the linearized perturbation equations.

Numerical experiments show that the geostrophic wind approximation is also effective in eliminating internal gravity-inertia waves from the non-linear equations. When applied in the selective way the resulting equations are the **quasi-geostrophic equations**.

Including only the major terms these equations are:

$$\frac{\partial \rho_g}{\partial t} + \vec{v}_g \cdot \nabla(\rho_g + f) = f \frac{\partial w}{\partial p}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial p}\right) + \vec{v}_g . \nabla\left(\frac{\partial \phi}{\partial p}\right) + \sigma w = 0 \qquad \text{---------- ------------ (1.5)}$$

Where

And

$$\vec{v}_g = \frac{1}{\rho}\vec{k} \times \nabla \phi$$

$$\zeta_g = \frac{1}{f}\nabla^2 \phi \qquad \text{---------------- ---------- ---------------- (1.6)}$$

It will be seen that (1.5) has only two dependent variables (φ and ω). It is therefore a closed system, and may be characterized as the simplest baroclinic prediction system.

Undesirable wave types been eliminated. It is conceivable that the system is too simplified or too restrictive. Nevertheless, the quasi-geostrophic equations have been used extensively in analytical and numerical studies of the large-scale flow in the atmosphere.

1.8 Numerical Advantages

The quasi-geostrophic equations are simpler than the unmodified equations. In addition, they possess additional advantages in connection with numerical integrations. This was one of the major reasons for their use in early numerical weather prediction experiments.

The numerical advantages arise from the fact that there is usually an upper limit to the time step. This occurs in a numerical integration when a differential equation is replaced by a finite difference equation. The upper limit it the time step is given by the formula

$$\Delta t \leq \frac{\Delta x}{c} \qquad \text{(1.7)}$$

Where

Δx = grid size.
C = maximum speed of propagation within the system.

This formula is known as the Friedrich – Courant – Levy criterion. According to (1.7) we can find the largest possible time step when we know the smallest grid size and the largest possible value of c.

To illustrate the application of (1.7), let us first assume that we where to integrate the original equations without any simplifications. The smallest grid size would then be in the vertical direction. If we use z =2 km (as original proposed by Richardson) and the largest speed of propagation ($\approx 330ms^{-1}$), then

$$\Delta t \leq 6\sec. \qquad \text{(1.8)}$$

A time step s small as this would be prohibitive in large-scale predictions.

Now assume that the hydrostatic assumption has been introduced into the equations. This filters out the vertical component of the sound waves. Of course, sound waves could still propagate in the horizontal direction. The main effect would be that Δx would be interpreted as the horizontal grid size. Hence:

$$\Delta t \leq 1155\sec. \qquad \text{(1.9)}$$

When use x=381 km and c=330 m/sec.

The effect of introducing the hydrostatic equation is therefore to increase the time step to about 20 minutes.

If we finally use the quasi-geostrophic equations, all fast-moving waves will be filtered out. The maximum speed of propagation is then of the order of magnitude of the wind speed. Assuming c ≤ 100 m/sec., we find:

$$\Delta t \leq 3810s \approx 1 hour$$

This shows that considerable advantage has bee obtained by filtering out the fast-moving waves.

The computational stability of solutions depends exclusively on the properties of the finite-difference equations that are actually solved. It dose not matter that the fast-moving waves may not be present in physical actuality. The equations do not distinguish between physically real fluctuations and random errors in the initial values. If Δt is taken greater than the time needed for the fastest wave to cross the grid interval, computational instability will arise. Both real and spurious disturbances will amplify at the expense of the meteorologically important waves.

The fastest of modern computing machines is certainly capable of solving the original hydrodynamical equations in their original unmodified form. However, practical and economic factors need to be taken into consideration. One such factor is the horizontal extent of the area for which a prediction is required or which is necessary for an accurate prognosis. In addition, the vertical and horizontal resolution, the speed of the computer and ultimately the computation time for the prediction need consideration.

Other things being equal, the modified (i.e. filtered) equations can be solved several times faster than the general equations. This is because the maximum permissible time step is several times larger.