Republic of Iraq
Ministry of Higher Education
And Scientific Research
Mustansiriyah University
College of Science

Advanced Atmospheric Radiation and Remote Sensing

PhD lectures

(2018-2019)

Reference Text Book by;
Fernerkundung atmosp¨arischer Zustandsgr¨oßen Bj¨orn-Martin Sinnhuber ,
University of Berlin, Berlin, Germany 2015

Contents

Chapter 1

Preface

This script summarizes the most relevant parts of the lecture 'Remote Sensing of the Atmosphere ('Fernerkundung atmosphärischer Zustandsgrößen') as given in the summer semester of 2013 at the Karlsruhe Institute of Technology. It is based on previous lectures given at the University of Bremen, partly in collaboration with Astrid Bracher (Alfred Wegener Institute and University of Bremen). Even though we have spent some effort in writing this script, it cannot replace a proper textbook. More importantly, it can also not replace the lecture. Many pictures, graphics and additional explanations are given in the lecture (e.g., in the form of slides shown) that are not included here.

The focus of this lecture is on the fundamentals of atmospheric remote sensing methods, with a particular focus on passive remote sensing techniques. Active techniques are covered only very briefly in the final chapter, that discusses the fundamentals of weather and precipitation radar remote sensing. As the focus of this lecture is on the fundamentals and underlying principles, the technical aspects of the remote sensing systems, like detectors, spectrometers and optical components are not covered in any detail.

Chapter 2

Electromagnetic Radiation

2.1 Maxwell Equations and Electromagnetic Waves

Electromagnetic waves do not need a medium to propagate (no ‘ether’) but are given by the changing electric and magnetic fields. They follow from the Maxwell equations:

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.1}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J \tag{2.2}$$

$$\nabla \cdot D = \rho \tag{2.3}$$

$$\nabla \cdot B = 0 \tag{2.4}$$

with:

E: Electric field
D: Displacement ($D = \epsilon_0 \epsilon_r E$)
H: Magnetic field
B: Induction ($B = \mu_0 \mu_r H$)
J: Current density ($J = \sigma E$)
ρ: Charge density
σ: Conductivity
μ: Permeability ($\mu = \mu_0 \mu_r$)

ϵ: Permittivity (dielectric constant, $\epsilon = \epsilon_0 \epsilon_r$)

Thus we can write:

$$\nabla \times E = -\mu \frac{\partial}{\partial t} H \tag{2.5}$$

$$\nabla \times H = \sigma E + \epsilon \frac{\partial}{\partial t} E \tag{2.6}$$

and with the assumption of $\nabla \cdot E = 0$ and $\nabla \times H = \epsilon \frac{\partial}{\partial t} E$ (because $J \to 0$ at vacuum = no free charges) it follows:

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \tag{2.7}$$

This is a three-dimensional wave equation. Solutions are propagating planewaves. E.g., for the y-component of the electric field we can write:

$$E_y = E_0 e^{-i(kx - \omega t)} \tag{2.8}$$

(and similarly for the x- and z-component) with:
E_0: Amplitude
k: (complex) wave number
ω: angular frequency

Inserting the wave ansatz (eq. 2.8) in vacuum ($\epsilon_r = 1$ and $\sigma = 0$) in eq. 2.7, leads to:

$$k^2 = \omega^2 \mu_0 \epsilon_0 \tag{2.9}$$

For the (phase) velocity of electromagnetic waves in vacuum it thus follows:

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \tag{2.10}$$

This is the speed of light!

In media the speed of light (= velocity of electromagnetic waves) will be reduced:

$$c_r = \frac{c}{\sqrt{\epsilon_r'}} \tag{2.11}$$

Inserting the wave ansatz to the wave equation outside of vacuum leads to:

$$k^2 = \omega^2 \mu \epsilon - i\omega\mu\sigma \tag{2.12a}$$

$$= \omega^2 \mu \left(\epsilon - i\frac{\sigma}{\omega} \right) \tag{2.12b}$$

Most materials of interest have $\mu_r \approx 1$, so we can write:

$$k^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_r' \tag{2.13}$$

with an effective dielectric constant:

$$\epsilon_r' = \epsilon_r - i\frac{\sigma}{\epsilon_0 \omega} \tag{2.14}$$

In vacuum $\epsilon_r = 1$ and $\sigma = 0$, so that

$$k^2 = \omega^2 \mu_0 \epsilon_0 \tag{2.15}$$

The ratio between the speed of light in vacuum and the speed of light in a medium is the refractive index:

$$n = \frac{c}{c_r} = \sqrt{\epsilon_r'} \tag{2.16}$$

The refractive index is thus a complex quantity that we can split into a real and an imaginary part:

$$n = \eta + i\chi \tag{2.17}$$

Writing for k:

$$k = \frac{\omega n}{c} = \frac{\omega \eta}{c} + \frac{i\omega\chi}{c} \tag{2.18}$$

and inserting into the wave ansatz.

$$E_y = E_0 e^{-i(kx - \omega t)} \tag{2.19}$$

gives:

$$E_y = E_0 e^{\frac{\omega\chi x}{c}} e^{-i\left(\frac{\omega\eta x}{c} - \omega t\right)} \tag{2.20}$$

I.e., we see that the imaginary part of the refractive index (χ) describes an attenuation (or damping) of the wave in the medium.

The distance d over which the electric field is fallen off to $\frac{1}{e}$ is thus given by

$$d = \frac{c}{\omega \mid \chi \mid} \tag{2.21}$$

and called the skin depth.

Example:
Sea water at 20°C has a dielectric constant at 10 GHz of $\epsilon_r' = 52 - 37i$:

$$\begin{aligned} n &= \sqrt{\epsilon_r'} = 7.6 - 2.4i \\ \chi &= -2.4 \\ d &= \frac{c}{\omega|\chi|} \\ &= 3 \cdot 10^8 \,\mathrm{m/s} 2\pi \cdot 10 \cdot 10^9 \mathrm{s}^{-1} \cdot 2.4 \\ &= 1.99\,\mathrm{mm} \end{aligned}$$

This means that sea water is virtually opaque at 10 GHz.

Summary electromagnetic waves:
Frequency $\nu = \frac{\omega}{2\pi}$
Wavelength $\lambda = \frac{c}{\nu}$
Wavenumber $k = \frac{2\pi n}{\lambda}$ (if using the refractive index n here, then λ is the vacuum wave length).

Note that wave number is proportional to frequency!

Often another definition of the wave number is introduced as:

$$\tilde{\nu} = \nu / c. \tag{2.22}$$

(The wavenumber $\tilde{\nu}$ is typically expressed in units of cm^{-1}.) This is related to k by $\tilde{\nu} = k/2\pi$. (Similarly as $\nu = \omega/2\pi$.)

Energy of a photon $E = h\nu$, with h Planck's constant. (Please don't confuse with electric field vector!)

Because

$$\tilde{\nu} = \frac{E}{hc} \tag{2.23}$$

wavenumber is also proportional to photon energy.

2.2 Polarization

For electromagnetic waves the vectors for the electric and magnetic fields are perpendicular to each other and also perpendicular to the direction of propagation:

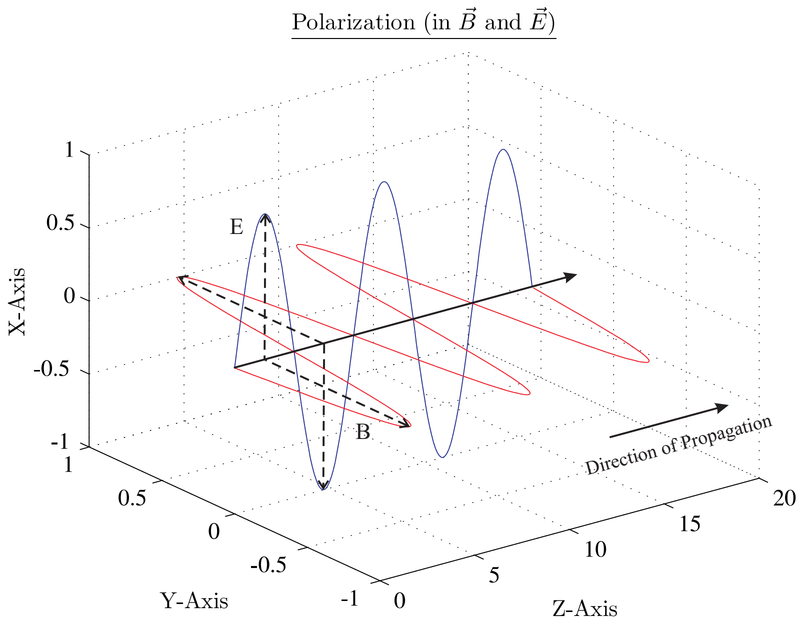

Figure 2.1: Polarization with electric and magnetic field.

The electromagnetic wave is thus linearly polarized (with the E-vector in the x-z-plane and the H-vector in the the y-z-plane and the propagation of the wave in z-direction here). Altough a single electromagnetic wave is linearly polarized, natural light is in general unpolarized (polarization is possible, but only a special case). Two waves with the same frequency and the same propagation direction but different polarization planes for the E-vector superpose to a resulting E-vector. Depending on the phase difference of the two waves the resulting wave will be either linearly, elliptically or circularly polarized. (Individual electromagnetic waves also have a clear phase relationship, i.e., they are coherent. In contrast, natural light is incoherent.)

2.2.1 Stokes Parameter

We can describe the state of polarization by the amplitudes of two orthogonal linear polarizations and the phase difference between them. A more convenient description uses the the so-called 4 Stokes parameters I, Q, U, and V. Assume that we have an instrument that measures light intensity (we will shortly define more precisely what 'intensity' means) using a polarizer where the angle by which we turn the polarizer is given by ϕ:

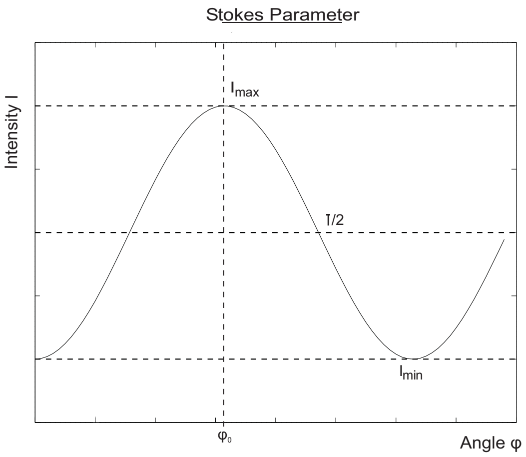

Figure 2.2: The special stokes parameter I.

$$I(\phi) = \frac{1}{2}\left(\bar{I} + \Delta I \cos 2\left(\phi - \phi_0\right)\right) \tag{2.24}$$

with:

$$\bar{I} = I_{\max} + I_{\min} \tag{2.25a}$$
$$\Delta I = I_{\max} - I_{\min} \tag{2.25b}$$

then:

$$Q = \Delta I \cos 2\phi_0 \tag{2.25c}$$
$$U = \Delta I \sin 2\phi_0 \tag{2.25d}$$

so that:

$$I(\phi) = \frac{1}{2}\left(\bar{I} + Q\cos 2\phi + U\sin 2\phi\right) \tag{2.26}$$

Now we introduce a wave plate with retardations ϵ and measure:

$$I(\phi,\epsilon) = \frac{1}{2}\left(\bar{I} + Q\cos 2\phi + (U\cos\epsilon - V\sin\epsilon)\sin 2\phi\right) \tag{2.27}$$

We can determine I, Q, U and V from four measurement of the intensity:

$$I(\phi = 0, \epsilon = 0) = 1/2(I + Q) \tag{2.28a}$$
$$I(\phi = \pi/2, \epsilon = 0) = 1/2(I - Q) \tag{2.28b}$$
$$I(\phi = \pi/4, \epsilon = 0) = 1/2(I + U) \tag{2.28c}$$
$$I(\phi = \pi/4, \epsilon = \pi/2) = 1/2(I - V) \tag{2.28d}$$

Q and U describe the degree of linear polarization and V describes the degree of circular polarization. If light is unpolarized then $Q = U = V = 0$.
We can define the degree of polarization as:

$$\frac{\sqrt{Q^2 + U^2 + V^2}}{I} \tag{2.29}$$

2.2.2 Radiometric Definitions

The intensity or more specifically the flux of energy carried by electromagnetic radiation can be described in several ways.

Side remark: Review of solid geometry and the solid angle:

The treatment of the radiation field requires us to consider the amount of radiant energy with a certain solid angle Ω.

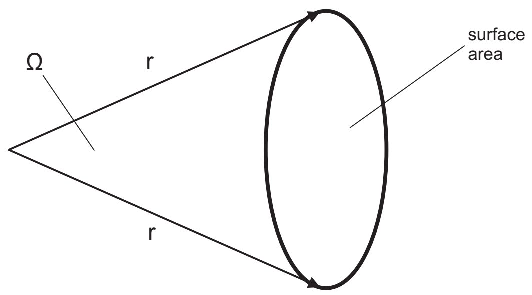

Figure 2.3: The solid angle over the surface area.

The solid angle is defined as,

$$\Omega = \frac{\text{area}}{r^2}$$

and given in sterradians (sr.). The solid angle spanned by a full sphere is $4\pi sr$. This is in analogy with the ordinary plane angle in radians (rad), where the angle ϕ can be defined as,

$$\phi = \frac{\text{length}}{r}$$

and where the angle over a full circle is 2π rad.

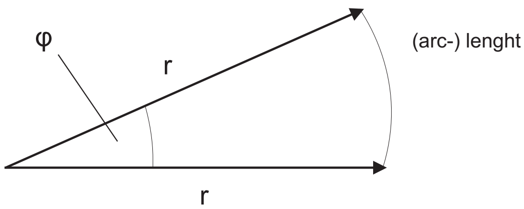

Figure 2.4: Location of the ϕ in plane.

2.2.3 The Radiant Flux

We start our discussion of radiometric definitions by introducing the radiant flux Φ. It gives the rate of energy transported towards or away from a surface in units of Watts ($W = J/s$). As an example, the radiant flux or power emitted by the sun is $\Phi = 3.9 \times 10^{26}$ W.

2.2.4 Monochromatic Radiance or Intensity

The intensity of electro-magnetic radiation or the monochromatic radiance describes the flux of photons of a given frequency per irradiated area and per solid angle.

$$I_\nu = \frac{dE_\nu}{dt\, dA\, d\Omega\, d\nu} \tag{2.30a}$$

$$= \frac{\text{energy}}{\text{time} \cdot \text{area} \cdot \text{solid angle} \cdot \text{frequency}} \tag{2.30b}$$

typically given in units of $\mathrm{J\,s^{-1}\,m^{-2}\,sr^{-1}\,Hz^{-1} = W\,m^{-2}\,sr^{-1}\,Hz^{-1}}$ with:

E_ν: (monochromatic) radiant energy (J)
t: time (s)
A: irradiated area ($\mathrm{m^2}$)
Ω: solid angle (sr)
ν: frequency ($\mathrm{Hz} = \mathrm{s^{-1}}$)

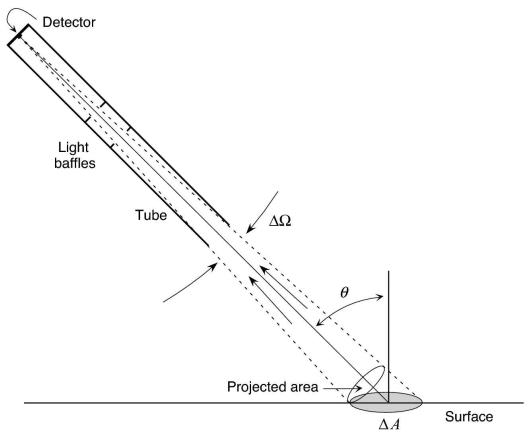

Figure 2.5: Schematic of a radiance meter viewing the surface. From Martin, An Introduction to Ocean Remote Sensing, Cambridge.

2.2.5 Monochromatic Irradiance

(Also known as flux density.)

$$F_\nu = \int_{2\pi} I_\nu \cos\theta \, d\Omega \tag{2.31}$$

with units of: $\mathrm{Wm^{-2}Hz^{-1}}$.

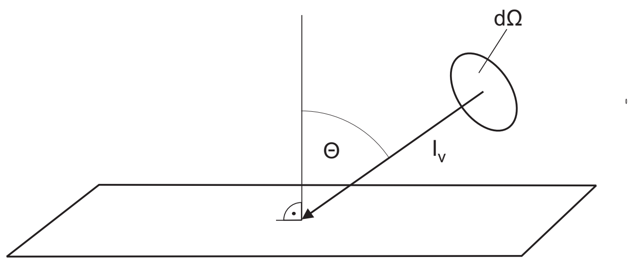

Figure 2.6: The orientaion of $d\Omega$ in the flux density.

I.e., the irradiance (flux density) is the normal component of the radiance (intensity) integrated over one hemisphere.

2.2.6 Total Flux Density

The total flux density (also called total irradiance) is obtained by integrating the flux density over all frequencies:

$$F = \int_0^\infty F_\nu \, d\nu \tag{2.32}$$

with units of: $\mathrm{W\,m^{-2}}$.

2.2.7 Total Flux

Finally, we can return at the total flux or radiant flux, as introduced above, by integrating the total flux density over the irradiated surface area:

$$\Phi = \int F \, dA \tag{2.33}$$

with units of W.

Caution: The definition of these quantities (radiance, irradiance, flux, etc.) may vary in different text books!

2.3 Black Body Radiation, Planck's Law

All bodies with $e \neq 0$ emit radiation, called thermal radiation. For a perfectly black body (i.e., $\alpha = e = 1$) the emitted radiance I_ν is given by Planck's law:

$$I_\nu = L_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{2.34}$$

typically with units of $\mathrm{W\,m^{-2}\,sr^{-1}\,Hz^{-1}}$, or expressed in terms of wavelength rather than frequency:

$$L_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{2.35}$$

in units of $\mathrm{W\,m^{-3}\,sr^{-1}}$. Here k is the Boltzmann constant

$$k = 1.38 \cdot 10^{-23}\,\mathrm{J\,K^{-1}} \tag{2.36}$$

and h is Planck's constant

$$h = 6.63 \cdot 10^{-34}\,\mathrm{Js} \tag{2.37}$$

(From $\nu = \frac{c}{\lambda}$ follows $d\nu = \frac{-c}{\lambda^2}\,d\lambda$ and thus

$$L_\nu\,d\nu = L_\lambda\,d\lambda \tag{2.38}$$

.

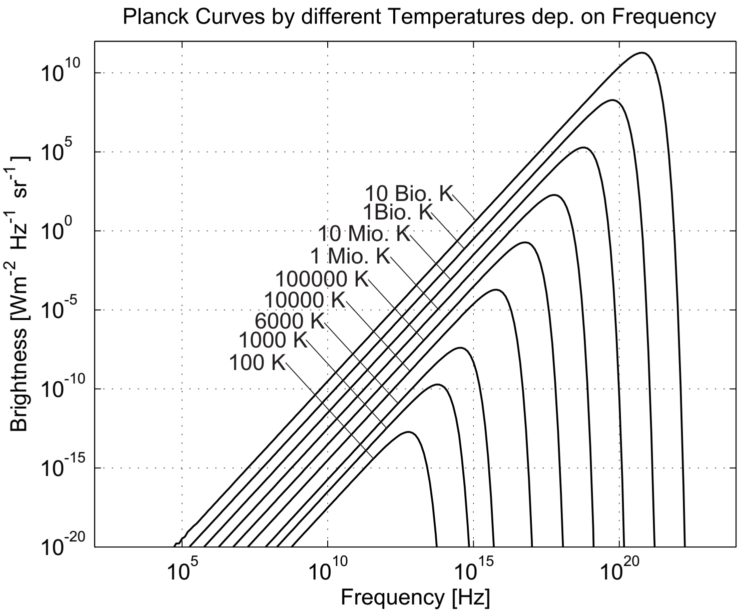

Figure 2.7: Planck curves for several temperatures in dependence to the frequency.

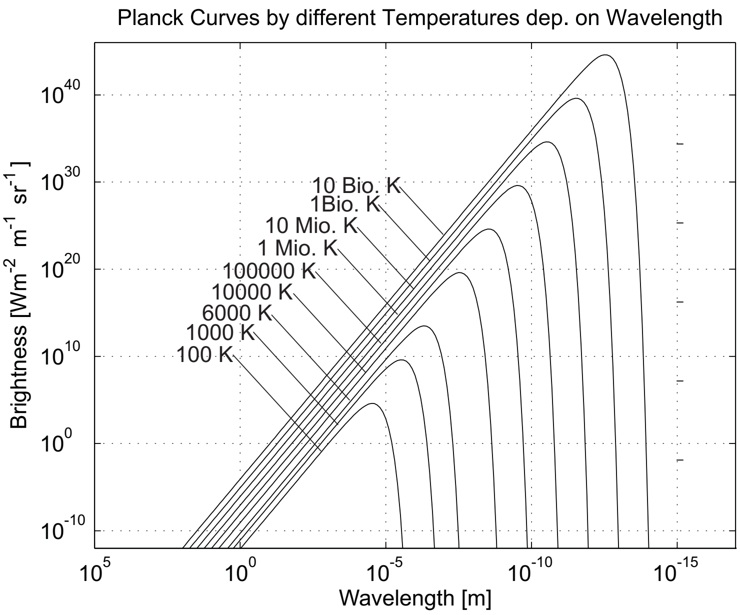

Figure 2.8: Planck curves for several temperatures in dependence to the wavelength.

The maximum of the Planck curve is at

$$\nu_{\max} = 0.941 \frac{3kT}{h} \text{ [Hz]} \tag{2.39}$$

or

$$\lambda_{\max} = \frac{2.897 \cdot 10^{-3}}{T} \text{ [m]} \tag{2.40}$$

This is Wien's displacement law.

Examples: For $T \approx 300\,\mathrm{K}$ (typical temperature of the Earth) we get $\lambda_{\max} \approx 10\,\mu\mathrm{m}$ and for $T \approx 6000\,\mathrm{K}$ (approximately the temperature of the sun) we get $\lambda_{\max} \approx 480, \mathrm{nm}$.

Integrating the Planck function over all wavelength (or frequencies) we get the Stefan-Boltzmann law:

$$L = \int_0^\infty L_\lambda \, d\lambda = \frac{2k^4\pi^4}{15c^2h^3} T^4 \tag{2.41}$$

and by integrating over all directions we get:

$$\int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta \cos\theta L \, d\theta = \sigma T^4 \tag{2.42}$$

(units: $\mathrm{W\,m^{-2}}$) with:

$$\sigma = 5.670 \cdot 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}} \tag{2.43}$$

.

2.4 Brightness Temperature

At microwave frequencies (more specifically for $h\nu \ll kT$) the exponential in the Planck function can be approximated by:

$$e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT} \tag{2.44}$$

so that:

$$L_\nu \approx \frac{2k\nu^2}{c^2} T \tag{2.45}$$

This is the Rayleigh-Jeans approximation.

Side remark: Note that the Rayleigh-Jeans law does not contain the Planck constant! This is historically of interest, as the Rayleigh-Jeans approximation was formulated before the Planck law.

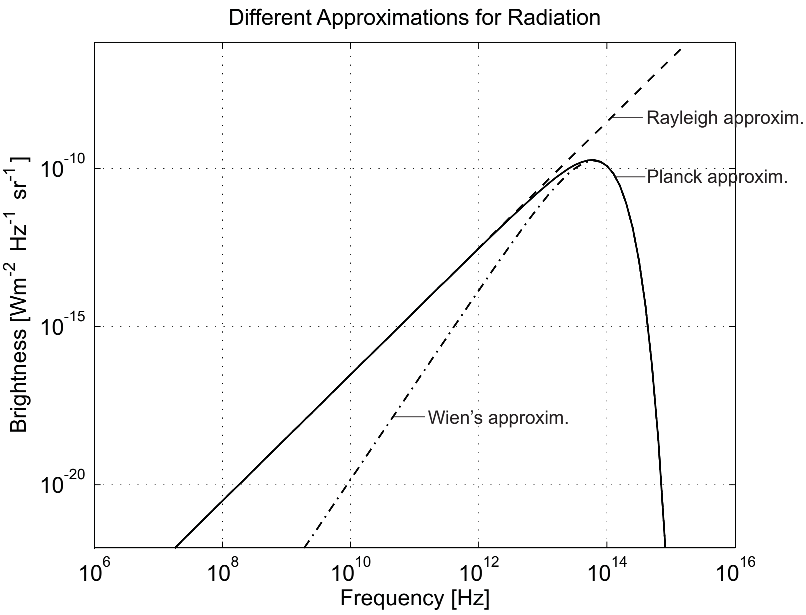

Figure 2.9: The different approximations for black-body radiation by Planck, Rayleigh and Wien.

In this regime ($h\nu \ll kT$) radiance is proportional to temperature. I.e., we can use temperature to measure radiance. By using the Rayleigh-Jeans approximation, we can define the so called black-body brightness temperature.

$$T_B = \frac{c^2}{2k\nu^2} L_\nu \tag{2.46}$$

This is the temperature a black body would have emitting the same radiance.

2.5 Interaction of Electromagnetic Radiation with Matter

Matter can react with electromagnetic radiation by:

- reflection
- absorption

- transmission
- emission

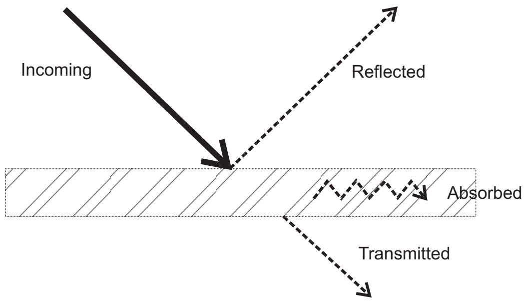

Figure 2.10: The interaction of electromagnetic radiation with matter.

The fraction of the incoming radiation being reflected is described by the reflectivity coefficient ρ that is dimensionless and between 0 and 1. E.g., a reflectivity of $\rho = 1$ means all incoming radiation is reflected. Similarly there are the absorptivity (α), emissivity (e) and transmissivity (τ) coefficients.

The sum of reflection, absorption and transmission is 1:

$$\alpha + \rho + \tau = 1 \tag{2.47}$$

Furthermore, from Kirchhoff's law follows that always the emissivity coefficient equals the absorptivity coefficient.

$$e = \alpha \tag{2.48}$$

I.e., a body can only emit radiation where it also absorbs radiation.