

(i) Given

$$(1) p \wedge q$$

$$(2) p \rightarrow \sim (q \wedge r)$$

$$(3) s \rightarrow r$$

$$\therefore \sim s$$

Solution:

1- $p \wedge q$	1 st hypothesis(premise)
2- p	Inf. (1) Properties of \wedge
3- q	Inf. (1) Properties of \wedge
4- $p \rightarrow \sim (q \wedge r)$	2 nd hypothesis(premise)
5- $\sim (q \wedge r)$	Inf. (2),(4)
6- $\sim q \vee \sim r$	De Morgan's Law on (5)
7- $\sim r$	Inf. (3),(6) and Domination Laws
8- $s \rightarrow r$	3 rd hypothesis(premise)
9- $\sim r \rightarrow \sim s$	Contrapositive Law
10- $\sim s$	Inf. (7),(9)

(ii) Given

$$(1) \sim (p \vee q) \rightarrow r$$

$$(2) \sim p$$

$$(3) \sim r$$

$$\therefore q$$

Solution:

$$1- \sim (p \vee q) \rightarrow r \quad 1^{\text{st}} \text{ hypothesis(premise)}$$

$$2- \sim r \quad 3^{\text{rd}} \text{ hypothesis(premise)}$$

$$3- \sim r \rightarrow (p \vee q) \quad \text{Contrapositive Law and Double Negation Law}$$

$$4- p \vee q \quad \text{Inf. (2),(3)}$$

$$5- \sim p \quad 2^{\text{nd}} \text{ hypothesis(premise)}$$

$$6- q \quad \text{Inf. (4),(5)}$$

(iii) Given

$$(1) \sim p \rightarrow (r \wedge s)$$

$$(2) p \rightarrow q$$

$$(3) \sim q$$

$\therefore r$

Solution:

$$1- p \rightarrow q \quad 2^{\text{nd}} \text{ hypothesis(premise)}$$

$$2- \sim q \quad 3^{\text{rd}} \text{ hypothesis(premise)}$$

$$3- \sim q \rightarrow \sim p \quad \text{Contrapositive Law on (1)}$$

$$4- \sim p \quad \text{Inf. (2),(3)}$$

$$5- \sim p \rightarrow (r \wedge s) \quad 1^{\text{st}} \text{ hypothesis(premise)}$$

$$6- r \wedge s \quad \text{Inf. (4),(5)}$$

$$7- r \quad \text{Inf. (6) Properties of } \wedge$$

(iv) Given

$$(1) p \rightarrow (\sim r \wedge \sim s)$$

$$(2) p \vee \sim q$$

$$(3) s$$

$$\therefore \sim q \wedge s$$

Solution:

$$1- p \rightarrow (\sim r \wedge \sim s) \quad 1^{\text{st}} \text{ hypothesis(premise)}$$

$$2- (r \vee s) \rightarrow \sim p \quad \text{Contrapositive Law on (1)}$$

$$3- p \vee \sim q \quad 2^{\text{nd}} \text{ hypothesis(premise)}$$

$$4- \sim p \rightarrow \sim q \quad \text{Implication Law on (3)}$$

$$5- (r \vee s) \rightarrow \sim q \quad \text{Inf. (2),(4)}$$

$$6- s \quad 3^{\text{rd}} \text{ hypothesis(premise)}$$

$$7- r \vee s \quad \text{Inf. (6)}$$

$$8- \sim q \quad \text{Inf. (5),(7)}$$

$$9- \sim q \wedge s \quad \text{Inf. (6),(8)}$$

(v) Given

$$(1) p \vee q$$

$$(2) q \rightarrow r$$

$$(3) \sim r$$

$$\therefore p$$

Solution:

1- $q \rightarrow r$	2 nd hypothesis(premise)
2- $\sim r \rightarrow \sim q$	Contrapositive Law on (1)
3- $\sim r$	3 rd hypothesis(premise)
4- $\sim q$	Inf. (2),(3)
5- $p \vee q$	1 st hypothesis(premise)
6- $(p \vee q) \wedge \sim q$	Inf. (4),(5)
7- $(p \wedge \sim q) \vee (q \wedge \sim q)$	Distributive Law on (6)
8- $(p \wedge \sim q) \vee F$	Contradiction Law (7)
9- $(p \wedge \sim q)$	Identity Law on (8)
10- p	Inf. (9) properties of \wedge