# CAPTER TWO: PRDICTION PROBLEM P.–SYSTEM

## *2.1 Introduction*

In the preceding chapter we should that the vertical velocity could be determined by the solution of Richardson's equation. This vertical velocity ( $w_R$ ) must exist in the atmosphere in order to maintain hydrostatic equilibrium everywhere.

You will recall that the P- system of coordinates is also based on the assumption of hydrostatic equilibrium. The transformation of the equations through the change of the vertical coordinates does not change the physical content of the equations.

In this chapter we shall formulate the prediction problem in the P-system of coordinates.

## *2.2 The Basic Equations in the P- System*

The two equations of motion for horizontal flow, the hydrostatic equation, the continuity equation and the thermodynamic equation in the P – system are as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial p} = -\frac{\partial \phi}{\partial x} + fv + F_x \quad \text{------------------ --------------------} \quad (2.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial p} = -\frac{\partial \phi}{\partial y} + fu + F_y \quad \text{----------------- -------------------} \quad (2.2)$$

$$\frac{\partial \phi}{\partial p} = -\frac{R}{p}\left(\frac{p}{p_\circ}\right)^{k}\theta \quad \text{---------------------- ------------------ -------------------} \quad (2.3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial p} \quad \text{----------------------- --------------------- -----------------} \quad (2.4)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial p} = \frac{1}{c_p}\left(\frac{p}{p_\circ}\right)^{-k} Q \quad \text{------------------- -----------------} \quad (2.5)$$

Note that: (1) $k = \frac{R}{c_p}$

(2) From the definition of potential temperature: $T = \theta\left(\frac{p}{p_\circ}\right)^{k}$ ------------ (2.6)

(3) The equation of stat leads to $\alpha = \frac{RT}{p}$ ---------------------- -------------- (2.7)

(4) Hence the hydrostatic equation in the P-system $\frac{\partial \phi}{\partial p} = -\alpha$ ------- ----- (2.8)

can be written from (2,3)

(5) The thermodynamic equation in the form $\frac{d(\ln\theta)}{dt} = \frac{1}{c_p T}Q$ ---------- (2.9)

Has been expanded to the form (2.5).

The five equations (2.1), (2.2), (2.3), (2.4) and (2.5) have the five variables u, v, w, φ, and θ. Three of these equations contain time derivatives and are therefore prognostic equations.

The remaining two equations are (2.3), and (2.4). these can used to obtain φ and w from the already computed values of u, v, and θ at the new time.

It will notice that both φ and w must be obtained from an integration of (p), i.e. in the vertical direction. Therefore it is necessary to specify boundary conditions.

## *2.3 Boundary Conditions*

The boundary condition on w at the outer limit of the atmosphere is:

$$\begin{matrix} w = 0 \\ p = 0 \end{matrix} \quad \text{--------------- ------------------ ----------------------} \quad (2.10)$$

This arises from the fact that the pressure (p) approaches zero at the limit.

Hence the pressure change $w = \frac{dp}{dt}$ for a parcel of air also becomes zero.

The boundary condition at the earth's surface was discussed in section (1.6). The normal component of the velocity vector must be zero. Hence:

$$w_s = \left(\frac{dp}{dt}\right)_s = \left(\frac{\partial p}{\partial t}\right)_s + \vec{V}_s . \nabla p_s \quad \text{-------- ---------------- ---} \quad (2.11)$$

Where:

s denoted the surface of the earth.

Vs wind comnonent along the earth's surface.

Ps = ps (x, y, t) is the pressure at the earth's surface.

We may also express the boundary condition at the earth's surface in two other forms;

1. the equation (2.11) may be re-written:

$$\frac{\partial p_s}{\partial t} \equiv \left(\frac{\partial p}{\partial t}\right)_s$$ ----------- is the local pressure tendency at the earth's surface.

Using the equation of continuity (2.4)

$$w_s = -\int_0^{p_s} \nabla_p . \vec{V} dp \quad \text{---------------------- ---------------------} \quad (2.13)$$

We may write (2.12) in the form

$$\frac{\partial p_s}{\partial t} = -\int_0^{p_s} \nabla_p \vec{V} dp - \vec{V}_{SH} . \nabla p_s \quad \text{--------- ----------------------} \quad (2.14)$$

Where: $\vec{V}_{SH}$ = horizontal component of Vs

$\vec{V}_{SH} . \nabla p_s = \vec{V}_s . \nabla p_s$ since $p_s$ is a function of x, y, and t only.

2. Another form of the lower boundary is obtaining by considering the isobaric surface through the point at earth's surface.

   The transformation equation from the z-system to the p-system is given by:

$$\frac{\partial p}{\partial t} = \rho g \left(\frac{\partial z}{\partial t}\right)_p$$

$$\nabla_H p = \rho g \nabla_p z$$

Then lead to:

$$\frac{\partial p_s}{\partial t} = \rho_s g \left( \frac{\partial z}{\partial t} \right)_{sp}$$

$$\nabla p_s = \rho_s g \nabla_{sp} z$$

$$i.e. \partial \phi = g \partial z, \ldots\ldots, \quad \partial p_s = \rho \partial \phi$$

$$\left( \frac{\partial \phi}{\partial t} \right)_{ps} = \frac{1}{\rho_s} \frac{\partial p_s}{\partial t}$$

$$\nabla_{sp} \phi = \frac{1}{\rho_s} \nabla p_s$$

Hence equation (2.14) becomes:

$$\left( \frac{\partial \phi}{\partial t} \right)_{sp} = -\alpha_s \int_0^{p_s} \nabla_p . \vec{V} dp - \vec{V}_{SH} . \nabla_{sp} \phi \quad \text{---------------- ---------------} \quad (2.15)$$

Note:

1. In using equation (2.15) it is usually assumed that $p_s \approx p_\circ \approx 1000 \quad mb$
2. In case $\phi_{sp}$ becomes the geopotential of 1000 mb isobaric surface $(\phi_\circ)$ .

$$\therefore \frac{\partial \phi_\circ}{\partial t} = -\alpha_s \int_0^{p_s} \nabla_p . \vec{V} dp - V_g . \nabla \phi_\circ \quad \text{---------- ------------------------} \quad (2.16)$$

3. Equation (2.16) can be used to calculate the new values of the geopotential at the lower (isobaric) boundary.

## *2.4 The Prediction Problems*

The equation (2.1) to (2.5) combined with (2.16) form the basis for the prediction system using pressure as the vertical coordinate.

Equation (2.1), (2.2), (2.5), and (2.16) are used to compute new values of u, v, θ, and φ by a system of finite differences of these equations.

The method may be summarized as follows:

1. The atmosphere is divided into elementary volumes by a three- dimensional grid.
2. All partial derivatives in space and time are replaced by finite differences of the values of the varnishes in the grid points.
3. These all terms, excess the time derivative in (2.1), (2.2), (2.5), and (2.16) can calculate from values of u, v, w, θ, and φ at a given time.
4. Hence the time derivative in the prognostic equations can be determined.
5. Using the time derivatives we may obtain values of u, v, θ, and φ called the new values.

## *To obtain a new of φ*

1. integrate equation (2.3)

$$\phi(p) = \phi_{\circ} + \int_{p}^{p_{\circ}} \frac{R}{p}\left(\frac{p}{p_{\circ}}\right)^{k} \theta dp \quad \text{------------- -----------------} \quad (2.17)$$

2. Substitute the new values of θ, and φ.
3. Replace the integrate term in (2.17) by a finite sum.

   Since the isobaric surface remain the same xxxx the time integration, the pressure dependent factor in the integrand does not change.

## *To obtain a new of w*

Integrate equation (2.4).

$$w(p) = -\int_{\circ}^{p} \nabla_{p} . \vec{V} dp \quad \text{------------------ ----------------------} \quad (2.18)$$

Hence we have a new value of the five variables u, v, w, θ, and φ at the new time instant.

## 2.5 *Special Aspects of the Prediction System*

The formulation of the lower boundary condition (2.16) in the form of a prognostic equation for φ is very important. It provides the value of the integration constant of equation (2.17) which was derived from the hydrostatic equation.

The upper boundary condition w=0, p=0 similarly provides the integration constant in the integration (2.18) of the continuity equation. The prediction system for P-system is equivalent to that developed by Richardson as described in chapter 1. Equation (2.18) is equivalent to Richardson's equation for the vertical velocity. However, it is much similar than the equation (1.25) for ( $w_R$ ).

The information mace spry in start the integrations are the fields of u, v, θ, and φ. These retirements are well suited to the present observation system. The horizontal wind vector can be determined at the variant pressure levels. In addition, the radiosonde provides temperature vxxxxxx function of pressure.

Thus it is easy to compute:

1. φ

$$\theta = T\left(\frac{p}{p_{\circ}}\right)^{\frac{-R}{c_p}} \quad \text{--------------------- -------------------------} \quad (2.19)$$

2. $\phi_{\circ}$

$$\phi_{\circ} = \phi_{s} + RT_{m} \ln\left(\frac{p_{s}}{p_{\circ}}\right) \quad \text{------------- ------------------------} \quad (2.20)$$

Where $\phi_s$ geopotential at the earth's surface.

## *2.6 The sigma system of coordinates*

There is a disadvantage in using height (z) or pressure (p) as the vertical coordinate. The lower boundary of the atmosphere is determined by the continental elevations and the ocean surface. This differs from the coordinate surface in the z-system and the p-system.

The horizontal surface in the z-system and isobaric surface in the p-system intersects the lower boundary surface in the region of the high elevations. This difference was evident in changing from equation (2.14) to equation (2.18).

Another difficulty arises when the isobaric surface is partly in the atmosphere and partly *under ground* in the vicinity of a grid point.

As result of these difficulties special care has to be taken in evaluating the pressure force, for example. Coordinate systems have now been designed to have the lower boundary as a coordinate surface. The vertical coordinate in such a system is the actual pressure divided by the surface pressure, i.e.

$$\sigma = \frac{p}{p_s} \qquad \text{---------------------- -------------------------} \quad (2.21)$$

Thus **σ=0** at the outer limit of the atmosphere and **σ =1** at the earth's surface. This is known as the σ-system. It is used in practical numerical weather predictions.