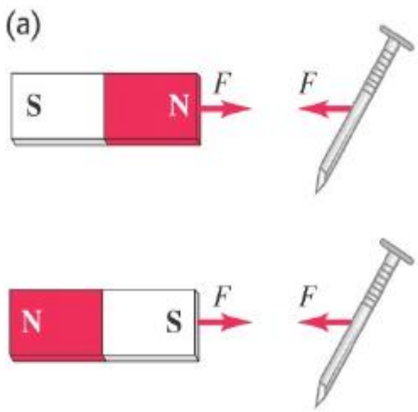
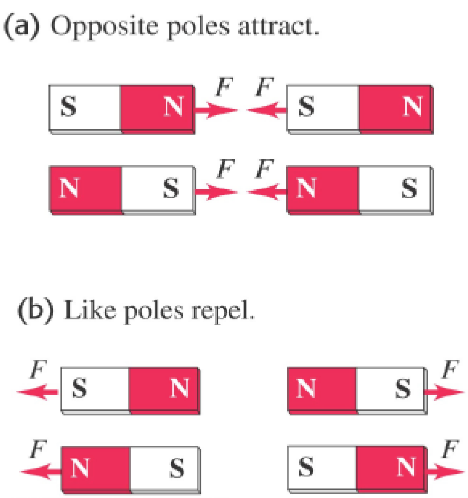
1-1 Magnetism

Magnets exert forces on each other just like charges. You can draw magnetic field lines just like you drew electric field lines. Magnetic north and south pole's behavior is not unlike electric charges. For magnets, like poles repel and opposite poles attract.
A permanent magnet will attract a metal like iron with either the north or South Pole.





Electric field:
1) A distribution of electric charge at rest creates an electric field E in the surrounding space.
2) The electric field exerts a force $\vec{F}_e = q\vec{E}$ on any other charges in presence of that field.
Magnetic field:
1) A moving charge or current creates a magnetic field in the surrounding space (in addition to E).
2) The magnetic field exerts a force $\vec{F}_m$ on any other moving charge or current present in that field.
- The magnetic field is a vector quantity associated with each point in space.

$\vec{F}_m = |q|\, v_\perp B$ ....(1)

$= |q|\, (\vec{v} \times \vec{B})$ ....(2)

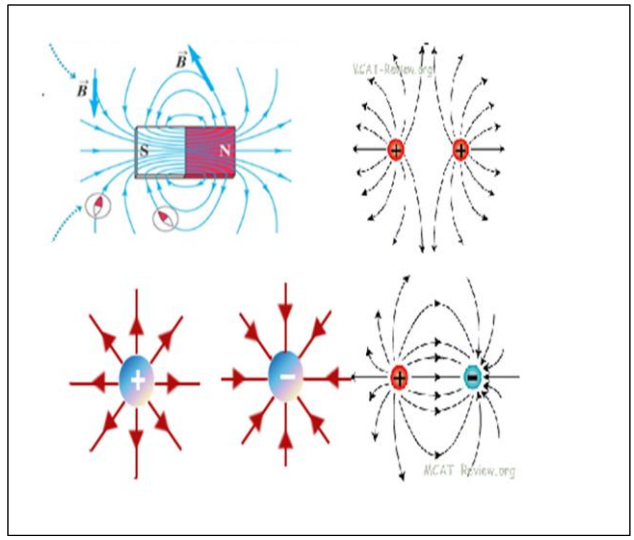
$= |q| vB \sin \Theta$ ……….(3)

Remember that in cross products: $\vec{a} \times \vec{b} = ab \sin \theta$

$\vec{F}_m = q\,(\vec{v} \times \vec{B}\,)$ ……..(4)

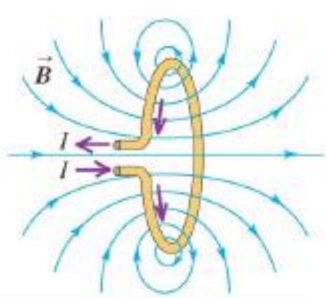
$\vec{F}_m$ *Is always perpendicular to* $((\vec{v}\,)$ *And* $(\vec{B}\,)$.

1-2 Magnetic Field Lines

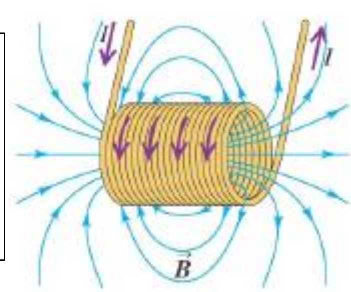


1- Magnetic field lines may be traced from N to S (similar to the electric field lines).
2- At each point they are tangent to magnetic field vector.
3- The more densely packed the field lines, the stronger the field at a point.
4- Field lines never intersect.
5- The field lines point in the same direction as a compass (from N to S).
6- Magnetic field lines are not "lines of force" Magnetic field lines have no ends, they continue through the interior of the magnet.

(c) Magnetic fields of a current-carrying loop and a current-carrying coil (solenoid)

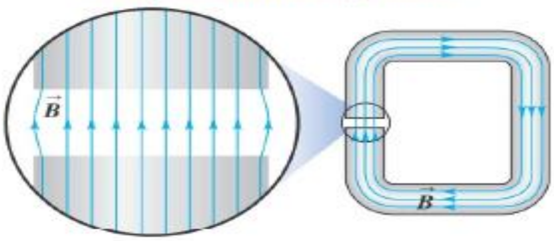


Notice that the filed of the loop and especially that of the coil look like the filed of a bar magnet.



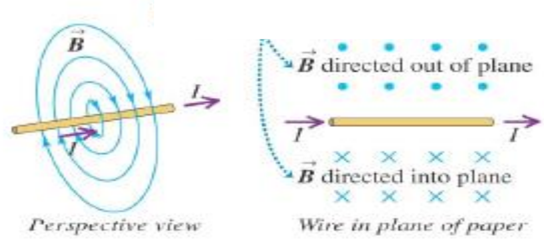
(a) Magnetic fied of C- shape magnet

Between flat parallel magnetic poles the magnetic filed is nearly uniform



(b) Magnetic filed of straight current carrying wier

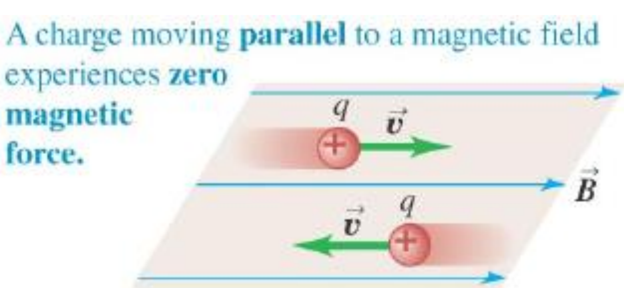
To represent the magnetic filed coming out of the plane of the paper we use dot and crosses respectively

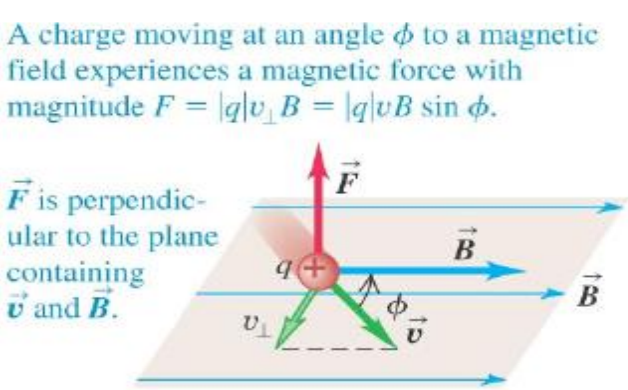


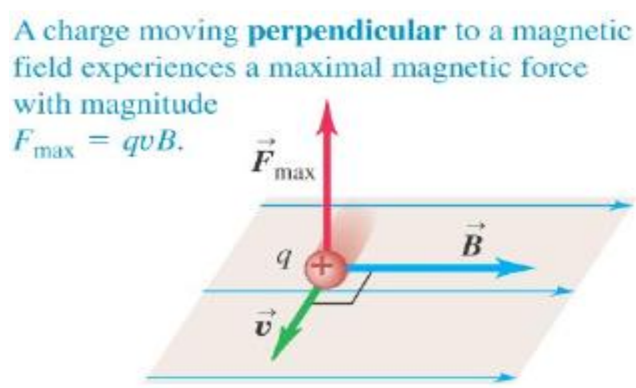
1-3 Magnetic Fields and Forces

Experiments on various charged particles moving in a magnetic field give the following results:

1. The magnitude F_B of the magnetic force applied to the particle is proportional to the charge q and to the speed v of the particle.
2. The magnitude and direction of F_B depend on the velocity of the particle and on the magnitude and direction of the magnetic field B.
3. When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.







Notes:

1.When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both v and B; that is, F_B is perpendicular to the plane formed by v and B.
2.The magnetic force applied to a positive charge is in the direction opposite the direction of the magnetic force applied on a negative charge moving in the same direction.
3. The magnitude of the magnetic force applied to the moving particle is proportional to sinθ, where θ is the angle the particle's velocity vector makes with the direction of B.

We can summarize these observations by writing the magnetic force in the form:

As $\vec{E} = \frac{\vec{F}}{q}$ ……….(5)

$\vec{B} = \frac{\vec{F}}{qv \sin \theta}$ ……..….(6)

$\vec{F} = \vec{B}(qv\ sin\theta)$

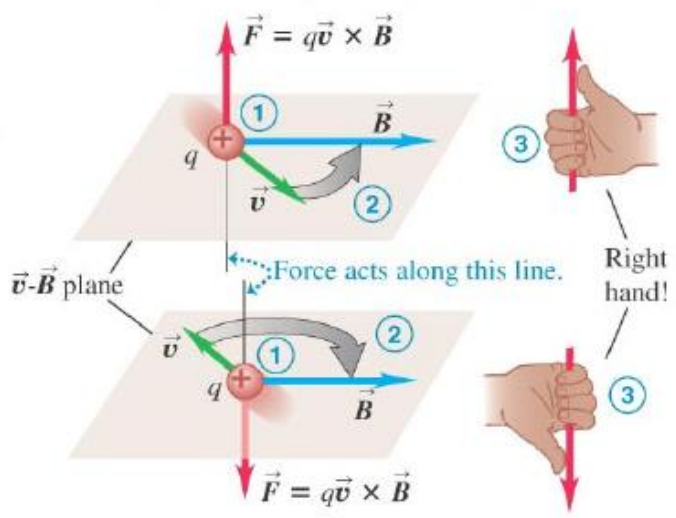
Or

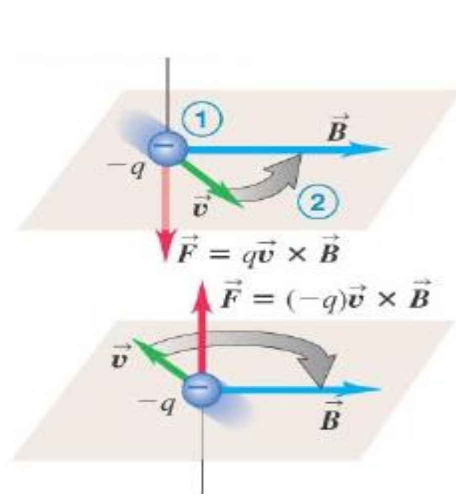
$\vec{F}_B = q(v \times \vec{B})$

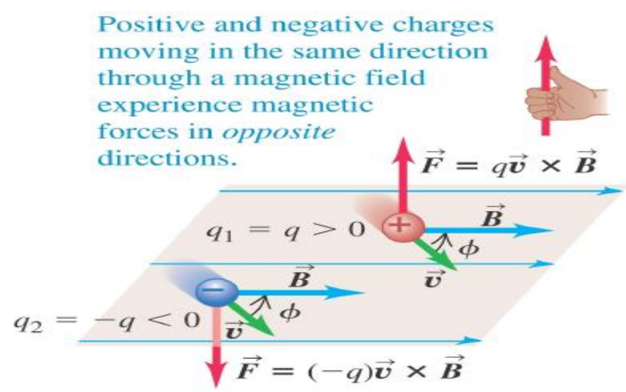
1-4 Right Hand Rule

Positive charge moving in magnetic field →direction of force follows right hand rule

Negative charge → F direction contrary to right hand rule.





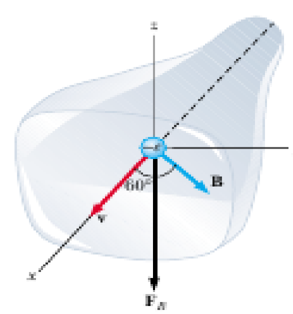


The magnitude of the magnetic force on a charged particle is:

$F_B = |q|\, v\, B\, sin\phi$

where ϕ is the smallest angle between v and B. From this expression, we see that F_B is zero when v is parallel or antiparallel to B (ϕ = 0 or 180°) and maximum when v is perpendicular to B (ϕ = 90°). In the SI unit $1T = \frac{N}{C.m/sec} = \frac{N}{Amp.m}$ 1Gauss = 10^{-4}T

Example1: An electron in a television picture tube moves toward the front of the tube with a speed of $8x10^6$ m/s along the x-axis show in the Figure. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the x- axis and lying in the x, y plane. Calculate the magnetic force on the electron.



Solution: $F_B = |q|\ v\ B\ sin\phi$

$F_B = 1.6 \times 10^{-19} C \times 8 \times 10^6\ m/sec \times 0.025T \times sin60°$

$$F_B = 2.8 \times 10^{-14} N$$

Example 2:

A regular magnetic field in which B = 0.12 T to the east, proton at a speed of $5x10^5$ m / Sec was thrown in the magnetic field. Find out the amount of magnetic force F_B Attached to the proton for the following cases:

a. Towards the south?

b. Westward?

c-Northward?

d-East?

e - Towards making the corner 60 to the east ?

Solution:

The proton charge is 1.6×10^{-19} C, and the magnetic force is $F_B = q\ v\ B\ sin\theta$

a) Towards the south, then the angle btween the magnetic field and the proton is 90^o

   $F_B = q\ v\ B\ sin\theta$

   $= (1.6\times10^{-19}) \times (5\times10^5 m/ Sec) \times (0.12T) \times sin90$

   $= 9.6 \times10^{-15}\ nt$

b) Westward, then the angle btween the magnetic field and the proton is 180^o

   $F_B = q\ v\ B\ sin\theta$

   $= (1.6\times10^{-19}) \times (5\times10^5 m/ Sec) \times (0.12T) \times sin180$

   $= 0$

c) Northward, then the angle btween the magnetic field and the proton is 90°

$F_B = q\,v\,B\,sin\theta$

= (1.6×10^{-19}) × (5×10^{5}m/ Sec) × (0.12T) × sin90

= 9.6 ×10^{-15}*nt*

d) East, then the angle btween the magnetic field and the proton is 90°

$F_B = q\,v\,B\,sin\theta$

= (1.6×10^{-19}) × (5×10^{5}m/ Sec) × (0.12T) × sin180

= 0

e) Towards making the corner 60 to the east?

$F_B = q\,v\,B\,sin\theta$

= (1.6×10^{-19}) × (5×10^{5}m/Sec) × (0.12T) × sin60

= 8.31×10^{-15} *nt*

H.W. Find the magnetic force for an electron moving with velocity 6×10^{7}m/Sec. In the same regular magnetic field?

1-5 Magnetic Flux and Gauss's Law for Magnetism

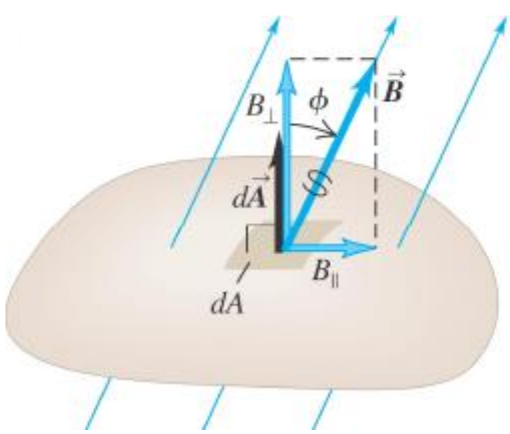
$\Phi_B = \int B_\perp\, dA = \int B\, cos\emptyset . dA = \int \vec{B} . d\vec{A}$

Magnetic flux is a scalar quantity.

If $\vec{B}$ is uniform:

$\Phi_B = B_\perp A = B\,A\,cos\emptyset$ ………(7)

1 Weber (1 Wb = 1 T.m^{2} = 1 N .m / A)



- Difference with respect to electric flux ⇒ the total magnetic flux through a closed surface is always zero. This is because there is no isolated magnetic charge ("monopole") that can be enclosed by the Gaussian surface.

$\Phi_B = \int \vec{B}\,.\,d\vec{A} = 0$……….(8)

$B = \frac{d\Phi_B}{dA_\perp}$ …………(9)

- The magnetic field is equal to the flux per unit area across an area at right angles to the magnetic field = magnetic flux density.

1-6 Motion of Charged Particles in a Magnetic Field

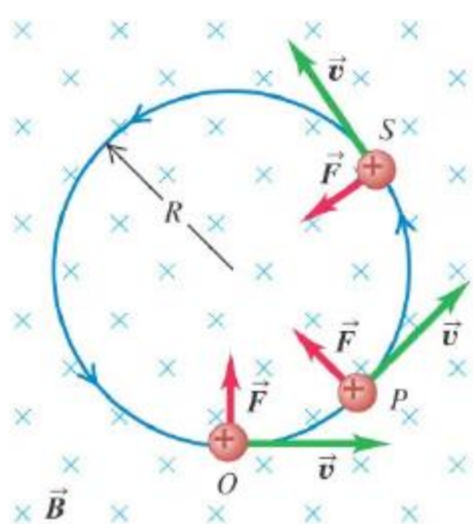
- Magnetic force perpendicular to $\vec{v}$ It cannot change the magnitude of the velocity, only its direction.

$\vec{F}_m = q\,\vec{v} \times \vec{B}$

- $\vec{F}$ does not have a component parallel to particle's motion ⇒ cannot do work.

- Magnitudes of F and v are constant (v perp. B) ⇒ uniform circular motion.

$\vec{F}_m = |q|.v.B = m\frac{v^2}{R}$ ………(10)



Radius of circular orbit in magnetic field:

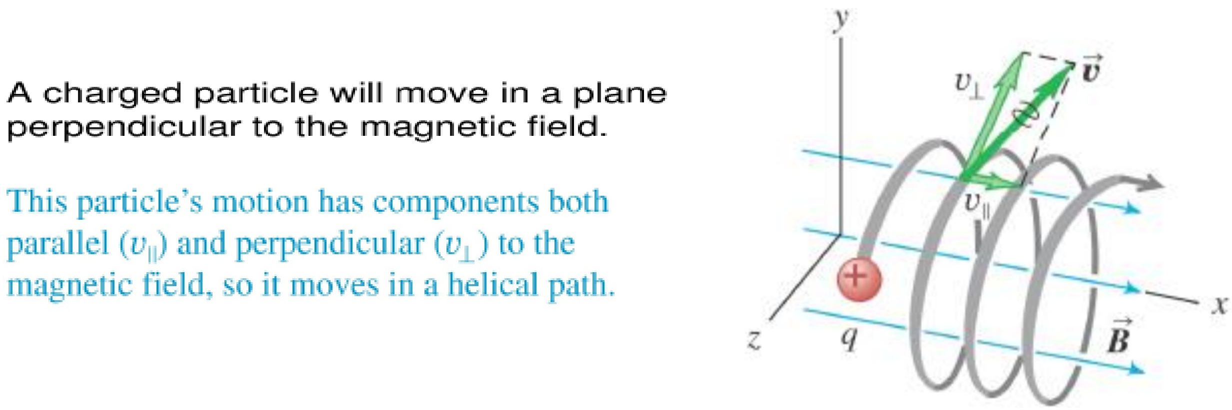
$$R = \frac{mv}{|q|B}$$

+ particle ⇒ counter-clockwise rotation.
- particle ⇒ clockwise rotation.

Angular speed:

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv}$$

-If v is not perpendicular to B ⟶ v parallel to B constant because F = 0 particle moves in a helix. (R same as before, with $v = v_\perp$).



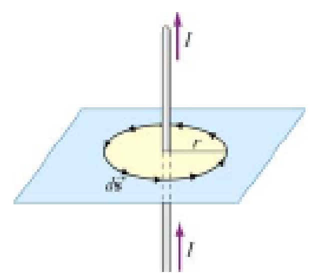
1-7 Ampere' Law: states that the line integral of B and dl over a closed path is μ_o times the current enclosed in that loop:

$$\oint \vec{B}.\overrightarrow{dl} = \mu_o.i_{enclosed} \quad \text{……….(11)}$$

$\mu_o = 4\pi \times 10^{-7}\mathrm{T.m/A}$ is the magnetic permeability of free space.

Example: Using Ampère's law to find the field around a long straight wire:

Sol:



$$\oint \vec{B}.\overrightarrow{ds} = \mu_o.i_{enclosed}$$

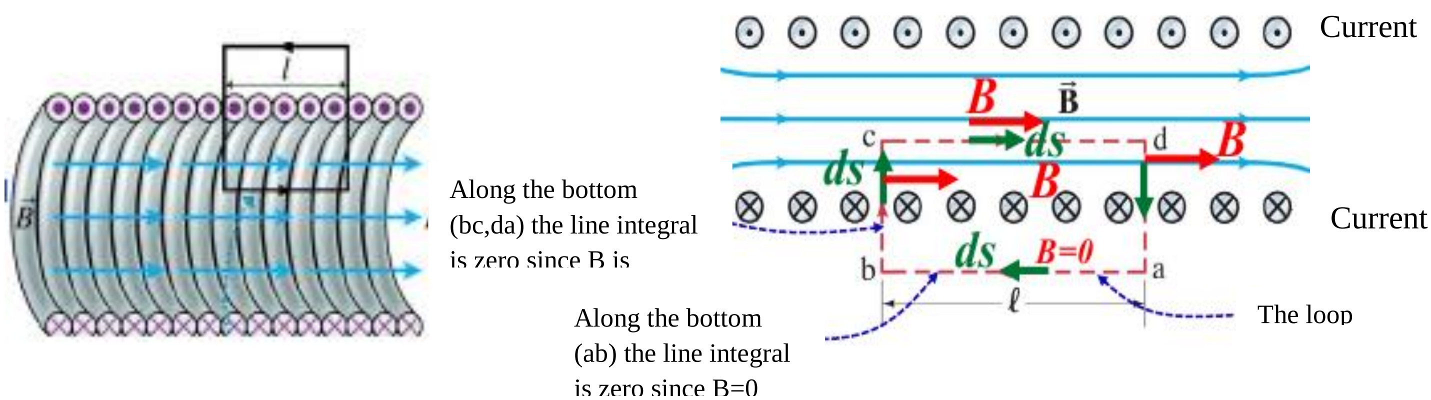
$$\oint \overrightarrow{ds} = 2\pi r \quad the\ circumference\ of\ the\ loop$$

$$B.2\pi r = \mu_o.i_{enclosed}$$

$$B = \frac{\mu_o.i_{enclosed}}{2\pi r} \quad \text{………..(12)}$$

1-8 Solenoid

A solenoid is a helical coil of wire with the same current I passing through each loop in the coil. A uniform magnetic field can be generated with a solenoid.



$$\oint \vec{B}\,.\,\overrightarrow{ds} = \mu_o.I \text{ ....(13)}$$

$$\oint_{abcda} \vec{B}\cdot d\vec{s} = \underbrace{\int_a^b \vec{B}\cdot d\vec{s}}_{\vec{B}=0} + \underbrace{\int_b^c \vec{B}\cdot d\vec{s}}_{\vec{B}\perp d\vec{s}} + \underbrace{\int_c^d \vec{B}\cdot d\vec{s}}_{\vec{B}\parallel d\vec{s},\ \vec{B}=const} + \underbrace{\int_d^a \vec{B}\cdot d\vec{s}}_{\vec{B}\perp d\vec{s}} = \mu_0 I_{in}$$

There are N loops with current I enclosed by an Amperian loop, so

$$I_{in} = N.I \text{ ....(13)}$$

$$B\int_c^d ds = \mu_o.N.I \text{ ....(14)}$$

$$Bl = \mu_o.N.I \Rightarrow B = \frac{\mu_o.N.I}{l} \qquad \textbf{where } n = \frac{N}{l}$$

n is the number of turns per unit length.

$$B_{solenoid} = \mu_o.n.I \text{ ....(15)}$$

Example2: What current is required in the windings of a long solenoid that has 1000 turns uniformly distributed over a length of 0.4m, to produce at the center of the solenoid a magnetic field of magnitude $1x10^{-4}$ T?

Solution:

$$\boldsymbol{B} = \frac{\boldsymbol{\mu_o}.\boldsymbol{N}.\boldsymbol{I}}{\boldsymbol{l}}$$

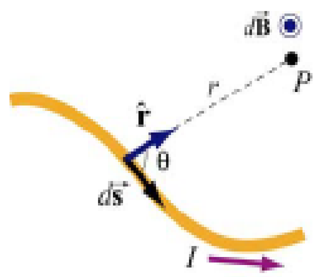
$$\boldsymbol{I} = \frac{\boldsymbol{B}.\boldsymbol{l}}{\boldsymbol{\mu_o}.\boldsymbol{N}.}$$

$$I = \frac{(1 \times 10^{-4}T) \times 0.4m}{\left(4\pi \times 10^{-7}\frac{T.m}{Amp}\right) \times 1000} = 31.8 \text{ mA}$$

1-9 Sources of Magnetic Fields

Biot-Savart Law

Currents, which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire and produce a current $\boldsymbol{I}$, the magnetic field at any point **P** due to the current can be calculated by adding up the magnetic field contributions, $d\vec{B}$, from small segments of the wire $d\vec{s}$, see the figure



The infinitesimal current source can then be written as $Id\vec{s}$

Let **r** denote as the distance from the current source to the field point **P**, and $\vec{r}$ The corresponding unit vector. The Biot-Savart law gives an expression for the magnetic field contribution, $d\vec{B}$, from the current source, $Id\vec{s}$

$$\boldsymbol{d\vec{B}} = \frac{\boldsymbol{\mu_o}}{\boldsymbol{4\pi}} \frac{\boldsymbol{Id\vec{s}\times\vec{r}}}{\boldsymbol{r^2}} \text{ .... } \textbf{(16)}$$

Where is $\boldsymbol{\mu_o}$ a constant called the permeability of free space:
$\mu_o = 4\pi \times 10^{-7}\text{T. m/A}$

Adding up these contributions to find the magnetic field at the point P requires integrating over the current source:

$$\vec{B} = \int_{wire} d\vec{B} = \frac{\mu_o I}{4\pi} \int_{wire} \frac{d\vec{s} \times \vec{r}}{r^2}$$

1-9 Magnetic Field of a Moving Charge

According to the definition of current

$$Id\vec{s} = \frac{dq}{dt} d\vec{s} = dq \frac{d\vec{s}}{dt} = dq\vec{v}$$

Where is the velocity at which the charge carriers move down the wire. In the limiting case where the segment contains just a single charge carrier carrying a charge q , dq becomes q and

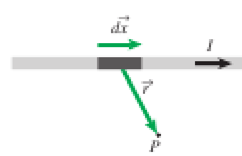
$dq\vec{v} = q\vec{v}$

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^2} \quad \ldots\ldots\ldots\ldots(17)\ single\ particale$$

1-11 Magnetic field of a moving charged particle

Let us now, use the Biot-Savart law to find an expression for the magnetic field caused by charged particles moving at constant velocity. Consider first a straight wire carrying a current of magnitude *I* and aligned with the *x* axis, as in the Figure. The magnetic field generated at point *P* by a small segment is given by:

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^2}$$



Where $\hat{r}$ Is a unit vector pointing from the segment to the point at which we wish to determine the magnetic field, and r is the distance between the segment and the point P. Suppose the segment contains an amount of charge dq. Let the charge carriers responsible for the current take a time interval dt to have displacement. According to the definition of current we have

$$I = \frac{dq}{dt}$$

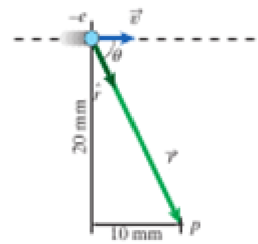
$$I d\vec{x} = \frac{dq}{dt} d\vec{x} = dq \frac{d\vec{x}}{dt} = dq\vec{v}$$

$$I d\vec{x} = q\vec{v}$$

Substituting this result into Biot-Savart law, we obtain an expression for the magnetic field of a single moving charged particle:

$$\boldsymbol{\vec{B} = \frac{\mu_o}{4\pi} \frac{q\vec{v}\times\vec{r}}{r^2}} \text{ ………(18)}$$

Example: An electron carrying a charge e= –1.6 x10 $^{-19}$ C moves in a straight line at a speed ν = 3x10^{7} m/s. What are the magnitude and direction of the magnetic field caused by the electron at a point **P** 10 mm ahead of the electron and 20 mm away from its line of motion?



The magnitude of $\vec{r}$ *is r and equal*

$$\vec{r} = \sqrt{10^2 + 20^2} = 22mm$$

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^2}$$

$$\vec{B} = \frac{(4\pi \times 10^{-7}\, m/A)}{4\pi} \frac{(1.6 \times 10^{-19} C)(3 \times 10^{7}\, m/sec) \times 0.89}{(3 \times 10^{-3} m)^2}$$

$$B = 8.6 \times 10^{-16}\text{T}$$

Problems

1- The North-Pole end of a bar magnet is held near a positively charged piece of plastic. Is the plastic (a) attracted, (b) repelled, or (c) unaffected by the magnet?

2-A charged particle moves with velocity v in a magnetic field B. The magnetic force on the particle is a maximum when v is (a) parallel to B, (b) perpendicular to B, (c) zero.

3- An electron moves in the plane of this paper toward the top of the page. A magnetic field is also in the plane of the page and directed toward the right. The direction of the magnetic force on the electron is (a) toward the top of the page, (b) toward the bottom of the page, (c) toward the left edge of the page, (d) toward the right edge of the page, (e) upward out of the page, (f) downward into the page.

4- A wire carries current in the plane of this paper toward the top of the page. The wire experiences a magnetic force toward the right edge of the page. The direction of the magnetic field causing this force is (a) in the plane of the page and toward the left edge, (b) in the plane of the page and toward the bottom edge, (c) upward out of the page, (d) downward into the page.

5- A charged particle is moving perpendicular to a magnetic field in a circle with a radius *r*. An identical particle enters the field, with v perpendicular to B, but with a higher speed *v* than the first particle. Compared to the radius of the circle of the first particle, the radius of the circle for the second particle is (a) smaller (b) larger (c) equal in size.

6- Charged particle is moving perpendicular to a magnetic field in a circle with a radius *r*. The magnitude of the magnetic field is increased. Compared to the initial radius of the circular path, the radius of the new path is (a) smaller (b) larger (c) equal in size.

7- Comper between the magnetic field and the electric field?

الفقره 1-1 في المحاضرات

8- State the magnetic field liens properties

الفقره 1-2 في المحاضرات

9- Define the magnetic flux and Gauss's law?

الفقره 1-5 من المحاضرات

10- Define the Amper law and give an expression for the magnetic field?

الفقره1-7 من المحاضرات

11- Define the Solenoid and give an expression for the magnetic field?

الفقره 1-8 من المحاضرات

12- State Biot- subvert law and give an expression for the magnetic field?

الفقره 1-9 من المحاضرات

13- A proton travels with a speed of 3.00×10^6 m/s at an angle of 37.0° with the direction of a magnetic field of 0.300 T in the $+y$ direction. What is (a) the magnitude of the magnetic force on the proton and (b) its acceleration?

(a) $$F_B = qvB\sin\theta = \left(1.60\times10^{-19}\ \text{C}\right)\left(3.00\times10^{6}\ \text{m/s}\right)\left(3.00\times10^{-1}\ \text{T}\right)\sin 37.0°$$

$$F_B = \boxed{8.67\times10^{-14}\ \text{N}}$$

(b) $$a = \frac{F}{m} = \frac{8.67\times10^{-14}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = \boxed{5.19\times10^{13}\ \text{m/s}^2}$$

14- A proton moves perpendicular to a uniform magnetic field B at 1.00×10^{7} m/s and experiences an acceleration of 2.00×10^{13} m/s^2 in the +*x* direction when its velocity is in the +*z* direction. Determine the magnitude and direction of the field.

$$F = ma = \left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{13}\ \text{m/s}^2\right) = 3.34\times10^{-14}\ \text{N} = qvB\sin 90°$$

$$B = \frac{F}{qv} = \frac{3.34\times10^{-14}\ \text{N}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{7}\ \text{m/s}\right)} = \boxed{2.09\times10^{-2}\ \text{T}}$$

The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction.

9-proton moving at 4.00×10^{6} m/s through a magnetic field of 1.70 T experiences a magnetic force of magnitude 8.20×10^{-13} N. What is the angle between the proton velocity and the field?

$F_B = qvB\sin\theta$ so $8.20\times10^{-13}\ \text{N} = \left(1.60\times10^{-19}\ \text{C}\right)\left(4.00\times10^{6}\ \text{m/s}\right)(1.70\ \text{T})\sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9°}$.

10-At the equator, near the surface of the Earth, the magnetic field is approximately 50.0 µT northward, and the electric field is about 100 N/C downward in fair weather. Find the gravitational, electric, and magnetic forces on an electron in this environment, assuming the electron has an instantaneous velocity of 6.00 x 10^6 m/s directed to the east.

Gravitational force: $F_g = mg = (9.11 \times 10^{-31}\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{8.93 \times 10^{-30}\ \text{N}}$

Electric force: $F_e = qE = (-1.60 \times 10^{-19}\ \text{C})(100\ \text{N/C}) = \boxed{1.60 \times 10^{-17}\ \text{N}}$.

Magnetic force: $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = (-1.60 \times 10^{-19}\ \text{C})(6.00 \times 10^{6}\ \text{m/s}) \times (50.0 \times 10^{-6}\ \text{N}\cdot\text{s/C}\cdot\text{m})$.

$F_B = \boxed{4.80 \times 10^{-17}\ \text{N}}$.

11-The magnetic field of the Earth at a certain location is directed vertically downward and has a magnitude of 50.0 µT. A proton is moving horizontally toward the west in this field with a speed of 6.20 x 10^6 m/s. (a) What is the direction and magnitude of the magnetic force the field exerts on this charge? (b) What is the radius of the circular arc followed by this proton?

(a) $B = 50.0 \times 10^{-6}$ T; $v = 6.20 \times 10^{6}$ m/s

Direction is given by the right-hand-rule: $\boxed{\text{southward}}$

$F_B = qvB\sin\theta$

$F_B = (1.60 \times 10^{-19}\ \text{C})(6.20 \times 10^{6}\ \text{m/s})(50.0 \times 10^{-6}\ \text{T})\sin 90.0°$

$= \boxed{4.96 \times 10^{-17}\ \text{N}}$

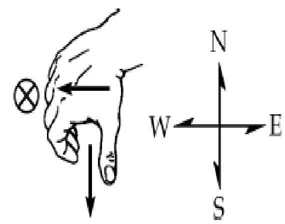
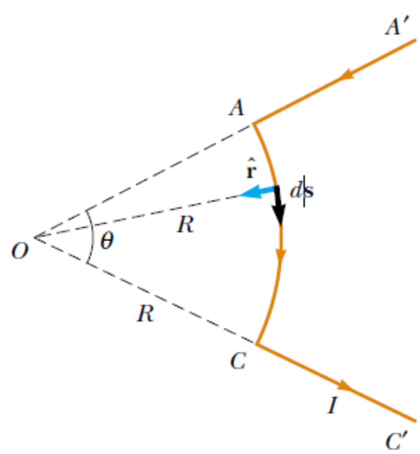


FIG. P29.29

(b) $F = \frac{mv^2}{r}$ so $r = \frac{mv^2}{F} = \frac{(1.67 \times 10^{-27}\ \text{kg})(6.20 \times 10^{6}\ \text{m/s})^2}{4.96 \times 10^{-17}\ \text{N}} = \boxed{1.29\ \text{km}}$

12-Calculate the magnetic field at point O of the current carrying wire segment shown in Figure. The wire consists of two straight portions and a circular arc of radius *R*, which subtends an angle θ. The arrowheads on the wire indicate the direction of the current.



$$dB = \frac{\mu_0 I}{4\pi} \frac{ds}{R^2}$$

Because I and R are constants in this situation, we can easily integrate this expression over the curved path AC:

$$B = \frac{\mu_0 I}{4\pi R^2} \int ds = \frac{\mu_0 I}{4\pi R^2} s = \frac{\mu_0 I}{4\pi R} \theta$$

where we have used the fact that $s = R\theta$ with θ measured in radians. The direction of $\mathbf{B}$ is into the page at O because $d\mathbf{s} \times \hat{\mathbf{r}}$ is into the page for every length element.

13-Consider the hemispherical closed surface in Figure 1. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the Magnetic flux through (a) the flat surface S1 and (b) the hemispherical surface S2.

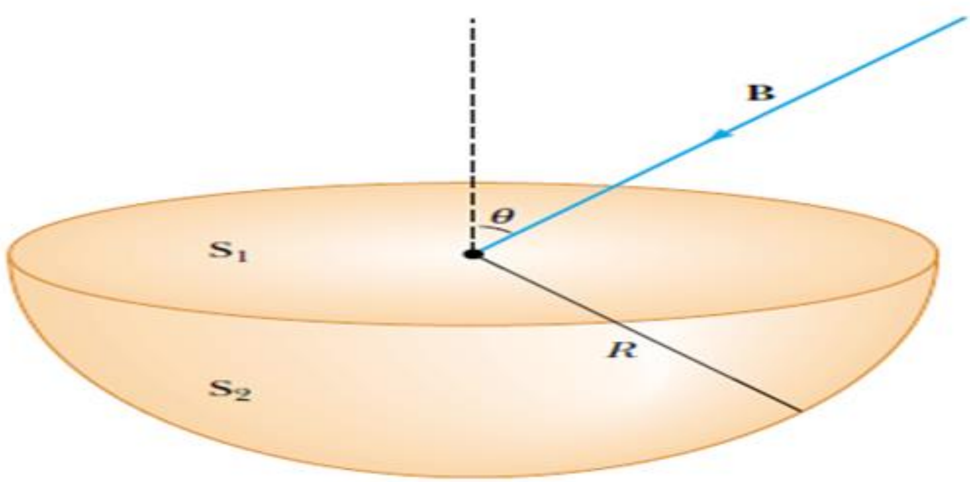


Figure 1

(a) $(\Phi_B)_{\text{flat}} = \mathbf{B}\cdot\mathbf{A} = B\pi R^2\cos(180-\theta) = \boxed{-B\pi R^2\cos\theta}$

(b) The net flux out of the closed surface is zero: $(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$.

$(\Phi_B)_{\text{curved}} = \boxed{B\pi R^2\cos\theta}$

14- cube of edge length l= 2.50 cm is positioned as shown in Figure P1. A uniform magnetic field given by B = (5ˆi + 4ˆj + 3ˆk) T exists throughout the region.

(a) Calculate the flux through the shaded face. (b) What is the total flux through the six faces?

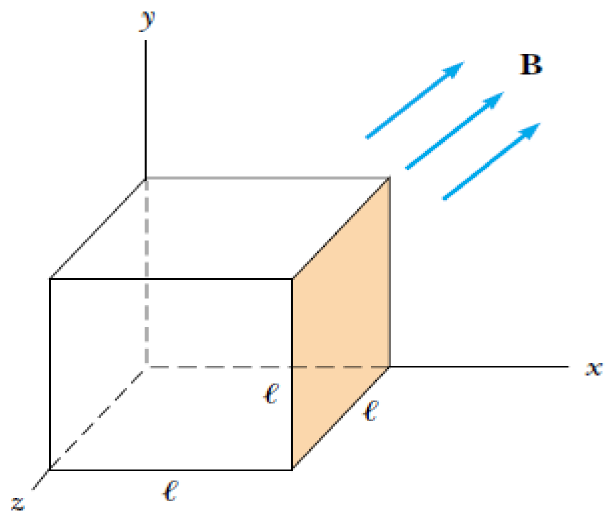


Figure 1

(a) $\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = \mathbf{B}\cdot\mathbf{A} = \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)\text{T}\cdot\left(2.50\times10^{-2}\text{ m}\right)^2\hat{\mathbf{i}}$

$\Phi_B = 3.12\times10^{-3}\text{ T}\cdot\text{m}^2 = 3.12\times10^{-3}\text{ Wb} = \boxed{3.12\text{ mWb}}$

(b) $(\Phi_B)_{\text{total}} = \oint \mathbf{B}\cdot d\mathbf{A} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)