Lecture 3

Variable Separation Method

The method of separation of variables combined with the principle of super position is widely used to solve initial boundary value problems involving Linear partial differential equations .

Usually , the dependent variable U (x, y) is expressed in the separable form

$U(x, y) = X(x)\ Y(y)$

Where X and Y are functions of x and y .

In many cases , the particle defferential equation reduces to two ordinary differential equations for X and Y .

The method is widely used in finding solutions of a large class of initial boundary value problems .

This method of solution is also as the method Eigen function expansion .

$U(x, y) = X(x)\ Y(y)$

$d\ U(x, y)/ dx – d\ U(x, y)/dy = 0$ by substitution where X is function of y and Y is function of y .

$dX(x)\ dY(y)/ dx – dX(x)\ dY(y)/dy = 0$ by divided $X(x)\ Y(y)$

$1/X(x)\ dX(x)/ dx – 1/ Y(y)\ dY(y)/dy = 0$

$1/X(x)\ dX(x)/ dx = C$

$dX(x)/X(x) = C\ dx$

$\ln X(x) = C\ X + G$

$X= e^{CX+G}$ ....$X = Ae^{CX}$

$1/ Y(y)\ dY(y)/dy = C$

$dY/Y(y) = C\ dy$

$\ln Y= Cy +R$

$Y= e^{CY+R}$.......$Y= Be^{CY}$

A, B are constant ....AB= D

$U(x, y) = X(x)\, Y(y)$

$U(x, y) = Ae^{CX}. Be^{CY} = AB\, e^{C(X+Y)}$

$U(x, y) =D\, e^{C(X+Y)}$

Classical Mechanics:

The classical mechanics describes the behavior of objects in terms of two equations .

One equation expresses the fact that the total energy is constant in the absence of external forces .

The other equation expresses the response of particles to the forces on them .

Velocity = $X_2-X_1/ t_2-t_1 = dx/ dt = \dot{X}$

Acceleration = $\dot{X}_2-\dot{X}_1/ t_2-t_1 = d^2X/dt^2 = d\dot{X}/ dt = \ddot{X}$

$F(x)= m \ddot{X}$

F= force , m= mass , $\ddot{X}$ = acceleration

$F(q) = m\, q..$