Cloud Physics Lab

LAB 5: Formation of Cloud Droplets – part (1)

<u>Purpose</u>:

Study the curvature and solute effects on the formation of cloud droplet

<u>Theory:</u>

A) <u>KELVIN’S EQUATION (Curvature effect)</u>

Saturation vapor pressure over a curved droplet is:

- The saturation vapor pressure over a curved water surface, $e_s(r)$, is greater than that over a flat surface, $e_s(\infty)$. This is expressed mathematically as:

$$e_s(r) = e_s(\infty)\exp\left[\frac{2\gamma}{R_v \rho_L T}\frac{1}{r}\right] = e_s(\infty)\exp(a/r) \qquad (1)$$

where:
γ: is the surface tension of the water-air interface (~ 0.075 N/m),
ρ_L: is the density of liquid water (~ 1000 kg/m^3),
r: is the radius of curvature (or radius of the droplet).

- For a droplet to be in equilibrium with environment (meaning the droplet will neither grow nor evaporate), then the environmental vapor pressure emust be equal to $e_s(r)$. If this is not true, the droplet will either grow or evaporate. This is summarized as:

Condition	The droplet will…
$e < e_s(r)$	evaporate
$e = e_s(r)$	remain the same size
$e > e_s(r)$	grow via condensation

- For a droplet to be in equilibrium then:

$$e = e_s(\infty)\exp(a/r) \qquad (2)$$

Dividing both sides of equation (2) by $e_s(\infty)$ we get:

$$S_{eq} = e/e_s(\infty) = \exp(a/r) \qquad (3)$$

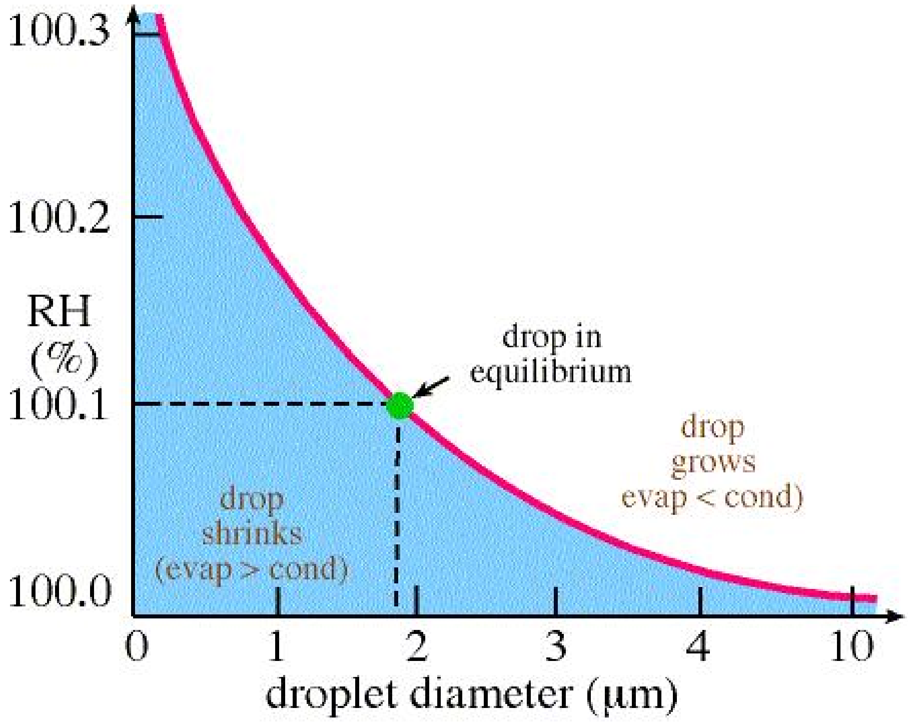
where:
S_{eq}: is the equilibrium saturation ratio.

- The saturation ratio is just relative humidity expressed as a ratio rather than a percent. The equilibrium saturation ratio is the saturation ratio required for the droplet to be in equilibrium.
- If the environmental saturation ratio is less than the equilibrium saturation ratio the droplet will evaporate.
- If the environmental saturation ratio is greater than the equilibrium saturation ratio the droplet will grow.

- For very small drops the equilibrium saturation ratio is extremely large.
- The increase of equilibrium saturation ratio with decreasing radius is known as the curvature effect.
- Homogeneous nucleation (condensation of pure water no dust or aerosols present) requires a relative humidity of 400- 500%. Though this can be achieved in a laboratory, such high relative humidity does not occur in the atmosphere. Therefore, homogeneous nucleation cannot explain the initial formation of cloud droplets.

In an environment with a vapor pressure, e_o only drops with a particular radius, which we call the critical radius, r_c will be in equilibrium. Note, however, that this is an unstable equilibrium. If the mass of the environment is large, so that e does not change if the drop grows a little, imagine a small perturbation that causes the drop to grow slightly. The equilibrium vapor pressure over the slightly larger drop will cause it to continue to grow. Similarly, a slight evaporation of the drop will cause it to evaporate. This means that any perturbation in the mass of the drop of critical radius will cause it to move away from its equilibrium. Hence, the equilibrium is unstable. The critical radius may be determined by setting $e_s(r_c) = e_o$ using Kelvin's equation, and solving for r_c:

$$r_c = \frac{2\sigma}{\rho_L R_v T \ln S} \qquad (4)$$



B) <u>RAOULT'S LAW (Solute Effect)</u>

Raoult's law is an empirical law that describes the reduction of equilibrium vapor pressure over the flat surface of a solution. It does not take into account the Kelvin (curvature effect), so we will subsequently combine Raoult's law and Kelvin's equation to determine the equilibrium vapor pressure over a solution drop.
The ratio of the saturation vapor pressure of the mixture over that of pure water is then:

$$\frac{e'}{e_s} = 1 - b/r^3 \qquad (5)$$

where:

$$b = \frac{3im_s}{4\pi\rho_L}\frac{M_w}{M_s} \qquad (6)$$

with:
i : ion factor
m_s : mass of solute
ρ_l : density of water
M_w : molecular weight of water
M_s : molecular weight of solute

- The reduction of saturation vapor pressure by introduction a solute is known as the solute effect.
- For droplets of small radius the saturation vapor pressure over the drop is much less than that over pure water. As radius increases the saturation vapor pressure of the solution approaches that of pure water.

Methodology:

1. Run the Matlab script *Lab5a.m* to plot a graph of saturation ratio versus cloud droplet radius for different Temperature (°C) and from the graphs determine the saturation ratio for the given critical radius.

Temperature (°C)	Critical Radius	Saturation Ratio
0	2e-2	
5	2e-2	
10	2e-2	
20	2e-2	

2. Run the Matlab script *Lab5b.m* to plot a graph of saturation ratio versus cloud droplet radius for different values of $NaCl_2$ mass (ms) and from the graphs determine the saturation ratio for the given critical radius.

Mass of $NaCl_2$, ms (g)	Droplet Radius	Saturation Ratio
2e-18	2e-2	
4e-18	2e-2	
6e-18	2e-2	

3. Discuss your results.