PHYS-2020: General Physics II
Course Lecture Notes
Section III

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Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 9th Edition* (2012) textbook by Serway and Vuille.

III. Current & Resistance

A. Electric Current.

1. **Current** is defined as the rate at which charge flows through a surface.

a) Mathematically:

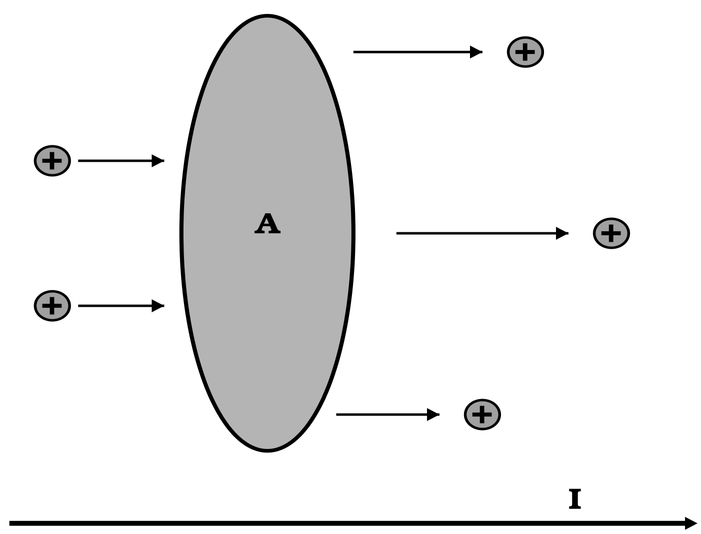
$$\boxed{I = \frac{\Delta Q}{\Delta t}\ ,} \qquad \text{(III-1)}$$

where I is the current, ΔQ is the amount of charge passing through an area of wire, and Δt is the time interval in which ΔQ is measured.

b) Current is measured in **amperes** in the SI unit system:

$$1 \text{ A} \equiv 1 \text{ C/s}\ . \qquad \text{(III-2)}$$

2. The <u>direction</u> of current is defined to be the direction at which a <u>positive</u> charge would flow through a wire.



a) In metals, it is electrons that flow and <u>not</u> positive charges $\Longrightarrow$ the electrons flow in the *opposite* direction of the current!

b) Moving charge (whether positive or negative) through a conductor is known as a mobile **charge carrier**.

3. Electrons flow in the opposite direction of the $\vec{E}$-field.

a) As an electron (or any charged particle) moves through a conductor, it collides with atoms (and/or molecules) in the conductor $\Longrightarrow$ causes a *zigzag* motion through the conductor.

b) The amount of charge passing through a wire can be determined as follows:

i) Let A be the cross-sectional area of a wire and Δx be a small slice along the length of the wire.

ii) The volume of this small segment of the wire is then $V = A\,\Delta x$ (**note** that V here is <u>volume</u> not potential).

iii) Let N be the number of charge carriers contained in this volume and q be the charge per carrier. Then,

$$n = \frac{N}{V} = \frac{N}{A\,\Delta x}\ ,$$

represents the number of carriers per unit volume.

iv) The total charge contained in this volume is thus

$$\Delta Q = Nq = (n\,A\,\Delta x)\,q\ . \qquad \text{(III-3)}$$

c) Although the electron makes a *zigzag* path through the wire, *on average*, it continues to move down the electric field (remember in the opposite sense) at an average speed called the **drift speed** v_d:

$$v_d = \frac{\Delta x}{\Delta t} \quad \Longrightarrow \quad \Delta x = v_d \, \Delta t \ .$$

We can then substitute this into Eq. (III-3) giving

$$\Delta Q = (n \, A \, v_d \, \Delta t) \, q \ . \qquad \text{(III-4)}$$

d) Dividing both sides by Δt gives

$$\frac{\Delta Q}{\Delta t} = I = n \, q \, v_d \, A \qquad \text{(III-5)}$$

or

$$v_d = \frac{I}{n \, q \, A} \ .$$

Since we normally talk about electrons in a metal wire, we can rewrite this as

$$v_d = \frac{I}{n \, |e| \, A} \ . \qquad \text{(III-6)}$$

e) If no current exists in a conductor, the electric field is zero inside the conductor. However, if current exists, an electric field exists inside the conductor (due to Maxwell's laws — see §IX of the notes).

Example III–1. Problem 17.7 (Page 611) from the Serway & Vuille textbook: A 200-km long high-voltage transmission line 2.0 cm in diameter carries a steady current of 1000 A. If the conductor is copper with a free charge density of 8.5×10^{28} electrons per cubic meter, how long (in years) does it take one electron to travel the full length of the cable? *Added question*: How long would it take a photon to travel the same distance?

Solution:

The drift speed of electrons in the line is (from Eq. III-6)

$$v_d = \frac{I}{n \, q \, A} = \frac{I}{n \, |e| \, (\pi D^2/4)} \ ,$$

where $I = 1000$ A $= 1000$ C/s is the current, $n = 8.5 \times 10^{28}$ electrons/m^3 is the charge carrier density, $|e| = 1.60 \times 10^{-19}$ C is the charge per electron, and $D = 2.0$ cm $= 0.020$ m is the diameter of the wire. Solving for the drift velocity gives

$$v_d = \frac{4(1000\text{ C/s})}{(8.5 \times 10^{28}\text{ m}^{-3})(1.60 \times 10^{-19}\text{ C})\pi(0.020\text{ m})^2} = 2.3\times10^{-4}\text{ m/s}.$$

The time to travel the length L of the 200 km $= 2.00 \times 10^5$ m wire is then

$$\Delta t = \frac{L}{v_d} = \frac{2.00 \times 10^5\text{ m}}{2.34 \times 10^{-4}\text{ m/s}} \left(\frac{1\text{ yr}}{3.156 \times 10^7\text{ s}}\right) = \boxed{27.1\text{ yr}.}$$

For the additional question, since light (*i.e.*, photons) travels at a speed of $c = 3.00 \times 10^8$ m/s, the time it takes a photon to travel this length is

$$\Delta t = \frac{L}{c} = \frac{2.00 \times 10^5\text{ m}}{3.00 \times 10^8\text{ m/s}} = 6.67 \times 10^{-4}\text{ s} = \boxed{667\mu\text{s}.}$$

Note that even though an individual electron in the flow of electricity takes a long time to travel through the wire, the E-field inside the wire propagates along the wire at the speed of light.

B. Resistance and Ohm's Law.

1. **Resistance** is the ratio of voltage difference to current:

$$R \equiv \frac{\Delta V}{I}. \qquad \text{(III-7)}$$

a) Measured in **ohms**:

$$\Omega = \frac{\text{V}}{\text{A}} = \frac{\text{volts}}{\text{amperes}}. \qquad \text{(III-8)}$$

b) Resistance measures how hard it is for electrons (or any charged particle) to flow through material $\Longrightarrow$ it essentially measures the number of internal collisions an electron (or any charged particle) has in a circuit.

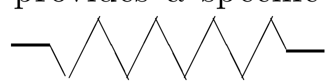
c) The higher the resistance, the more collisions with internal atoms/molecules that make up the wire or resistor. The larger the number of collisions per second, the larger the amount of heat generated in the wire or resistor.

2. **Ohm's Law**: Resistance that remains constant over a wide range of applied voltage differences such that the voltage difference is linearly dependent on current:

$$\boxed{\Delta V = I\,R\ .} \qquad \text{(III-9)}$$

a) Materials that obey this law are called *ohmic* $\Longrightarrow$ conductors are ohmic.

b) Materials that do not obey this law are called *non-ohmic* $\Longrightarrow$ semiconductors are non-ohmic.

3. A **resistor** is a simple circuit element that provides a specific resistance in an electric circuit. It's symbol is 

C. Resistivity.

1. Since resistance is related to the number of collisions an electron has with atoms/molecules of the wire, we can describe resistance in terms of the geometric properties of the conductor and the composition of the conductor (*i.e.*, ohmic material):

$$\boxed{R = \rho\,\frac{L}{A}\ ,} \qquad \text{(III-10)}$$

where L is the length of the conductor, A is the cross-sectional area of the conductor (both of these are the geometric part), and ρ is the resistivity of the material (which is related to the composition of the material — see Table 17.1 in the textbook).

2. Electric conductors have low resistivity, insulators have high resistivity.

3. Resistivities can change with temperature (see §III.D).

Example III–2. Problem 17.17 (Page 611) from the Serway & Vuille textbook: A wire 50.0 m long and 2.00 mm in diameter is connected to a source with a potential difference of 9.11 V, and the current is found to be 36.0 A. Assume a temperature of 20°C and, using Table 17.1, identify the metal of the wire.

Solution:

From Ohm's law (Eq. III-9) we can calculate the resistance of the wire to be

$$R = \frac{\Delta V}{I} = \frac{9.11 \text{ V}}{36.0 \text{ A}} = 0.253 \ \Omega \ .$$

The diameter of the wire is $D = 2.00$ mm $= 2.00 \times 10^{-3}$ m, so the resistivity (using Eq. III-10) of the metal is

$$\begin{aligned} \rho &= \frac{R\,A}{L} = \frac{R\,(\pi D^2/4)}{L} = \frac{(0.253 \ \Omega)\pi(2.00 \times 10^{-3} \text{ m})^2}{4(50.0 \text{ m})} \\ &= 1.59 \times 10^{-8} \ \Omega \cdot \text{m} \ . \end{aligned}$$

Comparing this value to those listed in Table 17.1 of the textbook, the metal must be $\boxed{\text{silver.}}$

D. Temperature Variation of Resistance.

1. For most metals, resistivity increases in an approximate linear fashion with temperature:

$$\rho = \rho_\circ \left[1 + \alpha \left(T - T_\circ\right)\right] \ . \tag{III-11}$$

a) ρ is the resistivity at some temperature T.

b) $\rho_\circ$ is the resistivity at some temperature $T_\circ$.

c) α is the **temperature coefficient of resistivity** (see Table 17.1).

2. If a wire is of constant cross-sectional area A and length L with respect to change in temperature, we can write

$$R = R_\circ \left[1 + \alpha \left(T - T_\circ\right)\right] \ . \tag{III-12}$$

3. Thermometers that measure temperature from resistance in their circuit are called **thermistors** (also called **thermocouples**).

Example III–3. At 40.0°C, the resistance of a segment of gold wire is 100.0 Ω. When the wire is placed in a liquid bath, the resistance decreases to 97.0 Ω. What is the temperature of the bath? (*Hint*: First determine the resistance of the gold wire at room temperature.)

Solution:

If $R = 100.0\ \Omega$ at $T = 40.0°\mathrm{C}$, then the resistance at $T_\circ = 20.0°\mathrm{C}$ (*i.e.*, room temperature) can be found with Eq. (III-12). The thermal resistance coefficient for gold at 20.0°C is $\alpha = 3.40 \times 10^{-3}$ $°\mathrm{C}^{-1}$, so

$$R_\circ = \frac{R}{1 + \alpha(T - T_\circ)}$$

$$= \frac{100.0\ \Omega}{1 + (3.40 \times 10^{-3}\ ^\circ\mathrm{C}^{-1})\,(40.0\ ^\circ\mathrm{C} - 20.0\ ^\circ\mathrm{C})}$$
$$= 93.6\ \Omega\ .$$

Now we once again use Eq. (III-12) to determine the temperature of the bath. After a little algebra, we get

$$T = T_\circ + \frac{R - R_\circ}{\alpha R_\circ} = 20.0\ ^\circ\mathrm{C}\ + \frac{97.0\ \Omega - 93.6\ \Omega}{(3.40 \times 10^{-3}\ ^\circ\mathrm{C}^{-1})\,(93.6\ \Omega)}$$
$$= \boxed{30.6\ ^\circ\mathrm{C}\ .}$$

E. Superconductors.

1. There is a class of metals and compounds whose resistance virtually goes to zero below a certain temperature, T_c, called the **critical temperature** (see Figure 17.11 in the textbook).

2. These materials are known as **superconductors**.

3. Once current has been set up in a superconductor, it persists without any applied voltage (since $R = 0$).

F. Electrical Energy and Power.

1. A battery can deliver **power** to an electric circuit. Power (as covered in General Physics I) is the amount of work exerted over an interval of time:

$$\mathcal{P} = \frac{W}{\Delta t}\ . \qquad \text{(III-13)}$$

2. Since work is equal to the change of potential energy (see Eq. II-3), we can write

$$W = \Delta\mathrm{PE}\ = q\,\Delta V$$

or

$$\mathcal{P} = q \, \frac{\Delta V}{\Delta t} \; . \qquad \text{(III-14)}$$

3. If we have a current of charges Δq across a voltage difference ΔV, we can rewrite Eq. (III-14) as

$$\mathcal{P} = \Delta q \, \frac{\Delta V}{\Delta t} = \Delta V \, \frac{\Delta q}{\Delta t} = (\Delta V) \, I$$

or

$$\boxed{\mathcal{P} = I \, \Delta V \; .} \qquad \text{(III-15)}$$

As we can see from this equation, the unit for power (W = watt) must be equal to amperes times volts, or

$$1 \text{ W} \;=\; \text{A} \cdot \text{V} \; . \qquad \text{(III-16)}$$

4. Using Ohm's law (Eq. III-9), we can also express Eq. (III-15) as

$$\mathcal{P} = I^2 \, R = \frac{(\Delta V)^2}{R} \; . \qquad \text{(III-17)}$$

a) The power delivered to a conductor of resistance R is often referred to as an I^2R *loss*.

b) Note that Eq. (III-17) applies only to resistors (and other ohmic devices) and not to non-ohmic devices like light bulbs and diodes.

c) Electric companies measure the amount of power you have used over the billing cycle (= total energy used). The units used by the electric company are **kilowatt-hour**:

$$1 \text{ kWh} \;=\; (10^3 \text{ W})(3600 \text{ s}) = 3.60 \times 10^6 \text{ J} \; . \qquad \text{(III-18)}$$

Example III–4. Problem 17.34 (Page 612) from the Serway & Vuille textbook: If the electrical energy costs 12 cents per kilowatt-hour, how much does it cost to (a) burn a 100-W lightbulb for 24 hr, and (b) operate an electric oven for 5.0 hr if it carries a current of 20.0 A at 220 V?

Solution (a):

The energy produced by the 100-W lightbulb in one day (24 hr) is

$$E = \mathcal{P} \cdot \Delta t = (100 \text{ W})(24 \text{ hr}) = (0.100 \text{ kW})(24 \text{ hr}) = 2.40 \text{ kWh} ,$$

where kWh = kilowatt-hour. At a rate of $\mathcal{R}$ = 12-cents (= \$0.12) per kWh, the cost of this energy is

$$\begin{aligned} \text{cost} &= E \cdot \mathcal{R} = (2.40 \text{ kWh}) \left(\frac{\$0.12}{\text{kWh}} \right) \\ &= \$0.29 = \boxed{29 \text{ cents.}} \end{aligned}$$

Solution (b):

Using Eq. (III-15), we see that the power of the electric oven is

$$\mathcal{P} = I \, \Delta V = (20.0 \text{ A})(220 \text{ V}) = 4400 \text{ W} = 4.40 \text{ kW} .$$

The energy to run this over for 5.0 hr is

$$E = \mathcal{P} \cdot \Delta t = (4.40 \text{ kW})(5.0 \text{ hr}) = 22 \text{ kWh} .$$

Using this with our cost-rate, we get

$$\begin{aligned} \text{cost} &= E \cdot \mathcal{R} = (22 \text{ kWh}) \left(\frac{\$0.12}{\text{kWh}} \right) \\ &= \boxed{\$2.64 \text{ .}} \end{aligned}$$