Q1: Show that

(1) $(p \to q) \wedge \sim q \Longrightarrow \sim p$.
(2) $p \wedge (p \to q) \to \sim q$ is a contingency using a truth table.
(3) $p \to (p \vee q)$ is a tautology using a truth table.
(4) $(p \wedge q) \to p$ is a tautology using the table of logical equivalences.
(5) $(p \wedge q) \to (p \vee q)$ is a tautology using the table of logical equivalences.
(6) $[p \to (q \to r)] \equiv [(p \wedge q) \to r]$ using a truth table and logical proposition.
(7) $[p \to (q \to r)] \equiv [q \to (p \to r)]$ using a truth table and logical proposition.
(8) $[(p \wedge q) \to p] \equiv [q \to (p \vee \sim p)]$ using a truth table and logical proposition.
(9) $[(p \to q) \wedge (p \to r)] \equiv [p \to q \wedge r)]$ using a truth table and logical proposition.

(10) $[(p \to q) \wedge (r \to q)] \equiv [(p \vee r) \to q]$ using a truth table and logical proposition.

(11) $(p \to q) \equiv (\sim q \to \sim p)$ using a truth table and logical proposition.

(12) $p \wedge (\sim p \vee q) \equiv p \wedge q$ using a truth table and logical proposition.
(13) $p \vee (p \wedge q) \equiv p$ using a truth table and logical proposition.

(14) Is $\veebar$ commutative or associative?

(15) Is $\veebar$ distributive over $\wedge$, $\vee$, or $\to$?

(16) Is this true $p \veebar q \equiv p \leftrightarrow \sim q$?

(17) $[(p \to q) \wedge (q \to r)] \Longrightarrow (p \to r)$ using a truth table.
(18) $[(p \vee q) \wedge \sim p]) \Longrightarrow q$ using a truth table.
(19) $[(p \to q) \wedge (r \to s)] \Longrightarrow [(p \vee r) \to (q \vee s)]$ using a truth table.

(20) $[(p \to q) \wedge (p \vee r)] \Longrightarrow (q \vee r)$ using a truth table.

Q2: Represent as propositional expressions:

Tom is a math major but not computer science major and use De Morgan's Laws to write the negation of the expression, and translate the negation in English.

Q3: Given the hypotheses:

(i) "It is not sunny this afternoon and it is colder than yesterday"
(ii) "We will go swimming only if it is sunny"
(iii) "If we do not go swimming, then we will take a canoe trip"
(iv) "If we take a canoe trip, then we will be home by sunset"

Does this imply that **“we will be home by sunset”?**

Q4: Represent as propositional expressions, and use De Morgan's Laws to write the negation of the expression, and translate the negation in English.

“Tom is a math major but not computer science major.”

Q5: Let

p = "John is healthy"
q = "John is wealthy"
r = "John is wise"

Represent symbolically:

(i) John is healthy and wealthy but not wise.

(ii) John is not wealthy but he is healthy and wise.

(iii) John is neither healthy nor wealthy nor wise.

Q6: Translate the sentences into propositional expressions:

"Neither the fox nor the lynx can catch the hare if the hare is alert and quick."

Q7: Represent as propositional expressions.

"You can either (stay at the hotel and watch TV) or (you can go to the museum and spend some time there)".

Q8: Given a sentence **"If we are on vacation we go fishing."** Then

(i) Translate the sentence into a logical expression,

(ii) Write the negation of the logical expression and translate the negation into English,

(iii) Write the converse of the logical expression and translate the converse into English,

(iv) Write the inverse of the logical expression and translate the inverse into English,

(v) Write the contrapositive of the logical expression and translate the contrapositive into English.

Q9: Write the contrapositive, converse and inverse of the expressions:

$p \to q$,

$\sim p \to q$,

$q \to \sim p$.

Q10: Determine whether the following arguments are valid or invalid:

(i) Premises:

(a) If I read the newspaper in the kitchen, my glasses would be on the kitchen table.

(b) I did not read the newspaper in the kitchen.

Conclusion: My glasses are not on the kitchen table.

(ii) Premises:

(a) If I don't study hard, I will not pass this course

(b) If I don't pass this course I cannot graduate this year.

Conclusion: If I don't study hard, I won't graduate this year.

(iii) Premises:

(a) You will get an extra credit if you write a paper or if you solve the test problems.

(b) You don't write a paper, however you get an extra credit.

Conclusion: You have solved the test problems.

Q11: Find the DNF of (p →q)→ ~r.

Q12: Find the DNF of p $\underline{\vee}$ q.

Q13: Find an expression equivalent to p →q that uses only ∧ and ~.

Q14: Convert the following statement into CNF. (p → q)→ (~r ∧ q).

Q15: Negate the following sentences.
(i) The number x is positive, but the number y is not positive.
(ii) If x is prime, then $\sqrt{x}$ is not a rational number.
(iii) For every prime number p, there is another prime number q with $q > p$.
(iv) There exists a real number a for which $a + x = x$ for every real number x.
(v) Every even integer greater than 2 is the sum of two primes.
(vi) The integer x is even, but the integer y is odd.
(vii) At least one of the integers x and y is even.
(viii) The numbers x and y are both odd.
(ix) For every real number x there is a real number y for which $y^3 = x$.
(x) I don't eat anything that has a face.
Q16: Write the following propositions with quantifiers and then give its negation with translations into words.

(i) Some counting numbers are greater than five

(ii) Every element of set D is less than 7.

(iii) Some elements of set D are less than 13.

(iv) Every counting number greater than 4 is greater than 2.
(v) Some counting numbers are even.
(vi) Every counting number which is divisible by 2 is even.
(vii) Every counting number is even or odd.

(viii) For every x in D_x and for every $y \in$ in D_x, x plus y less than 3.

(ix) At least one politician isn't a logician.

(x) Only nonlogicians are politicians.

Q17: Prove that $\sqrt{2}$ is irrational using contradiction method.