PHYS-2010: General Physics I
Course Lecture Notes
Section II

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Edition 2.5

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2010: General Physics I taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 9th Edition* (2012) textbook by Serway and Vuille.

II. Mathematical Techniques

A. Dimensional Analysis.

1. Always make sure that all terms in an equation have the same dimensions (*i.e.*, units).

2. Then try to reduce a parameter in an equation to a combination of the three basic concepts: **length** [**L**], **mass** [**M**], and **time** [**T**].

3. For example, the acceleration of a body in a gravitational field is proportional to the mass of the primary body and inversely proportional to the square of the distance:

$$a = G\,\frac{M}{r^2}\ ,$$

where G is a constant. From this formula, find the dimensions of G.

$$[a] = \ \mathbf{L}\,\mathbf{T}^{-2} \qquad [M] = \ \mathbf{M} \qquad [r] = \ \mathbf{L}\ ,$$

where **L** represents *length*, **M** represents *mass*, and **T** represents *time*. Then

$$G = \frac{ar^2}{M} \quad \Longrightarrow \quad [G] = \frac{[a]\ [r]^2}{[M]} = \frac{\mathbf{L}\,\mathbf{T}^{-2}\,\mathbf{L}^2}{\mathbf{M}} = \mathbf{L}^3\,\mathbf{M}^{-1}\,\mathbf{T}^{-2}$$

or the dimensions of G in the *basic* (*i.e.*, *fundamental*) units in SI are $\mathrm{m^3/kg/s^2}$.

4. When a symbol or variable has *square brackets* around it, this means: *what are the dimensions (i.e., units) of this symbol or variable*?

B. Algebra Review.

1. Cross multiplication: $mx = ny \Longleftrightarrow \dfrac{x}{y} = \dfrac{n}{m}$.

2. Factoring: $y = mx + mb \Longleftrightarrow y = m(x + b)$.

3. Powers & Roots:

a) $a \times a \times a \times a \times a \times a \times a = a^7$

or

$a \times a \times \cdots (m - \text{times}) \cdots \times a = a^m$

"a" is raised to the "m^{th}" power.

b) $a^{1/m} = \sqrt[m]{a} \Longrightarrow$ "m^{th}" root of "a."

c) $a^0 \equiv 1$ (note that the "$\equiv$" symbol means "*defined to be*").

d) $a^{-m} = \dfrac{1}{a^m}$.

e) $(ab)^m = a^m b^m, \quad \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m} = a^m b^{-m}$.

f) $a^m a^n = a^{m+n}, \quad \dfrac{a^m}{a^n} = a^{m-n}$.

g) $(a^m)^n = a^{mn}, \quad \sqrt[n]{a^m} = a^{m/n}$.

4. Exponentials and Logarithms:

$$\begin{aligned} y &= a^x && \text{(base "}a\text{" to power "}x\text{")} \\ x &= \log_a y && \text{(the exponent of "}a\text{" that yields "}y\text{")} \end{aligned}$$

a) Product: $\log_a(xy) = \log_a x + \log_a y$.

b) Quotient: $\log_a \left(\dfrac{x}{y}\right) = \log_a x - \log_a y$.

c) Power: $\log_a (y^n) = n \log_a y$.

d) Two common bases:

i) Base 10 ⇒ **common logarithms**:

$$\boxed{\log_a = \log_{10} \equiv \log}$$

$$x = \log y \quad \Longleftrightarrow \quad y = 10^x \ .$$

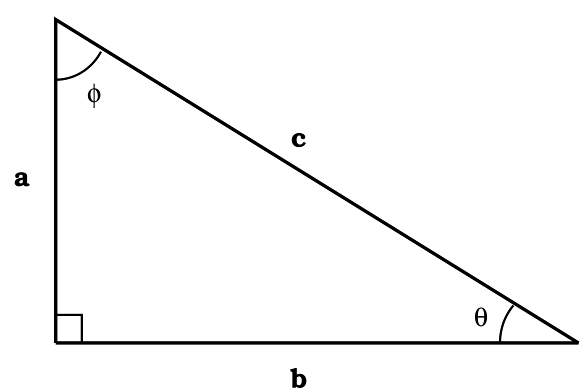
ii) Base $e = 2.71828...$ ⇒ **natural logarithms**:

$$\boxed{\log_a = \log_e \equiv \ln}$$

$$x = \ln y \quad \Longleftrightarrow \quad y = e^x \ .$$

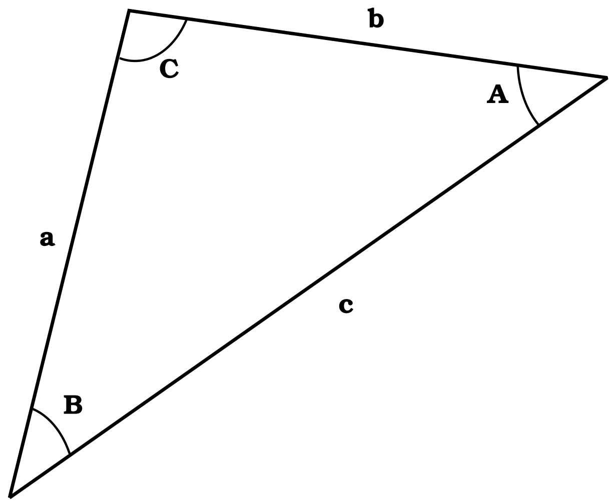
C. Basic Trigonometry.

1. Right-angle triangle relationships:



$$\sin\theta = \frac{a}{c}, \quad \cos\theta = \frac{b}{c}, \quad \tan\theta = \frac{a}{b} = \frac{\sin\theta}{\cos\theta}$$
$$a^2 + b^2 = c^2 \qquad \text{or} \qquad \sin^2\theta + \cos^2\theta = 1$$
$$\theta + \phi \quad = 90^\circ = \quad \frac{\pi}{2} \text{ radians.}$$

2. Generic triangle relationships:



a) Law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \ .$$

b) Law of cosines:

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \ , \\ b^2 &= a^2 + c^2 - 2ac \cos B \ , \\ c^2 &= a^2 + b^2 - 2ab \cos C \ . \end{aligned}$$

3. Additional useful trigonometric identities.

a) Angle-sum and angle-difference relations:

$$\begin{aligned} \sin(\alpha + \beta) &= \sin\alpha \, \cos\beta + \cos\alpha \, \sin\beta \\ \sin(\alpha - \beta) &= \sin\alpha \, \cos\beta - \cos\alpha \, \sin\beta \\ \cos(\alpha + \beta) &= \cos\alpha \, \cos\beta - \sin\alpha \, \sin\beta \\ \cos(\alpha - \beta) &= \cos\alpha \, \cos\beta + \sin\alpha \, \sin\beta \\ \tan(\alpha + \beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \, \tan\beta} \\ \tan(\alpha - \beta) &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \, \tan\beta} \end{aligned}$$

b) Double-angle relations:

$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha\,\cos\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha \\ &= \frac{1-\tan^2\alpha}{1+\tan^2\alpha} \\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}\end{aligned}$$

D. Scalars and Vectors.

1. A **scalar** has magnitude but no directional information (*e.g.*, 'v' is a scalar).

a) 4 kg and 600 K are scalars.

b) 420 km/s is a scalar (*i.e.*, speed).

2. A **vector** has both magnitude and directional information (*e.g.*, '$\vec{v}$' is a vector).

a) 420 km/s to the NW (northwest) is a vector (*i.e.*, velocity).

b) 420 km/s NW is not equal to 420 km/s SE (southeast)!

c) Note that in these course notes I will always represent a vector with an arrow over the variable letter (*e.g.*, $\vec{A}$), whereas your textbook indicates a vector with a boldface letter (*e.g.*, $\boldsymbol{A}$).

d) Arithmetic for scalars and vectors are handled differently with respect to each other. We will describe vector arithmetic in §IV of these notes.