Lecture 4

Balanced Motion Part 2

4.1 The Gradient Wind

The gradient wind is defined as the wind existing if the trajectory of a particle (or air parcel) is circular and we have a balance among the pressure gradient force, the Coriolis force and the centrifugal force.

A. Cyclonic flow (low pressure)

In this case, a Coriolis force and the centrifugal force act in the same direction. In order to have a balance, the pressure gradient force must act in the opposite direction and we have a low pressure in the center (see case a in Figure 4.1).

If we take the effect of curvature into account, we have to expand the horizontal momentum equation to include the centrifugal term:

$$PGF = CF + CeF \qquad (4.1)$$

and by using geostrophic balance $f\,V_g = -\frac{1}{\rho}\frac{\partial p}{\partial n}$, equation 4.1 becomes:

$$\mathrm{f\,V_g = f\,V_G + \frac{V_G^2}{R}} \qquad (4.2)$$

Here R is the radius of curvature. V_G is the Gradient wind.

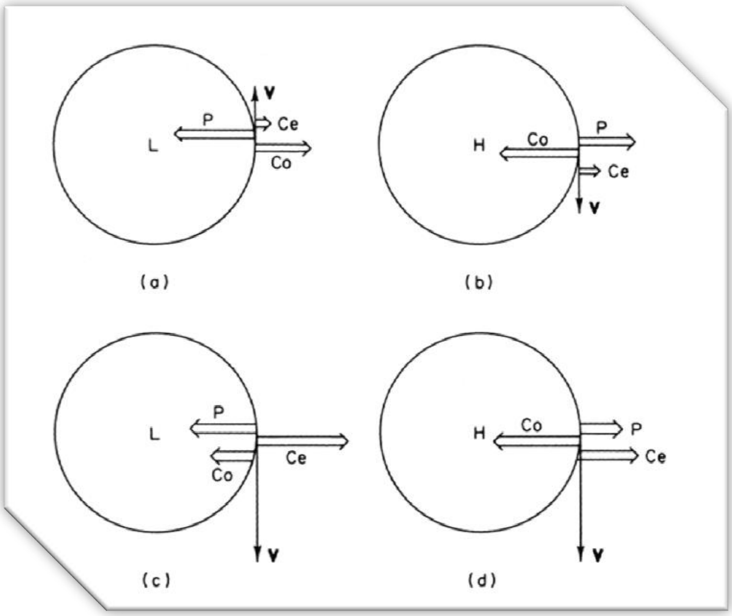


Fig. 4.1

Four balances in the Northern Hemisphere for the four types of gradient flow.

P is the pressure gradient force (PGF)

Co is Coriolis force (CF)

Ce is the centrifugal force

H is short of High, L short of Low

The gradient wind speed is obtained by solving equation (4.2) for V_G to yield:

$$f\,V_g = f\,V_G + \frac{V_G^2}{R}$$

Dividing by V_G^2,

$$f\,\frac{V_g}{V_G^2} = \frac{f}{V_G} + \frac{1}{R}$$

$$fV_g\left(\frac{1}{V_G}\right)^2 - f\left(\frac{1}{V_G}\right) - \frac{1}{R} = 0$$

By using quadratic formula to solve,

$$x = \frac{b \mp \sqrt{b^2 + 4\,a\,c}}{2\,a}$$

we get,

$$a = f\,V_g \qquad b = f \qquad c = \frac{1}{R} \qquad x = \frac{1}{V_G}$$

$$\frac{1}{V_G} = \frac{f \mp \sqrt{f^2 + 4\,\frac{f\,V_g}{R}}}{2\,f\,V_g}$$

Dividing the numerator and the denominator of the right side on $(2\,f)$ we get,

$$\frac{1}{V_G} = \frac{\frac{1}{2} \mp \sqrt{\frac{1}{4} + \frac{V_g}{R\,f}}}{V_g}$$

$$\therefore \qquad V_G = \frac{V_g}{\frac{1}{2} \mp \sqrt{\frac{1}{4} + \frac{V_g}{R\,f}}}$$

This equation tells us that $V_G < V_g$ in all cases because the denominator is larger than one. The difference between V_G & V_g becomes larger at smaller R, and at smaller f . To illustrate this difference we consider:

At $V_g = 10\,\frac{m}{s}$ $\quad and \quad$ $f = 10^{-4}\,s^{-1}$

$if \quad R = 1000\ km$, we find $V_G = 9.16\ m/s$ and the difference between V_G & V_g is small.

When R becomes much smaller the difference between V_G & V_g will be large.
If we assume that $f = 10^{-4}\ s^{-1}\ and\ \ V_g = 10\frac{m}{s}$ we may calculate the value of R necessary to make $V_G = \frac{1}{2}V_g$, we find from the equation that the radius of $R = 200\ km$

B. Anticyclonic flow (high pressure)

In this case, a pressure gradient force and the centrifugal force are in the same direction. In order to have a balance the Coriolis force must act in the opposite direction, we have a high pressure in the center (case b and d in Fig. 4.1).

$$PGF + Ce\,F - CF = 0$$

$$f\,V_g + \frac{V_G^2}{R} - f\,V_G = 0$$

In the same previous manner,

$$\therefore \qquad V_G = \frac{V_g}{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{V_g}{R\,f}}}$$

We see that $V_G > V_g$ in all cases.
In the special case where $\frac{V_g}{R\,f} = \frac{1}{4}$, $V_G = 2V_g$, the maximum wind in the anticyclonic case is therefore twice the geostrophic wind. If we assume that, $f = 10^{-4}\ s^{-1}$ and $V_g = 10\ m/s$, the radius of curvature is equal to 400 km, and thus quite small.

4.2 The Cyclostrophic Flow

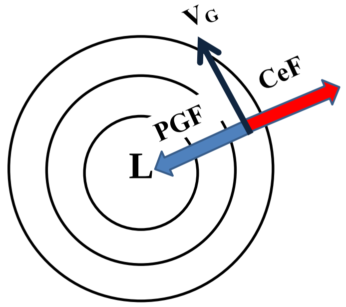
Cyclostrophic balance occurs when the pressure gradient force and centrifugal force are equal and in opposite direction. This is the situation near the equator

$$GF = Ce\ F$$

$$f\ V_g = \frac{V_G^2}{R}$$

$$V_G^2 = f\ V_g R$$

$$\therefore \quad V_G = \sqrt{f\ V_g R}$$



4.3 The Inertial Flow

In inertial flow, there is no pressure gradient force, there are two forces only, Coriolis and centrifugal that may balance each other.

$$CF = Ce\ F$$

$$f\ V_G = \frac{V_G^2}{R}$$

$$V_G = Rf$$