Coding Theory

Sheet 6

Spring 2014

1. Which Hamming codes are MDS (maximum distance separable) ?

2. Write out a parity-check matrix and a corresponding generator matrix for

   (a) $\mathrm{Ham}(2,3)$; (b) $\mathrm{Ham}(2,4)$; (c) $\mathrm{Ham}(3,3)$;
   (d) $\mathrm{Ham}(3,4)$; (e) $\mathrm{Ham}(3,5)$; (f) $\mathrm{Ham}(4,2)$.

3. * Use a parity-check matrix for $\mathrm{Ham}(4,2)$, with the columns in lexicographical order, and syndrome decoding to decode

   (a) 00000 00000 11111;

   (b) 00000 11111 11111;

   (c) 11111 11111 11111.

4. * Let $\mathbf{F}_4 = \{0, 1, \omega, \bar{\omega} \mid \bar{\omega} = \omega + 1 = \omega^2\}$. Use $\mathrm{Ham}(3,4)$, with a parity-check matrix having columns in lexicographical order, to decode

   (a) 1111111 1111111 1111111;

   (b) 1111111 $\omega\omega\omega\omega\omega\omega\omega$ $\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}$.

5. For $x, y \in V(n,2)$, let
   $$x \cap y = (x_1 y_1, \ldots, x_n y_n).$$
   Show that $w(x+y) = w(x) + w(y) - 2w(x \cap y)$.

6. If a binary $[n,k]$ code C has parity-check matrix H, show that the extended code C' constructed in Exercise 8 of Sheet 4 has parity check matrix H', where
   $$H' = \begin{bmatrix} H & z^T \\ u & 1 \end{bmatrix},$$
   with $z = 00\cdots 0$ of length $n-k$ and $u = 11\cdots 1$ of length n.

7. If $C = \mathrm{Ham}(r,2)$, show that every non-zero word of $C^\perp$ has weight 2^{r-1}.

   (Hint: Let $H = [h_1, \ldots, h_r]^{\mathrm{T}}$ be a parity check matrix of C with rows $h_1, \ldots, h_r$, and let $h = \sum \lambda_i h_i$ be an element of $C^\perp$; consider the j-th coordinate of h.)

As part of the course assessment, hand in at the School Office solutions to the starred questions, namely 3 and 4, by 2.00 p.m. on Thursday, 20th March. Solutions to all questions will be placed online on Friday, 21st March.