Chapter 3

Atmospheric Radiative Transfer

3.1 Scattering

Scattering changes only the direction of a photon, but does not destroy photons nor does it create any new photons. For the radiance in a given direction, scattering can be lead to a reduction, similar to absorption. For absorption:

$$\left(\frac{\partial I}{\partial s}\right)_{\text{absorption}} = -\alpha \cdot I \tag{3.1}$$

Similar for scattering out:

$$\left(\frac{\partial I}{\partial s}\right)_{\text{scattering out}} = -\alpha_s \cdot I \tag{3.2}$$

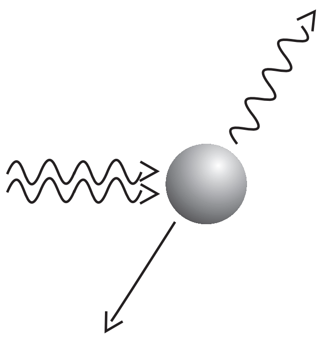

Figure 3.1: Scattering out at particle.

However, scattering can also lead to an intensification of the radiance in a given direction by scattering in of radiation from other directions.
For scattering in:

$$\left(\frac{\partial I}{\partial s}\right)_{\text{scattering in}} = +\alpha_s \cdot \frac{1}{4\pi} \int_0^{4\pi} I(\Omega')P(\Omega, \Omega')d\Omega' \tag{3.3}$$

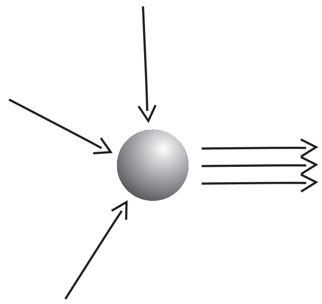

Figure 3.2: Scattering in at particle.

With P the so-called phase function, describing the probability for scattering in a given direction. Instead of giving $P(\Omega, \Omega')$ (i.e. as a function of incident and scattering solid angle) one can describe $P(\Theta)$, where Θ is the angle between an incident (incoming) and scattered light.

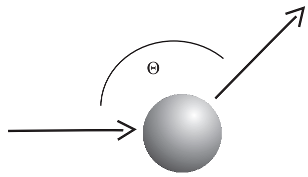

Figure 3.3: Scattering at particle with angle Θ.

Different regimes for scattering exist, depending on the wavelength of the radiation and the size of the scattering particles (air molecules, aerosols, rain drops, etc.).

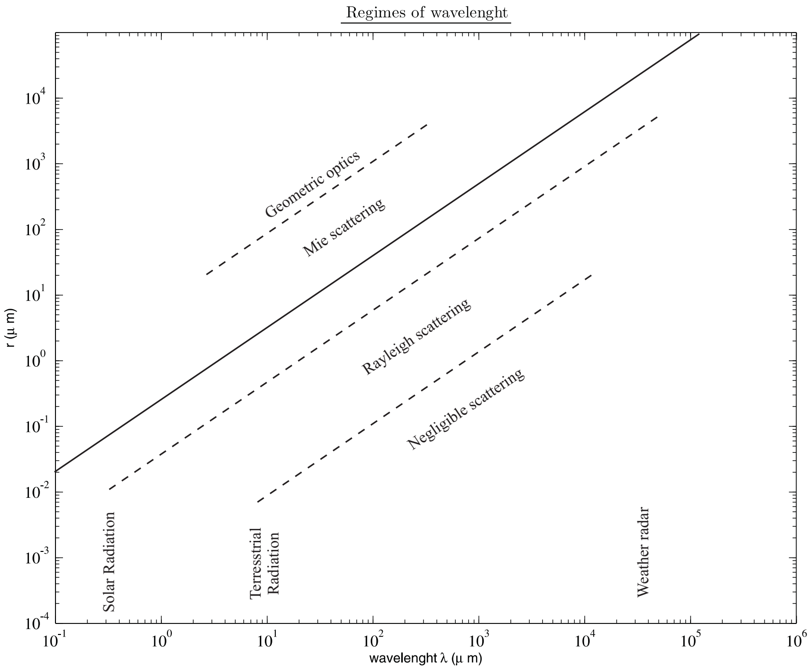

Figure 3.4: Different Scattering Regimes.

Define the size parameter:

$$\alpha = \frac{2\pi r}{\lambda} \tag{3.4}$$

with: λ wavelength and r size (radius) of particle.

For $\alpha \ll 1$ Rayleigh scattering
For $\alpha > 1$ Mie scattering (Mie-theory works for special particles only)
For $\alpha \gg 1$ Geometric optics

3.2 Rayleigh Scattering

Rayleigh scattering is the limiting case for the Mie-theory for $\alpha \ll 1$.

The cross section for Rayleigh scattering is given as:

$$\sigma_{\text{Rayleigh}} = N \frac{2\pi^5}{3} \frac{d^6}{\lambda^4} \left(\frac{n^2 - 1}{n^2 + 2} \right)^2 \tag{3.5}$$

with n the refractive index of the medium, λ the wavelength of radiation, d the diameter of the particles ($d = 2r$) and N the number density of scattering particles (number of particles per unit volume). Note the strong λ^{-4} wavelength dependence of the Rayleigh cross section. Because there is about a factor of two in wavelength between blue/violet ($\approx$ 400nm) light and red light ($\approx$ 800nm), the blue radiation will be scattered about 16 times more effective! ($\Rightarrow$ blue sky)

The phase function of Rayleigh scattering is given by:

$$p_{\text{Rayleigh}}(\Theta) = \frac{3}{4}(1 + \cos^2 \Theta) \tag{3.6}$$

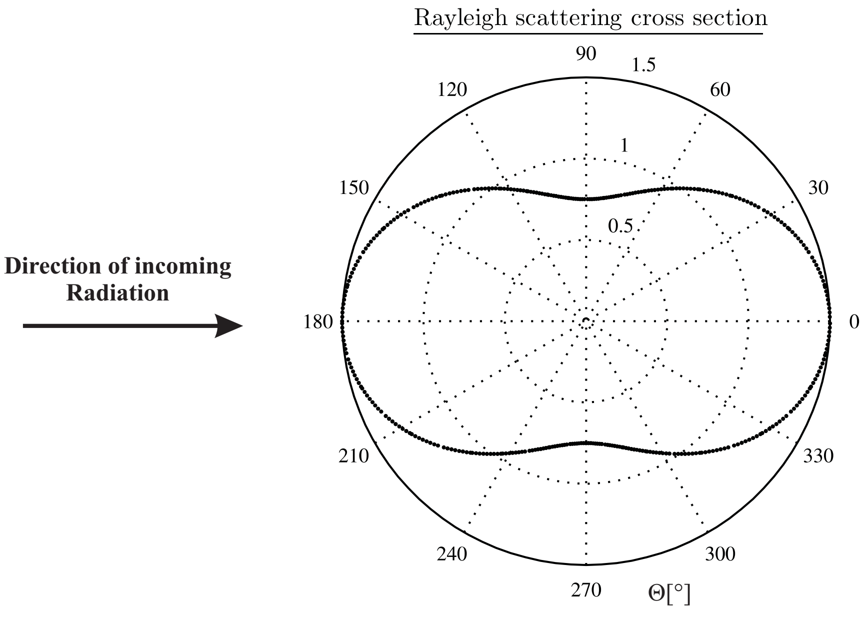

Figure 3.5: Phase Function of Rayleigh scattering.

Note the symmetry between forward and backward scattering.

3.3 Mie Scattering

Mie-theory is applicable for spherical particles only. Mie-theory provides scattering cross section and phase function as a function of:

1. refractive index
2. size parameter: $\alpha = \frac{2\pi r}{\lambda}$
3. scattering angle "Θ"

For typical atmospheric particles (aerosols, clouds) the scattering cross section σ_{Mie} has only a slight wavelength dependence. ($\Rightarrow$ white/gray clouds)

The phase function shows a strong forward peak for longer particles.

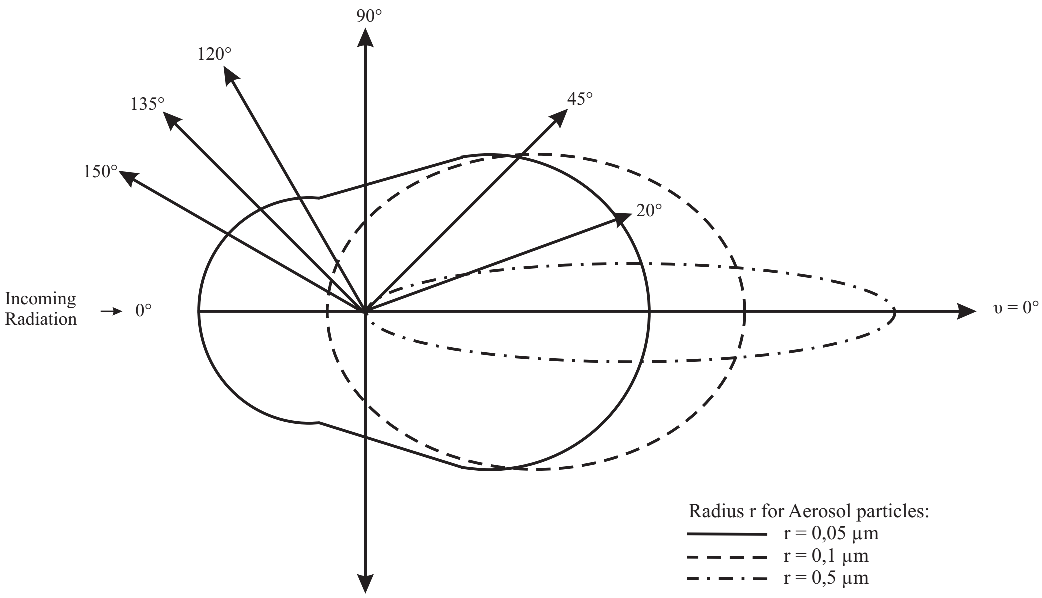

Figure 3.6: Scattering angles in Aerosols depending on the radius.

The asymmetry parameter g describes the ratio of forward to backward scattering:

$$g = \int_{-1}^{+1} p(\Theta) \cos\Theta \; d\cos\Theta \tag{3.7}$$

For pure forward scattering ($p(\Theta) = \delta(\Theta)$):

$$g = \int \delta(\Theta) \cos\Theta \; d\cos\Theta = \cos(0) = +1 \tag{3.8}$$

For pure backward scattering $g = -1$.
For Rayleigh scattering $g = 0$.

3.4 Radiative Transfer Equation

- absorption:
$$\left(\frac{dI}{ds}\right)_{\text{absorption}} = -\alpha_{\text{abs}} \cdot I \tag{3.9}$$

- emission:
$$\left(\frac{dI}{ds}\right)_{\text{emission}} = +\alpha_{\text{abs}} \cdot \underbrace{J}_{\text{source}} \tag{3.10}$$

  In particular for thermal emission in Local Thermodynamic Equilibrium (LTE):
$$\left(\frac{dI}{ds}\right)_{\text{thermal}} = +\alpha_{\text{abs}} \cdot \underbrace{L_\lambda(T)}_{\text{Planck function}} \tag{3.11}$$

- scattering out:
$$\left(\frac{dI}{ds}\right)_{\text{out}} = -\alpha_{\text{scat}} \cdot I \tag{3.12}$$

- scattering in:
$$\left(\frac{dI}{ds}\right)_{\text{in}} = +\alpha_{\text{scat}} \cdot \underbrace{\frac{1}{4\pi}\int_0^{4\pi} I \; p \; d\Omega}_{\text{J}_{scattering} \text{ source function}} \tag{3.13}$$

Taking these processes together leads to:

$$\frac{dI}{ds} = -(\alpha_{\text{abs.}} + \alpha_{\text{scatt.}}) \cdot I + \alpha_{\text{abs.}} \cdot L_\lambda(T) + \alpha_{\text{scatt.}} \cdot \frac{1}{4\pi}\int_0^{4\pi} I \cdot p \cdot d\Omega \tag{3.14}$$

This is the Radiative Transfer Equation (RTE).

3.5 Radiative Transfer without Scattering - Applications for Passive Microwave Sensing of Atmospheric Constituents

Without scattering, the Radiative Transfer Equation can be written as:

$$\frac{dI}{ds} = \underbrace{-\alpha}_{\text{abs. coeff.}} \cdot \underbrace{I}_{\text{radiance}} + \alpha \cdot \underbrace{L}_{\text{Planck function}} = -\alpha\,(I - L) \quad (3.15)$$

which is a good approximation for microwave or infra-red radiation.

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If we ignore the thermal emission for a moment, the RTE further simplifies to:

$$\frac{dI}{ds} = -\alpha \cdot I \quad (3.16)$$

Which can be solved as:

$$I = I_0 \cdot \exp(-\alpha \cdot s) \qquad \text{(Beer-Lambert-law)} \quad (3.17)$$

if α is constant (independent of the light path s).

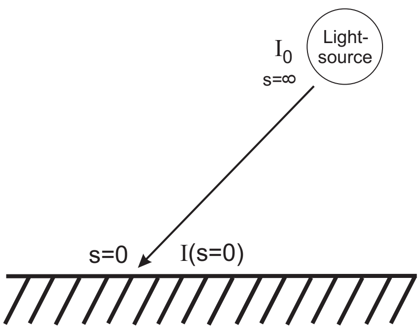

Figure 3.7: The way of radiation and the intensity.

If α is not constant:

$$\frac{dI}{ds} = -\alpha(s) \cdot I \tag{3.18a}$$

$$\rightarrow \int \frac{dI}{I} = -\int \alpha(s)\ ds \tag{3.18b}$$

$$\rightarrow \ln \frac{I}{I_0} = -\int \alpha(s)\ ds \tag{3.18c}$$

$$\rightarrow I = I_0 \cdot \exp\left(-\int_0^s \alpha(s')\ ds'\right) \tag{3.18d}$$

The term

$$\tau(s) = \int_0^s \alpha(s')\ ds'$$

is called the optical thickness or <u>optical depth</u>. The factor

$$T_\nu(s) = \exp\left(-\int_0^s \alpha(s')ds'\right)$$
$$= e^{-\tau(s)}$$

is called the <u>transmission</u>. We can also write the transmission as

$$T_\nu(s, s') = \exp\left(-\int_s^{s'} \alpha(s'')ds''\right)$$

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Including the effect of thermal emission, the RTE becomes

$$I(s) = \underbrace{I_0 \cdot e^{-\tau(s)}}_{\text{Background radiation transmitted through the whole atmosphere.}} + \int_0^s \underbrace{\alpha(s') \cdot L_\nu(T(s')) \cdot e^{-\tau(s')} \cdot ds'}_{\text{Thermal emission from layer s', transmitted through the atmosphere between layer s' and the observer at s.}} \tag{3.19}$$

It is convenient to replace the distance s by the altitude z.

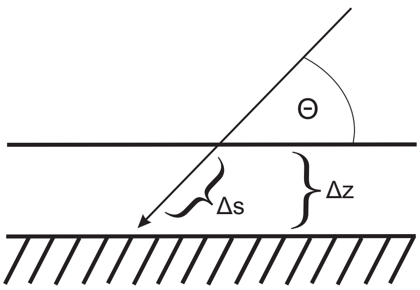

Figure 3.8: The radiance through a plane parallel atmosphere with the altitude z.

For a plane parallel to the atmosphere:

$$ds = \mu dz \tag{3.20}$$

with $\mu = 1/\sin\Theta$. Here μ is called the geometric air mass factor.

(For a spherical atmosphere, a good approximation is given by:

$$\mu(z) = \frac{1 + z/a}{\sqrt{\sin^2\Theta + (2z/a) + (z^2/a^2)}}$$

while a is the earth's radius.)

With this the RTE changes to:

$$I(z) = I_0 \cdot e^{-\tau(z)} + \int_0^z \alpha(z') \cdot \mu(z') \cdot L_\nu(z') \cdot e^{-\tau(z')} \cdot dz'$$

and

$$\tau(z) = \int_0^z \alpha(z') \cdot \mu(z') \cdot dz'$$

Note that because the derivative of

$$T_\nu = \exp\left(-\int \alpha \cdot \mu \cdot dz'\right)$$

is given by

$$\frac{dT_\nu}{dz} = -\alpha \cdot \mu \cdot \exp\left(-\int \alpha \cdot \mu \cdot dz'\right),$$

the RTE can now be written as:

$$I(z) = I_0 \cdot T_\nu(0,z) + \int_0^z L_\nu(T(z')) \cdot \frac{dT_\nu(0,z')}{dz'} \cdot dz' \tag{3.21a}$$

$$= I_0 \cdot T_\nu(0,z) + \int_{T_\nu=0}^{T_\nu=1} L_\nu \cdot dT_\nu \tag{3.21b}$$

This form of RTE can be easily discretized and further be solved, i.e. numerically. Define absorption coefficients and Planck-Function on a grid of N discrete levels.

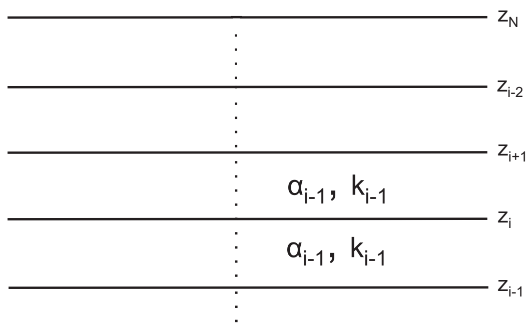

Figure 3.9: The N discrete levels of z_i.

The transmission T_ν can the be calculated as:

$$T_i = \exp\left(-\sum_{j=0}^{i-1} \alpha_j \mu_j \Delta z\right) \tag{3.22}$$

and

$$I(z) = \underbrace{I_0) \cdot Tr(z=\infty)}_{= I_N \cdot \tau_N} + \sum_{i=1}^{N} L_i \left(Tr_i - Tr_{i-1}\right) \tag{3.23}$$

3.6 Weighting Function for Microwave and Infrared Satellite Nadir Sounding

As a simple example we will consider in this section the retrieval of atmospheric temperature by a nadir looking microwave satellite sounder. I.e., the instrument measures brightness temperatures emitted by the Earth surface and the atmosphere. In order to simplify the example, we will consider a case (i.e., a spectral region) where the atmospheric absorption is primarily due to well mixed cases such as O_2 or CO_2.

Remember the RTE without scattering:

$$\frac{dI_\nu}{dz} = -\alpha_\nu I_\nu + \alpha_\nu L_\nu \tag{3.24a}$$

$$= -\alpha(I_\nu - L_\nu) \tag{3.24b}$$

(Also known as "Schwarzschild" equation.)

With $T_\nu(z, z')$ as the transmission between z and z' is given by:

$$T_\nu(z, z') = \exp\left(-\int_z^{z'} \alpha(z")dz"\right) \tag{3.25}$$

Now, the RTE can be solved to give the upward directed radiance at the altitude z'. This equation is given by:

$$I_\nu(z) = I_\nu(0)\ T_\nu(0, z)\ + \int_0^z L_\nu(z')\frac{\partial T_\nu(z, z')}{\partial z'}dz' \tag{3.26}$$

This is what the a satellite nadir sounder in the microwave or infrared spectral regin will observe (for $z \to \infty$). For a nadir looking satellite sensor this can be written as:

$$I_\nu(\infty) = I_\nu(0)\ T_\nu(0, \infty)\ + \int_0^\infty L_\nu(z)K_\nu(z)dz \tag{3.27}$$

with the Weighting Functions $K(z)$ defined as:

$$K_\nu(z) = \frac{\partial T_\nu(z, \infty)}{\partial z}. \tag{3.28}$$

We can write the measured intensity and the radiative transfer equation also in terms of brightness temperature:

$$T_{b,\nu}(\infty) = \epsilon T_{\text{surface}} T_\nu(0, \infty) + \int_0^\infty T(z)K_\nu(z)\,dz \tag{3.29}$$

where $T_{b,\nu}(\infty)$ is the measured brightness temperature at frequency ν, T_{surface} the (actual, physical) temperature of the Earth surface and ϵ the emissivity of the Earth surface ($\epsilon = 1$ if the surface is a black body). $T(z)$ is the temperature of the atmosphere at hight z, while $T_\nu(0, \infty)$ is the total transmission of the atmosphere at frequency ν between the surface ($z = 0$) and the top of the atmosphere ($z = \infty$).

This means that the observed signal will be the sum of a contribution from the emission of the surface (attenuated by the atmospheric transmission) plus the contributions from the individual atmospheric heights, weighted by the corresponding Weighting Function. This is the meaning of the Weighting Function: It will determine how much one atmospheric layer (the temperature of this layer in this example here) contributes to the measured signal (the measured brightness temperature in this example).

For the idealized example discussed in this section we can find an analytical expression for the weighting functions, as we will see in the following. In more realistic cases, however, the weighting functions can only be calculated numerically from the discretized radiative transfer equation.

If the absorption results form a uniformly mixed compound (such as CO_2 or O_2), the absorption coefficient can be assumed to be of the form :

$$\alpha_\nu(z) = \sigma_\nu n(z) \tag{3.30}$$

With the number density profile $n(z)$ decreasing exponetially with the height, given by:

$$n(z) = n(0) \exp\left(-\frac{z}{H}\right) \tag{3.31}$$

(scale height H, typically about 7km)

That means:

$$\alpha_\nu(z) = \alpha_\nu(0) \exp\left(-\frac{z}{H}\right) \tag{3.32}$$

then the transmission will be given by:

$$T_\nu(z, \infty) = \exp(-\alpha_\nu(z)H) \tag{3.33}$$

and the Weighting Function are given by:

$$K_\nu(z) = \frac{\partial T_\nu(z, \infty)}{\partial z} = \alpha_\nu(z) \exp(-\alpha_\nu(z)H) \tag{3.34}$$

These functions have their maximum at z_{max} when:

$$\alpha(z_{max})H = 1 \quad \Rightarrow \quad z_{max} = H \ln(\alpha_\nu(0)H) \tag{3.35}$$

At the maximum the Weight Function have the value:

$$K_\nu(z_{max}) = \frac{1}{eH} \approx 0,05/km \tag{3.36}$$

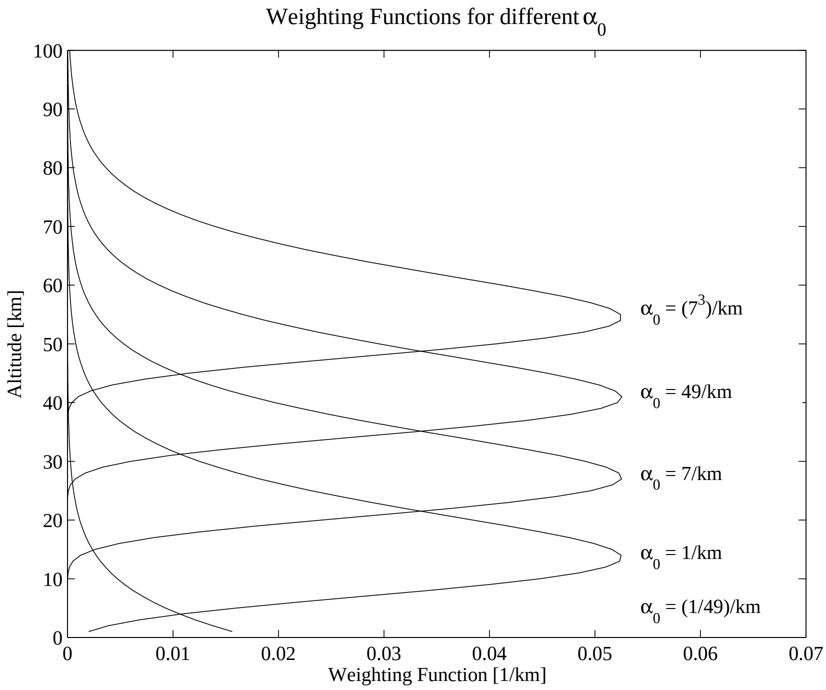

Figure 3.10: Idealized weighting functions for a nadir sounder, as given by eq. (4.11) with a scale height of $H = 7$km.