Lecture 1

Some Introductory Principles

1.1 Introduction

Dynamic meteorology is the study of those motions of the atmosphere that are associated with weather and climate. For all such motions, the discrete molecular nature of the atmosphere can be ignored, and the atmosphere can be regarded as a continuous fluid medium, or continuum. A “point” in the continuum is regarded as a volume element that is very small compared with the volume of atmosphere under consideration, but still contains a large number of molecules. The expressions air parcel and air particle are both commonly used to refer to such a point. The various physical quantities that characterize the state of the atmosphere (e.g., pressure, density, temperature) are assumed to have unique values at each point in the atmospheric continuum. Moreover, these field variables and their derivatives are assumed to be continuous functions of space and time. The fundamental laws of fluid mechanics and thermodynamics, which govern the motions of the atmosphere, may then be expressed in terms of partial differential equations involving the field variables as dependent variables and space and time as independent variables.

The general set of partial differential equations governing the motions of the atmosphere is extremely complex; no general solutions are known to exist. To acquire an understanding of the physical role of atmospheric motions in determining the observed weather and climate, it is necessary to develop models based on systematic simplification of the fundamental governing equations. As shown in later chapters, the development of models appropriate to particular atmospheric motion systems requires careful consideration of the scales of motion involved.

1.2 Vector Analysis

i. Some definitions:

- Spherical Coordinates System: The spherical coordinate system divides the Earth into longitudes (meridians) and latitudes (parallels). The Prime Meridian, which runs through Greenwich, United Kingdom, is defined to have longitude 0^o. The Equator is defined to have latitude 0^o. On the spherical coordinate grid, the west–east distance between meridians is the greatest at the Equator and converges to zero at both poles.

ii. Conversion Spherical to Cartesian coordinates:

Conversions between increments of distance in Cartesian coordinates and increments of longitude or latitude in spherical coordinates, along the surface of Earth, are obtained from the equation for arc length around a circle. In the west–east and south–north directions, these conversions are:

$$dx = (R_e \, cos\phi) d\lambda_e \qquad dy = R_e \, d\phi \tag{1.1}$$

where $R_e = 6371$ km is the radius of the Earth, $d\lambda_e$ is a west-east longitude increment (radians), $d\phi$ is a south-north latitude increment (radians).

Example 1.1: If a grid cell has dimensions $d\lambda_e = 5^o$ and $= 5^o$, centered at $\phi = 30^o N$ latitude, find dx and dy at the grid cell latitudinal center.

Solution: $dx = (R_e \, cos\phi) d\lambda_e \qquad dy = R_e \, d\phi$

First, $d\lambda_e = d\phi = 5^o \times \pi/180^o = 0.0873$ radians. Substituting these values into eq. 1.1 gives $dx = (6371)(0.866)(0.0873) = 482 km$ and $dy = (6371)(0.0873) = 556 km$. (Write in details)

iii. **Wind Velocity**: Winds are described by three parameters-velocity, the scalar components of velocity, and speed. Velocity is a vector that quantifies the rate at which the position of a body changes over time:

$$\vec{V} = iu + jv + kw \quad (total\ vector) \tag{1.2}$$

$$\vec{V}_h = iu + jv \quad (horizontal\ vector) \tag{1.3}$$

And,

$$u = \frac{dx}{dt}\ ,\ v = \frac{dy}{dt}\ ,\ w = \frac{dz}{dt} \tag{1.4}$$

are the scalar components of velocity. They have magnitude only. The magnitude of the wind is its speed. The total and horizontal wind speeds are defined as:

$$|\vec{V}| = \sqrt{u^2 + v^2 + w^2} \quad , \quad |\vec{V}_h| = \sqrt{u^2 + v^2} \tag{1.5}$$

iv. Vector Multiplication:

There two types of multiplication namely:

- *The dot product*: It is a product of two vectors gives a scalar.

Let $\vec{A}$ and $\vec{B}$ are two vectors,

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{(How?)} \tag{1.6}$$

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|cos\theta \tag{1.7}$$

Example 1.2: Let $\vec{A} = 2i - 1/2j - 3k$, and $\vec{B} = -3i + j - 1/2k$

Find $\vec{A}.\vec{B}$ and the angle between the two vectors.

Solution:

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}.\vec{B} = -6 + (-1/2) + 3/2 \quad = -6.5 + 1.5 = -5$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{4 + 1/4 + 9} = \sqrt{13.75}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{9 + 1 + 1/4} = \sqrt{10.25}$$

Using eqn. 1.7,

$$cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{-5}{\sqrt{13.25}\sqrt{10.25}} = \frac{-5}{11.52} = -0.434$$

$$\theta = cos^{-1}\theta = 115$$

- *The Cross Product*: it is product of vectors gives a vector:

$$\vec{A} \times \vec{B} = \vec{C} \qquad (1.8)$$

$$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right| sin\theta \qquad (1.9)$$

The direction of $\vec{C}$ is perpendicular on the plane of $\vec{A}$ and $\vec{B}$.

$$\vec{A} \times \vec{B} = \vec{C} = (A_y B_z - A_z B_y)i + (A_z B_x - A_x B_z)j + (A_x B_y - A_y B_x)k \qquad (1.10)$$

v. Gradient, Divergence and Curl Del Operator

Del Operator: is a vector differential operator denoted by the symbol $\vec{\nabla}$:

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} k\frac{\partial}{\partial z} \qquad (1.11)$$

This operator can be used in three differential ways:

a. Gradient of Scalar (pressure)

$$\vec{\nabla} p = i\frac{\partial p}{\partial x} + j\frac{\partial p}{\partial y} k\frac{\partial p}{\partial z} \qquad (vector) \qquad (1.12)$$

b. Divergence of a Vector (velocity)

$$\vec{\nabla}.\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \qquad (scalar)\ (How?) \quad (1.13)$$

c. Curl of a Vector (velocity)

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) i + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) j + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) k \quad (vector) \ (1.14)$$

vi. Laplacian Operator

If Q is any quantity then,

$$\vec{\nabla}.\vec{\nabla}Q \equiv \vec{\nabla}^2 Q \qquad (1.15)$$

where $\vec{\nabla}^2$ (del squared) is the scalar differential operator:

$$\vec{\nabla}^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (1.16)$$

$\vec{\nabla}^2 Q$ is called the Laplacian of Q and appears in several important partial equations of mathematical physics.

1.3 The Total Derivative

Meteorological variables such as $P, T, \vec{V}$ etc. can vary both in space and time, i.e. functions of four independent variables, *x*, *y*, *z* and *t*.

The differential of any of these variables (e.g., T) has the form

$$dT = \frac{\partial T}{\partial t}dt + \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz \qquad (1.17)$$

Dividing by dt,

$$\frac{dT}{dt} = \frac{\partial T}{\partial t}\frac{dt}{dt} + \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} + \frac{\partial T}{\partial z}\frac{dz}{dt}$$

By definition: $\frac{dx}{dt} \equiv u$; $\frac{dy}{dt} \equiv v$; $\frac{dz}{dt} \equiv w$

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

which can also be written as,

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \vec{V}.\nabla T \qquad (1.18)$$

This means

$$Total\ derivative = Partial\ derivative + \boldsymbol{Advection\ operator}$$

The total derivative ($\frac{dT}{dt}$) represents the change relative to a reference frame attached to the air parcel and moving with it (Lagrangian derivative). The term ($\frac{\partial T}{\partial t}$) represents the change from a coordinate system fixed x, y, and z coordinates. This is called the local derivative, or the Eulerian derivative. The term ($\vec{V}.\nabla T$) represents that part of the local change that is due to advection (transport of a property due to the mass movement of the fluid).

Rewrite equation (1.18) in the following form:

$$\frac{\partial T}{\partial t} = \frac{dT}{dt} - \vec{V}.\nabla T \qquad (1.19)$$

Enable us to understand the change we measure with our instruments:

The local measure that we make may be due to either a change within the fluid itself ($\frac{dT}{dt}$), or due to movement of the fluid with different property over our instrument, represented by ($-\vec{V}.\nabla T$) term.

Example: The temperature at our station has been decreasing. This may be due the entire air mass losing heat due to radiation or conduction ($\frac{dT}{dt}$) or due to the wind blowing colder air into our area, ($-\vec{V}.\nabla T$).