Lecture 5

The Thermal Wind

5.1 The Thermal Wind

If the geostrophic wind is increasing with height, then the horizontal pressure gradient must also be increasing with height. If the pressure gradient is increasing in positive x-direction, the temperature gradient must be increasing in the direction, then there must be horizontal thickness gradient, which means that the pressure surface not be parallel and will have different slopes, so that the geostrophic wind will differ on two pressure surface. From Fig. 5.1, the thickness of atmosphere at point x_2 is greater than the thickness at the point x_1.

The mean temperature between pressure levels p_1 and p_2 must be greater in the x_2 column than the x_1 column. This causes the horizontal pressure gradient to change rapidly over x_2 as height increases, resulting in stronger geostrophic wind (dark arrows).

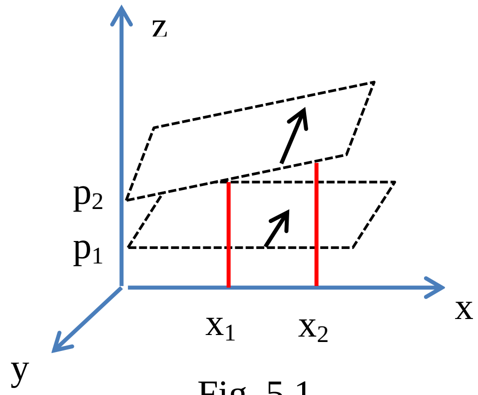

Fig. 5.1

- The thermal wind is defined as the vector difference between the geostrophic winds at two levels.
- The thermal wind is not really a wind; it is the shear of the geostrophic wind.

$$\frac{\partial \vec{V}_T}{\partial x} = \frac{g}{fT}\frac{\partial T}{\partial x}$$

$$\frac{\partial \vec{V}_T}{\partial z} = \frac{g}{f}\frac{\nabla T}{T}$$

- The vector $\frac{\partial \vec{V}_g}{\partial z}$ is parallel to the isothermal with cold air to the left and warm air to the right (see Fig. 5.2).

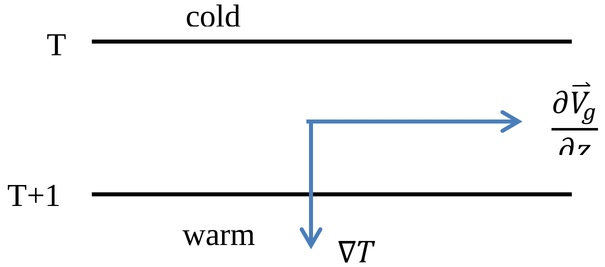

Fig. 5.2

- The thermal wind equation is very important because we may calculate the wind increase in the vertical direction from the temperature gradient and vice versa.

5.2 The Thermal Wind Equation

$$p = \rho\, R\, T \qquad (5.1)$$

$$\frac{\partial p}{\partial z} = -\rho\, g \qquad (5.2)$$

$$f\, V_g = \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (5.3)$$

From equation (5.1),

$$\rho = \frac{p}{R\,T} \rightarrow \frac{\partial \rho}{\partial z} = \frac{1}{R\,T}\frac{\partial p}{\partial z} \qquad (5.4) \qquad and,$$

$$p = \rho\, R\, T \;\rightarrow\; \frac{\partial p}{\partial x} = R\,(\rho\frac{\partial T}{\partial x} + T\,\frac{\partial \rho}{\partial x})$$

$$\frac{\partial p}{\partial x} = R\,\rho\frac{\partial T}{\partial x} + R\,T\,\frac{\partial \rho}{\partial x} \qquad (5.5)$$

$$f\, V_g = \frac{1}{\rho}\frac{\partial p}{\partial x} \;\rightarrow f\frac{\partial V_g}{\partial z} = \frac{\partial}{\partial z}(\frac{1}{\rho}\frac{\partial p}{\partial x})$$

$$f\frac{\partial V_g}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial x}\frac{\partial p}{\partial z} - \frac{1}{\rho^2}\frac{\partial \rho}{\partial z}\,\frac{\partial p}{\partial x}$$

By using equations (5.2) for the first right term and (5.4) for the second right term:

$$f\frac{\partial V_g}{\partial z} = -\frac{g}{\rho}\frac{\partial \rho}{\partial x} + \frac{1}{\rho^2}(\frac{1}{RT}\frac{\partial p}{\partial z}\,\frac{\partial p}{\partial x})$$

By substituting for $\frac{\partial p}{\partial z}$ from equation (5.2), and substituting for $\frac{\partial p}{\partial x}$ from (5.5) we get:

$$f\frac{\partial V_g}{\partial z} = -\frac{g}{\rho}\frac{\partial \rho}{\partial x} + \frac{g}{\rho\,R\,T}(R\,\rho\frac{\partial T}{\partial x} + R\,T\frac{\partial \rho}{\partial x})$$

$$f\frac{\partial V_g}{\partial z} = -\frac{g}{\rho}\frac{\partial \rho}{\partial x} + \frac{g\rho R}{\rho RT}\,\frac{\partial T}{\partial x} + \frac{gRT}{\rho RT}\,\frac{\partial \rho}{\partial x}$$

$$f\frac{\partial V_g}{\partial z} = \frac{g}{T}\frac{\partial T}{\partial x}$$

$$\boldsymbol{\frac{\partial V_g}{\partial z} = \frac{g}{fT}\,\frac{\partial T}{\partial x}} \qquad \text{, the thermal wind equation}$$

Question. Is the thermal wind stronger at the pole (90^{o} N) or at 30^{o} N?

Sol.

$$\frac{\partial V_g}{\partial z} = \frac{g}{fT}\frac{\partial T}{\partial x}$$

at the pole, $f = 2\,\Omega \sin 90 = 2\Omega$

at 30^{o} N, $f = 2\,\Omega \sin 30 = \Omega$

$$\vec{V}_T(pole) = \frac{g}{fT}\frac{\partial T}{\partial x} = \frac{g}{2\Omega\, T}\frac{\partial T}{\partial x} \qquad (1)$$

$$\vec{V}_T(30^o) = \frac{g}{f\,T}\frac{\partial T}{\partial x} = \frac{g}{\Omega\, T}\frac{\partial T}{\partial x} \qquad (2)$$

$$\frac{\vec{V}_T(pole)}{\vec{V}_T(30^o)} = \frac{\frac{g}{2\Omega\, T}\frac{\partial T}{\partial x}}{\frac{g}{\Omega\, T}\frac{\partial T}{\partial x}} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\therefore\ \vec{V}_T(pole) = \frac{1}{2}\,\vec{V}_T(30^o)$$

$$\therefore\ \vec{V}_T\ is\ stronger\ at\ the\ pole$$