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PhD-Lecture (III):EMW Interaction with Matter

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Harmonic Electromagnetic Fields

1. The wave equation

a) For source free regions ($q = 0$ and $\mu = 0$) the wave equation given by:

$$\nabla^2 E = j\omega\mu\sigma E - \omega^2\mu\epsilon E = \gamma^2 E \qquad (1)$$

$$\nabla^2 H = j\omega\mu\sigma H - \omega^2\mu\epsilon H = \gamma^2 H \qquad (2)$$

Where

$$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon = j\omega\mu\,(\sigma + j\omega\varepsilon)$$

$\gamma = \alpha + j\beta$ = propagation constant
α = attenuation constant (Np/m)
β = phase constant (rad/m)

b) For source free and lossless media ($\sigma = 0$) the wave equation given by:

$$\nabla^2 E = -\omega^2\mu\varepsilon E = -\beta^2 E \qquad (3)$$

$$\nabla^2 H = -\omega^2\mu\varepsilon H = -\beta^2 H \qquad (4)$$

Where $\beta^2 = \omega^2\mu\varepsilon$, β is the phase constant is also represented by(K).

2. Uniform Plane Waves in an Unbounded Lossless Medium

a) Electric and Magnetic Fields

Let us assume that a time-harmonic uniform plane wave is traveling in an unbounded lossless medium (ε, μ) in the z direction (either positive or negative), as shown in figure (1). In addition, for simplicity, let us assume the electric field of the wave has only an x component. We want to write expressions for the electric and magnetic fields associated with this wave.

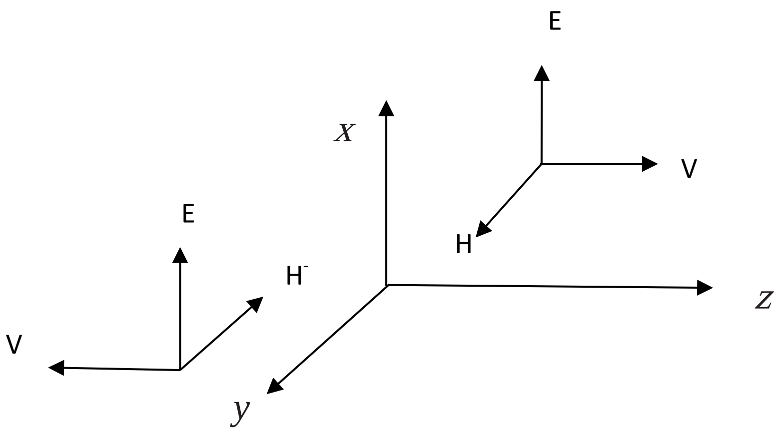

Figure (1): A plane wave traveling in the z direction

$$E^{+} = E_{\circ}^{+} \cos(wt - \beta z) \qquad (5)$$

$$H_{y}^{+} = \frac{E_{\circ}^{+}}{\eta} \cos(wt - \beta z) \qquad (6)$$

At t = 0

$$E^{+} = E_{\circ}^{+} \cos(-\beta z) \qquad (7)$$

$$H^{+} = \frac{E_{\circ}^{+}}{\eta} \cos(-\beta z) \qquad (8)$$

3. Wave Impedance

Since each term for the magnetic field (A/m) in eq. (1) and (2) is individually identical to the corresponding term for the electric field (V/m), the factor $\sqrt{\frac{\mu}{\epsilon}}$ in the denominator in (1) and (2) must have units of ohms (V/A). Therefore the factor $\sqrt{\frac{\mu}{\epsilon}}$ is known as the *wave impedance*, Z_w, denoted by the ratio of the electric to magnetic field, and it is usually represented by η.

$$H_{y}^{+} = \frac{1}{\sqrt{\frac{\mu}{\varepsilon}}} E_{x}^{+} \qquad (9)$$

$$H_y^- = -\frac{1}{\sqrt{\frac{\mu}{\varepsilon}}} E_x^- \quad (10)$$

$$z_w = \frac{E}{H} = \frac{E^+}{H^+} = \frac{E^-}{H^-} = \sqrt{\frac{\mu}{\epsilon}} = \eta \quad , \eta_\circ = \sqrt{\frac{\mu_\circ}{\epsilon_\circ}} = 377 \text{ ohm}$$

problem1: The electric field of a uniform plane wave traveling in free space is given by $E = \hat{a}_y(E_\circ^+ e^{-j\beta z} + E_\circ^- e^{j\beta z})$

$$E = \hat{a}_y(E_y^+ + E_y^-)$$

Where $E_\circ^+$ and $E_\circ^-$ are constants. Find the corresponding magnetic field.

Solution*:* For the electric field component that is traveling in the +z direction, the corresponding magnetic field component is given by

$$H^+ = -a_x \frac{E_\circ^+}{\eta_\circ} e^{-j\beta z}$$

And

$$H^- = a_x \frac{E_\circ^-}{\eta_\circ} e^{j\beta z}$$

$$H = H^+ + H^- = \hat{a}_x \frac{1}{\eta_\circ} = (-E_\circ^+ e^{-j\beta z} + E_\circ^- e^{j\beta z})$$

4. Phase and Energy Group Velocities for real part

$$E_x(z,t) = E_x^+ + E_x^- = R_e(E_\circ^+ e^{J(\omega t - \beta z)} + E_\circ^- e^{j(\omega t - \beta z)}) \quad (11)$$

$$E_x(z,t) = E_\circ^+ cos(\omega t - \beta z) + E_\circ^- \cos(\omega t + \beta z)$$

$$H_x(z,t) = \sqrt{\frac{\epsilon}{\mu}} (E_\circ^+ cos(\omega t - \beta z) - E_\circ^- \cos(\omega t + \beta z)) \quad (12)$$

Phase velocity given by:

$$V_P^+ = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} \quad (13)$$

The power associated with positive traveling waves:

$$P_e^+ = \frac{1}{2}\epsilon {E_x^+}^2 = \frac{1}{2}\epsilon {E_\circ^+}^2 cos^2(\omega t - \beta z) \quad (14)$$

$$P_m^+ = \frac{1}{2}\mu {H_y^+}^2 = \frac{1}{2}\mu(\frac{\epsilon}{\mu} {E_\circ^+}^2 cos^2(\omega t - \beta z)) \quad (15)$$

$$P^+ = P_e^+ + P_m^+ = \epsilon {E_\circ^+}^2 cos^2(\omega t - \beta z)$$

Where $P_e^+$and P_m^+ represented the power of electric and the power of magnetic field respectively.

Poynting vector:

$$S^+ = E^+ \times H^+ = \hat{a}_x E_\circ^+ \cos(\omega t - \beta z) \times \hat{a}_y \sqrt{\frac{\epsilon}{\mu}} E_\circ^+ \cos(\omega t - \beta z)$$

$$= \hat{a}_z S^+ = \hat{a}_z \sqrt{\frac{\epsilon}{\mu}} {E_\circ^+}^2 cos^2(\omega t - \beta z) \quad (16)$$

Energy (group) velocity v_e^+:

$$v_e^+ = \frac{S^+}{P^+} = \frac{S^+}{P_e^+ + P_m^+} = \frac{\sqrt{\frac{\epsilon}{\mu}} {E_\circ^+}^2 \, COS^2(\omega t - \beta z)}{\epsilon {E_\circ^+}^2 COS^2(\omega t - \beta z)}$$

$$v_e^+ = \frac{1}{\sqrt{\mu \varepsilon}} \text{ Velocity of the wave energy} \quad (17)$$

The speed of light

$$(v^+)^2 = v_p^+ v_e^+ = \frac{1}{\mu \epsilon} \text{ the same holds for negative traveling waves}$$

5. Standing waves

When waves of the same frequency and amplitude traveling in opposite directions meet, a standing wave is produced. A standing wave is a wave in which certain points (nodes) appear to be standing still and other points (anti-nodes) vibrate with maximum amplitude above and below the axis.

Looking at the standing wave produced on the right, we can see a total of five nodes in the wave, and four anti-nodes. For any standing wave pattern, you will always have one more node than anti-node. Standing waves can be observed in a variety of patterns and configurations and are responsible for the functioning of most musical instruments. Guitar strings.

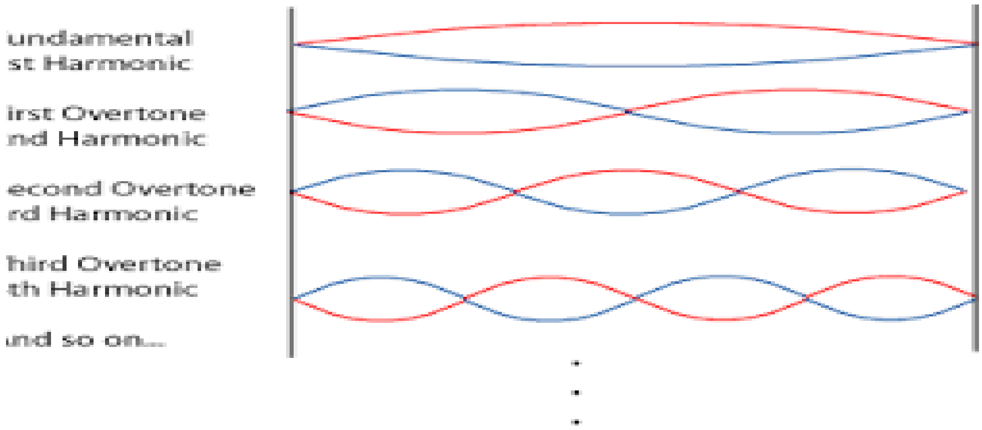

Figure to show standing waves

$$E_x(z) = E_\circ^+ e^{-j\beta z} + E_\circ^- e^{j\beta z} \quad (18)$$

$$= E_\circ^+(cos\beta z - jsin\beta z) + E_\circ^-(cos\beta z + jsin\beta z)$$

$$= (E_\circ^+ + E_\circ^-)cos\,\beta z - j(E_\circ^+ - E_\circ^-)sin\beta z$$

$$E_x(z) = \sqrt{{E_\circ^+}^2 + {E_\circ^-}^2 + 2E_\circ^+ E_\circ^- \cos(\beta z)}\; e^{-jtan^{-1}(\frac{E_\circ^+ + E_\circ^-}{E_\circ^+ + E_\circ^-}\tan\beta z)} \quad (19)$$

$|E_X(z)|_{max} = |E_\circ^+| + |E_\circ^-|$ when $\beta z = m\pi, m = 0,1,2,3, \dots\dots\dots$ bright point

$|E_X(z)|_{min} = |E_\circ^+| - |E_\circ^-|$ when $\beta z = (\frac{2m+1}{2})\pi, m = 0,1,2,3, \dots\dots\dots$dark point

The standing wave ratio (SWR) :

$$SWR = \frac{|E_X(z)|_{max}}{|E_X(z)|_{min}} = \frac{|E_\circ^+| + |E_\circ^-|}{|E_\circ^+| - |E_\circ^-|} \quad (20)$$

$$= \frac{1+\frac{|E_\circ^-|}{|E_\circ^+|}}{1+\frac{|E_\circ^-|}{|E_\circ^+|}} = \frac{1+|\ \Gamma\,|}{1-|\,\Gamma\ |} \quad (21)$$

where Γ is the reflection coefficient.

$|\text{SWR}|_{\min} = 1$ occurs when | Γ | = 0 no interference is found.

When $E_\circ^{+} = E_\circ^{-}$:

$|E_X(z)|_{E_\circ^{+} + E_\circ^{-}} = 2E_\circ^{+}|cos\beta z|$

$= 2E_\circ^{-}|cos\beta z|$

6. Skin depth

Wave impedance (intrinsic impedance Z_w):

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta_c = Z = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad \text{for lossy medium} \qquad (22)$$

The distance the EMW must travel in lossy medium to reduce its value to (e^{-1} = 0.368 =36%) is defined as the skin depth.

$$\delta = \frac{1}{\alpha} \qquad (23)$$

$$\alpha = \omega\sqrt{\mu\epsilon}\sqrt{\frac{1}{2}[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1]}$$

$$\delta = \frac{1}{\omega\sqrt{\mu\epsilon}\sqrt{\frac{1}{2}[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1]}} \qquad (24)$$

In summary, the attenuation constant α, phase constant β, wave Z_w and intrinsic η_c impedances, wavelength λ, velocity *v*, and skin depth δ for a uniform plane wave traveling in a lossy medium are listed in the second column of Table below. The same expressions are valid for plane and TEM waves. Simpler expressions for each can be derived depending upon the value of the $(\sigma/\omega\varepsilon)^2$ ratio. Media whose $(\sigma/\omega\varepsilon)^2$ is much less than unity $[(\sigma/\omega\varepsilon)^2 \ll 1]$ are referred to as good dielectrics and those whose $(\sigma/\omega\varepsilon)^2$ is much greater than unity $[(\sigma/\omega\varepsilon)^2 \gg 1]$ are referred to as good conductors.

In general: -

$\left(\frac{\sigma}{\omega\epsilon}\right)^2 \ll 1 \Rightarrow$ referred to as a good dielectric

$(\frac{\sigma}{\omega\epsilon})^2 \gg 1 \Rightarrow$ referred to as a good conductor

Good dielectric: I source free lossy media EM for Ampere's law given by:-

$$\nabla \times H = \sigma E + j\omega\epsilon E$$

$$\nabla \times H = (\sigma + j\omega\epsilon)E \qquad (25)$$

Good conductor:-

$$\alpha = \omega\sqrt{\mu\epsilon}\sqrt{\frac{1}{2}[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1]}$$

$$= \omega\sqrt{\mu\epsilon}\,[\frac{1}{2}(\frac{\sigma}{\omega\epsilon}[1+\frac{1}{\left(\frac{\sigma}{\omega\epsilon}\right)^2}]^{1/2} - 1]^{1/2}$$

$$= \omega\sqrt{\mu\epsilon}\,[\frac{1}{2}\left(\frac{\sigma}{\omega\epsilon} + \frac{1}{2}\frac{1}{\frac{\sigma}{\omega\epsilon}} - \frac{1}{8}\frac{1}{(\frac{\sigma}{\omega\epsilon})^3} + \cdots\right)]^{1/2}$$

$$\therefore \alpha \approx \sqrt{\frac{\omega\mu\sigma}{2}} \qquad \text{also} \qquad \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

The skin depth is measured by length unit.

EMW in lossy media

	Exact eq.	Dielectric	conductor
Attenuation constant	$\alpha = \omega\sqrt{\mu\epsilon}\sqrt{\frac{1}{2}[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1]}$	$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\alpha \approx \sqrt{\frac{\omega\mu\sigma}{2}}$
Phase constant	$\beta = \omega\sqrt{\mu\epsilon}\sqrt{\frac{1}{2}[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1]}$	$\beta \approx \omega\sqrt{\epsilon\mu}$	$\beta \approx \sqrt{\frac{\omega\mu\sigma}{2}}$
Wave impedances	$\eta_c = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$	$\eta_c \approx \sqrt{\frac{\mu}{\epsilon}}$	$\eta_c \approx \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$
Wavelength	$\lambda = \frac{2\pi}{\beta}$	$\lambda \approx \frac{2\pi}{\omega\sqrt{\mu\epsilon}}$	$\lambda \approx 2\pi\sqrt{\frac{2}{\omega\mu\sigma}}$
velocity	$\vartheta = \frac{\omega}{\beta}$	$\vartheta \approx \frac{1}{\sqrt{\mu\epsilon}}$	$\vartheta \approx \sqrt{\frac{2\omega}{\mu\sigma}}$

Skin depth	$\delta = \frac{1}{\alpha}$	$\delta \approx \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$	$\delta \approx \sqrt{\frac{2}{\omega\mu\sigma}}$

Problem 2: If the earth receives $2\frac{cal}{\min cm}$ solar energy. What are the amplitudes of electric and magnetic field of radiation.

Solution:

from pointing theorem, the energy flux per unit area per second is $|S| = |E \times H| = EHsin\,90 = EH$

The energy flux per unit area per second at earth is given

$$2\frac{cal}{\min cm} = \frac{2 \times 4.2 \times 10^4}{60} \quad \frac{joules}{m^2\ sec^2} = 1400$$

$$EH = 1400$$

$$\frac{E}{H} = \sqrt{\frac{\mu_\circ}{\varepsilon_\circ}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = 376.6$$

$$E = \sqrt{1400 \times 376.6} \;=\; 726.1\frac{vol}{m}$$

From (2)

$$H = \frac{1400}{E} = \frac{1400}{726.1} = 1.928\ \frac{A}{m}$$

problem 3: A plan electromagnetic wave travelling in positive z- direction in an unbounded lossless dielectric medium with relative permeability $\mu_r = 1$ and relative permittivity $\varepsilon_r = 3$ has a peak electric field intensity $E_\circ = 6\frac{v}{m}$ find.

(i) The speed of the wave .

(ii) The impedance of the medium.

(iii) The magnetic field.

(iv) Pointing vector.

Solution:

$$E_\circ = \sqrt{E_x^2 + E_y^2} = 6\frac{v}{m}$$

(i) The speed

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r\,\mu_\circ\varepsilon_r\varepsilon_\circ}}$$

$$\frac{1}{\sqrt{\mu_r\mu_\circ}}\cdot\frac{1}{\sqrt{\varepsilon_r\varepsilon_\circ}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{3\times10^8}{\sqrt{1\times3}} = 1.73\times10^8\,\frac{m}{s}$$

(ii) impedance of medium

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_\circ}{\varepsilon_\circ}}\,\cdot\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$= \sqrt{\frac{4\pi\times10^{-7}}{8.86\times10^{-12}}}\,\cdot\sqrt{\frac{1}{3}} = \frac{376.6}{\sqrt{3}} = 217.6\,ohm$$

(iii) The magnetic field.

$$H_\circ = \frac{E_\circ}{Z} = \frac{6}{217.6} = 2.76\ \times10^{-2}\,\frac{A}{m}$$

(v) Pointing vector.

$$S = E\times H$$

Pointing vector $= E_\circ\,H_\circ = \frac{E_\circ^2}{Z} = \frac{6^2}{217.6} = 0.165\,\frac{w}{m^2}$

Problem 4: EMW spreads in dielectric medium of frequency f = 100MHz & $\lambda = 1m$, if the medium is lossless and nonmagnetic, find the characteristic of this medium.

$(\mu = ?\,, \in = ?\,)$

Solution:

- For lossless medium $\lambda f = v = \frac{1}{\sqrt{\mu \in}}$
- For nonmagnetic medium $\mu = \mu_o$

$$\therefore \lambda f = \frac{1}{\sqrt{\mu_o \in}}$$

$\lambda f = \frac{1}{\sqrt{\mu_o \in_o \in_r}} = \frac{\in}{\in_o}, \in_r = \frac{\in}{\in_o}$

$1 \times 10^8 = \frac{3\times10^8}{\sqrt{\in_r}} \rightarrow \in_r = 9$

$\in_r = 9 \in_o \quad F/m$

$\mu = \mu_o$ H/m

$\delta = 0\ (\Omega m)^{-1}$

Problem 5: EMW spreading free source air of H-field given by $H_Z = H_\circ e^{-J(wt-5\pi y)} \frac{A}{m}$ find 1-F and 2- E –field

Solution:

a) $K = 5\pi$

$$K = w\sqrt{\mu_\circ \epsilon_\circ} = 2\pi f \sqrt{\sqrt{\mu_\circ \epsilon_\circ}} = \frac{2\pi f}{3\times10^8} = 5\pi \therefore f = \frac{15\times10^8}{2} = 750MHZ$$

b) $\overline{\nabla} \times \overline{H} = Jw\epsilon_\circ E = E_X = -\eta_\circ H_\circ e^{-J(wt-5\pi y)} \frac{V}{m}$