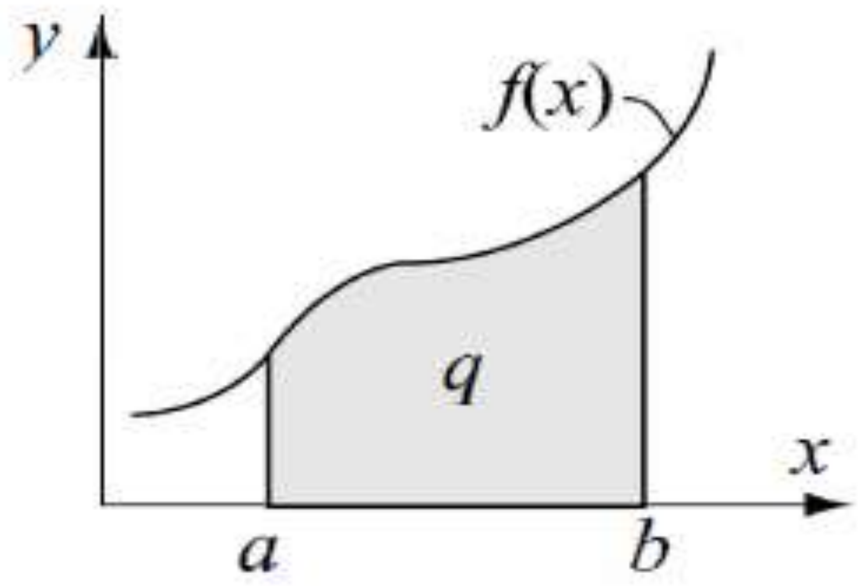
Chapter FIVE

Numerical Integration

NUMERICAL INTEGRATION

- Integration is a common mathematical operation in science and engineering.
- Calculating area and volume, velocity from acceleration, and work from force and displacement are just a few examples where integrals are used.
- Integration of simple functions can be done analytically, but more complex functions are difficult or impossible to integrate analytically.
- In applications of science and engineering the integrand can be a function or a set of data points.
- A definite integral of a function f(x) from a to b has the form:

$$q = \int_a^b f(x)dx$$

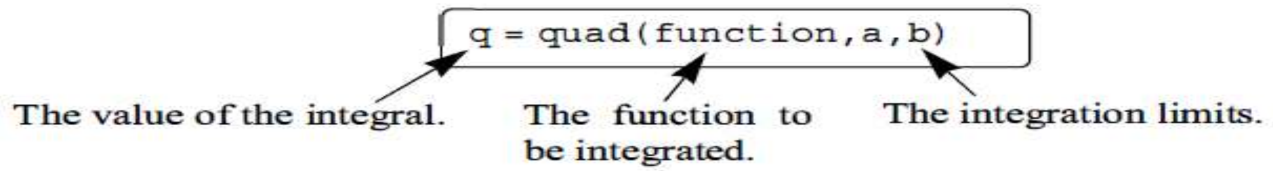


- The function f(x) is called the integrand, and the numbers a and b are the limits of integration.
- Graphically, the value of the integral q is the area between the graph of the function, the x axis, and X the limits a and b.
- When the integral is calculated numerically f(x) can be a function or a set of points.
- In numerical integration the total area is obtained by dividing the area into small sections, calculating the area of each section, and adding them up.
- Use the three MATLAB built-in integration functions quad, quadl, and trapz.

➢ The quad and quadl commands are used for integration when f(x) is a function, and trapz is used when f(x) is given by data points.

The quad command:

✓ The form of the quad command, which uses the adaptive Simpson method of integration, is:



✓ The function can be entered as a string expression or as a function handle.

✓ The function f(x) must be written for an argument x that is a vector (use element-by-element operations).

✓ quad calculates the integral with an absolute error that is smaller than l.0e-6. This number can be changed by adding an optional tol argument to the command:

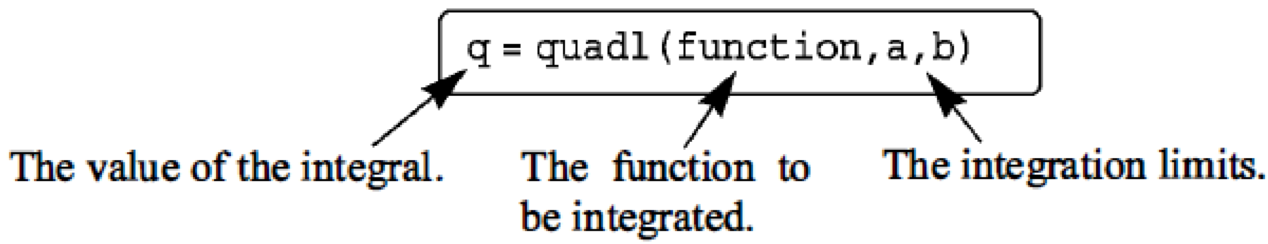
```
q = quad('function',a,b,tol)
```

Where

tol is a number that defines the maximum error. With larger tol the integral is calculated less accurately but faster.

The quadl command:

❑ The form of the quadl (the last letter is a lowercase L) command is exactly the same as that of the quad command:



❑ The difference between the two commands is the numerical method used for calculating the integration.

❑ The quadl command uses the adaptive Lobatto method, which can be more efficient for high accuracies and smooth integrals.

The trapz command:

- The trapz command can be used for integrating a function that is given as data points. It uses the numerical trapezoidal method of integration. The form of the command is

```
q = trapz(x,y)
```

Where

x and y are vectors with the x and y coordinates of the points, respectively. The two vectors must be of the same length.

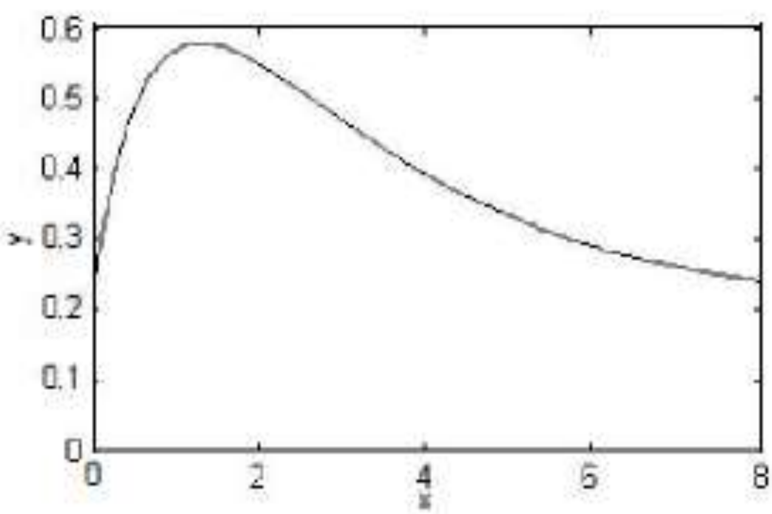
Example 2: Numerical integration of a function

Use numerical integration to calculate the following integral:

$$\int_0^8 (xe^{-x^{0.8}} + 0.2)dx$$

Solution

A plot of the function for the interval $0 \le x \le 8$ is shown on the right.



```
>> quad('x.*exp(-x.^0.8)+0.2',0,8)
ans =
    3.1604
```

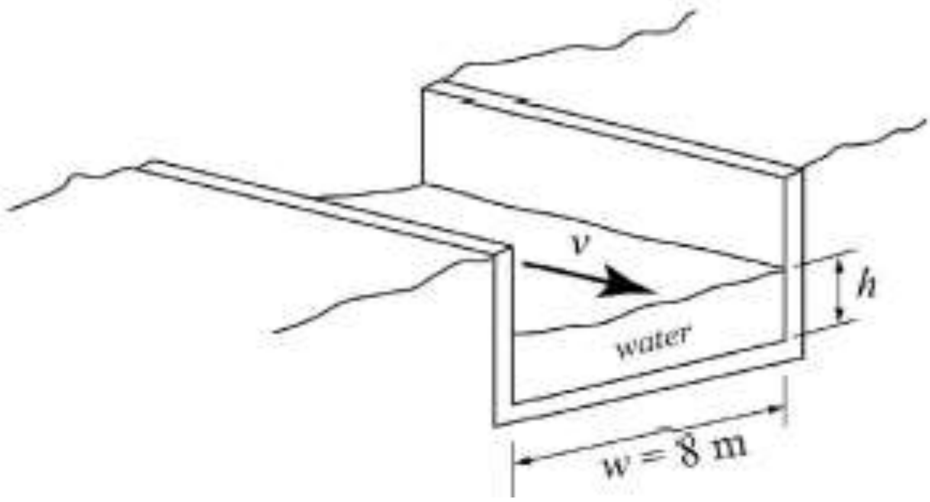
The second method is to first create a user-defined function that calculates the function to be integrated.

```
function y=quadrature(x)
y=x.*exp(-x.^0.8)+0.2;
>> q=quad(@quadrature,0,8)
   q =
      3.1604
```

Example 2 : Water flow in a river

To estimate the amount of water that flows in a river during a year, a section of the river is made to have a rectangular cross section as shown. In the beginning of every month (starting at January 1st) the height h of the water and the speed v of the water flow are measured. The first day of measurement is taken as 1 , and the last day-which is January 1st of the next year-is day 366. The following data was measured:

Day	1	32	60	91	121	152	182	213	244	274	305	335	366
h (m)	2.0	2.1	2.3	2.4	3.0	2.9	2.7	2.6	2.5	2.3	2.2	2.1	2.0
v (m/s)	2.0	2.2	2.5	2.7	5	4.7	4.1	3.8	3.7	2.8	2.5	2.3	2.0



Use the data to calculate

1. the flow rate, and

2. the total amount of water that flows in the river during a year.

Solution:

The flow rate, Q (volume of water per second) is given by:

$$Q = vwh \quad (\mathrm{m^3/s})$$

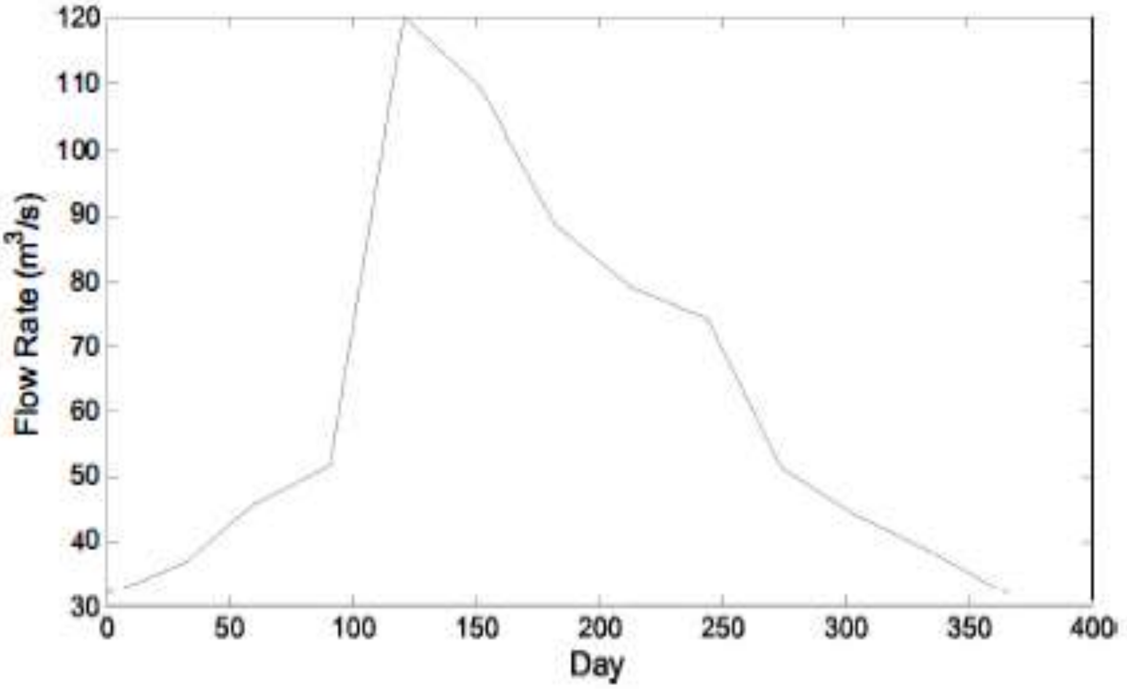
The total amount of water that flows is estimated by the integral:

$$V = (60 \cdot 60 \cdot 24)\int_{t_1}^{t_2} Q dt$$

```
w=8;
d=[1 32 60 91 121 152 182 213 244 274 305 335 366];
h=[2 2.1 2.3 2.4 3.0 2.9 2.7 2.6 2.5 2.3 2.2 2.1 2.0];
speed=[2 2.2 2.5 2.7 5 4.7 4.1 3.8 3.7 2.8 2.5 2.3 2];
Q=speed.*w.*h;
Vol=60*60*24*trapz(d,Q);
fprintf('The estimated amount of water that flows in the
river in a year is %g cubic meters.',Vol)
plot(d,Q)
xlabel('Day'), ylabel('Flow Rate (m^3/s)')
```

```
The estimated amount of water that flows in the river in a
year is 2.03095e+009 cubic meters.
```


```
The estimated amount of water that flows in the river in a
year is 2.03095e+009 cubic meters.
```



Use MATLAB to calculate the following integral:

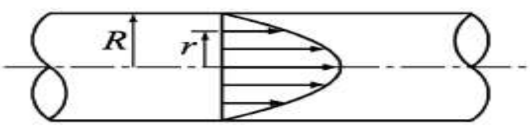
(a) $\int_2^{10} \frac{0.5x^3}{1+2\sqrt{x}}\,dx$ **(b)** $\int_0^9 \left(0.5 + \frac{\cos 1.2x}{(x+2)^2}\right) dx$

Use MATLAB to calculate the following integrals:

(a) $\int_1^8 \frac{e^x}{x^3}\,dx$ **(b)** $\int_0^{4\pi} \cos(x) e^{\sqrt{x}}\,dx$

The flow rate Q (volume of fluid per second) in a round pipe can be calculated by:

$$Q = \int_0^r 2\pi v r\,dr$$



For turbulent flow the velocity profile can be estimated by: $v = v_{max}\left(1 - \frac{r}{R}\right)^{1/n}$. Determine Q for $R = 0.25$ in., $n = 7$, $v_{max} = 80$ in./s.

A cross section of a river with measurements of its depth at intervals of 40 ft is shown in the figure. Use numerical integration to estimate the cross-sectional area of the river.