1.8. Quantifiers

Definition 1.8.1.
(i) A **predicate** or **propositional function** is a statement (formula) containing variables and that may be true or false depending on the values of these variables.

- That is, a property or relationship between objects represented symbolically.
- We represent a predicate by a letter followed by the variables enclosed between parenthesis: $P(x)$, $Q(x,y)$, etc.

(ii) An ***example*** for $P(x)$ is: value of x for which $P(x)$ is true.
(iii) A ***counterexample*** $P(x)$ is: value of x for which $P(x)$ is false.
(ii) The set, X which contain all possible value that satisfy the formula P is called a **universal set**.
(iii) A set Y which contains all values x belong to set X such that $P(x)$ is true is called a **solution set.**

$$Y = S_P = \{x \in X : P(x) \text{is true}\}$$

Example 1.8.2.

(i) $P(x) = x \leq 5 \wedge x > 3$ is true for $x = 4$ and false for $x = 6$ (counterexample).

Compare this with the statement

(ii) $P(x) = x \leq 5 \wedge x > 3$, for every real numbers, x which is definitely false.

(iii) There exists an x such that $P(x) = x \leq 5 \wedge x > 3$," which is definitely true.

(iv) Given the statement "**Ahmad is a logician**".

Let P represent **'is a logician'** and let x represent **'Ahmad'**. The predicate form of this statement is $P(x)$. That is, $P(x) =$ Ahmad is a logician.

(v) Let r: x is married to y.

Let M represent **"married".** Then, $\text{r} = M(x,y)$.

(vi) Let r: The numbers x and y are both odd.
This statement means (x is odd) $\wedge$ (y is odd).

Let P represent **'is a odd'** and let x, y represent **'numbers'**. The predicate form of this statement is $P(x) \wedge P(y)$.

Definition 1.8.3.

(i) The phrase **"for all x"** (**"for every x"**, **"for each x"**) is called a **universal quantifier** and is denoted by $\forall\, x$.

(ii) The phrase **"for some x"** (**"there exists an x"**) is called an **existential quantifier** and is denoted by $\exists\, x$.

Definition 1.8.4. (The Universal Quantifier Proposition)

(i) Let $f(x)$ be a proposition function which depend only on x. A sentence $\forall\, x, f(x)$ read "For all x, $P(x)$" mean

"For all values x in X(universal set), the assertion $f(x)$ is true.", that is;

$$\frac{\forall x, f(x)}{\therefore\ f(a)}$$

(ii) A sentence $\forall x\ (f(x) \longrightarrow \sim Q(x))$ read "No $f(x)$ are $Q(x)$"

Example 1.8.5.

(i) r: The square of all real numbers are positive.

$$\boxed{\text{r: } \forall x \in \mathbb{R}, \quad (x^2 \geq 0).}$$

(ii) r: The commutative law of addition of real numbers is holed.

$$\boxed{\text{r: } \forall\, x, \forall\, y \in \mathbb{R}, \quad (x + y = y + x).}$$

(iii) r: The associative law of addition of real numbers is holed.

$$\boxed{\text{r: } \forall\, x, \forall\, y, \forall\, z \in \mathbb{R}, \ \big((x + y) + z = x + (y + z)\big).}$$

(iv) r: All logicians are exceptional.
Let L represent **'set of logician'** and let E represent **'is exceptional'**. The predicate form of this statement is $\boxed{\text{r: } \forall x \in L,\ E(x)}$.

(v) r: All cars are red.
Let $X \coloneqq$ **Set of cars**, $f := $ **is red**. Then, r : $\forall x \in X, f(x)$.

Remark .1.8.6.

(i) The "**all**" form, the existential quantifier, is frequently encountered in the following context: $\forall x\ (f(x) \longrightarrow Q(x)),$

which may be read,

"For all x in a universal set X satisfying $f(x)$ also satisfy $Q(x)$".

For example:

(a) r: All logicians are exceptional.

Let L represent **'is a logician'** and let E represent **'is exceptional'**. Then

- Predicate Logic: r: $\forall x(L(x) \longrightarrow E(x))$
- In logical English: "For all x, if x is a logician, then x is exceptional."

(b) r: The square of all real numbers are positive.
Let P represent: $\in \mathbb{R}$ and let Q represent **" is positive"**.

- Predicate Logic: r: $\forall x(P(x) \longrightarrow Q(x))$; that is,

  $$\boxed{\text{r: } \forall x(\text{ if } x \in \mathbb{R} \longrightarrow (x^2 \geq 0).}$$

- In logical English: "For all x, if x is real number, then x is exceptional."

(c) Every(each, any) integer is even (or: Integers are even).

Let P represent: $\in \mathbb{Z}$ and let E represent **" is even"**.

- Predicate Logic: r: $\forall x(P(x) \longrightarrow E(x))$; that is,

  $$\boxed{\text{r: } \forall x(\text{ if } x \in \mathbb{Z} \longrightarrow E(x)).}$$

- In logical English: "For all x, if x is an integer, then x is even."

(ii) Parentheses are crucial here; be sure you understand the difference between the "all" form and $\boxed{\forall x, f(x) \longrightarrow \forall x, Q(x)}$ and $\boxed{(\forall x,\ f(x)) \longrightarrow Q(x)}$.

Definition 1.8.7. (The Existential Quantifier Proposition)

(i) A sentence $\exists x, f(x)$ read "For some x, $P(x)$" or "For some x such that $P(x)$" mean

"For some $x \in X$(universal set), the assertion $f(x)$ is true"; that is,

$$\frac{f(a)}{\therefore \exists x \in X, f(x)}$$

(ii) A sentence $\exists\, x, \big(f(x) \wedge \sim P(x)\big)$ read "Some $f(x)$ are not $P(x)$ ".

Example 1.8.8.

(i) $\exists\, x: (x \geq x^2)$ is true since $x = 0$ is a solution. There are many others.

(ii) r: Some logicians are exceptional.
Let L represent **'set of logician'** and let E represent **'is exceptional'**. The predicate form of this statement is r: $\exists x \in L,\ \ E(x)$.

(iii) r: There is a car which is red.
Let $X \coloneqq$ **Set of cars**, $f := $ **is red**. Then, r : $\exists x \in X, f(x)$.

Remark .1.8.9.

(i) The "**some**" form, the universal quantifier, is frequently encountered in the following context: $\exists x\ (f(x)\ \wedge\ Q(x))$,

which may be read,

some x in a universal set X satisfying $f(x)$ and satisfy $Q(x)$".

For example:

(a) r: Some logicians are exceptional.

Let L represent **'is a logician'** and let E represent **'is exceptional'**. Then

- Predicate Logic: r: $\exists x(L(x) \wedge\ E(x))$
- In logical English: "For some x, x is a logician and x is exceptional."

(b) r: The square of some integers numbers are two (or: There is an integer for which its square is two)
Let P represent: $\in\ \mathbb{Z}$ and let Q represent **" is 2"**.
- Predicate Logic: r: $\exists x(P(x) \wedge\ Q(x))$; that is,

$$\boxed{\text{r: } \exists x(x \in \mathbb{Z} \ \wedge \ x^2 = 2).}$$

- In logical English: "For some x, x is an integer number and $x^2 = 2$."

(c) At least one integer number is even (or: Some Integers are even).

Let P represent: $\in \mathbb{Z}$ and let E represent **" is even"**.

- Predicate Logic: r: $\exists x(P(x) \wedge E(x))$; that is,

$$\boxed{\text{r: } \exists x(x \in \mathbb{Z} \wedge E(x)).}$$

- In logical English: "For some x, x is an integer number and x is even."

Negation Rules of quantifiers 1.8.10.
(i) When we negate a quantified statement, we negate all the quantifiers first, from left to right (keeping the same order), then we negative the statement.
(ii) $\sim (x = y) = (x \neq y)$.
(iii) $\sim (x \equiv y) = (x \not\equiv y)$.
(iv) $\sim (x < y) = (y \leq x)$.
(v) $\sim (x \in Y) = (x \notin Y)$.
(vi) ~ (Even number) = Odd number.

Now define the a formal universal quantifier proposition using negation.

Definition 1.8.11.
$\forall x, f(x) = \sim\exists\, x, \sim f(x)$.

Example 1.8.12.
r: All logicians are exceptional.

Let L represent **'set of logician'** and let E represent **'is exceptional'**.

- Predicate Logic: $\boxed{\text{r: } \forall x \in L,\ \ E(x) = \sim\exists\, x, \sim E(x)}$.
- In logical English: "There is no x is a logician, for which x is not exceptional."

Equivalent Definitions 1.8.13.
(i) $\exists\, x, f(x) \equiv \sim\forall x, \sim f(x)$.
(ii) $\sim\big(\forall x, f(x)\big) \equiv \exists\, x, \sim f(x)$.
(iii) $\sim\big(\exists\, x, f(x)\big) \equiv \forall\, x, \sim f(x)$.

(iv) $\sim [\forall x\, (f(x) \longrightarrow Q(x))] \equiv \exists x\, (f(x) \wedge \sim Q(x))$
$\equiv$ Some $f(x)$ are not $Q(x)$

(v) $\sim \left(\exists\, x, (f(x) \wedge Q(x))\right) \equiv \forall\, x, \sim f(x) \vee \sim Q(x) \equiv \forall x\, (f(x) \longrightarrow \sim Q(x))$
$\equiv$ No $f(x)$ are $Q(x)$

Example 1.8.14.

(i) Express each of the following sentences in the form $\forall x, P(x)$ and then give its negation in both cases $\forall x, P(x)$ and in words.

(a) r: The square of every real number is non-negative.
Solution.

- **$\forall x, P(x)$ form:** r: $\forall x \in \mathbb{R}, x^2 \geq 0$.
- **Negation:** ~r: $\sim (\forall x \in \mathbb{R}, x^2 \geq 0) \equiv \exists x \in \mathbb{R}, \sim (x^2 \geq 0) \equiv \exists x \in \mathbb{R}, x^2 < 0$.
- **Negation in words:** ~r:There exists a real number whose square is negative.

(b) r: For all x, there exists y such that $xy = 1$.
Solution.

- **$\forall x, P(x)$ form:** r: $\forall\, x, \exists\, y$ such that $xy = 1$.
- **Negation:** ~r: $\sim (\forall\, x, \exists\, y$ such that $xy = 1)$
$\equiv \exists x, \sim (\forall\, y$ such that $(xy = 1))$
$\equiv \exists x, \forall\, y$ such that $xy \neq 1$.
- **Negation in words:** ~r: There exists x for all y such that $xy \neq 1$.

(ii) Let **r: Student who is intelligent will succeed**. Write " r " in predicate logic and English logic, and then give its negation in both cases.
Solution.
Let P: Student
Q: intelligent.
S: Succeed

- **Predicate Logic:** r**:** $\forall\, x((\mathrm{P}(x) \wedge \mathrm{Q}(x)) \rightarrow \mathrm{S}(x))$
- **Negation:** ~r: $\sim \left[\forall\, x\left((\mathrm{P}(x) \wedge \mathrm{Q}(x)) \rightarrow \mathrm{S}(x)\right)\right]$
$\equiv \sim \left[\forall\, x\left(\sim (\mathrm{P}(x) \wedge \mathrm{Q}(x)) \vee \mathrm{S}(x)\right)\right]$ Implication Low.
$\equiv \exists x\left((\mathrm{P}(x) \wedge \mathrm{Q}(x)) \wedge \sim \mathrm{S}(x)\right)$ De Mover's Law.
- **English logic:** ~r: There exist student who is intelligent and not succeed.

(iii) r: Some integer numbers are even but not odd.

Let $\mathbb{Z} \coloneqq$ **Set of Integers**, $E:=$ **is even**, $O:=$ **is odd**.

- Predicate Logic: r : $\exists x \in \mathbb{Z}, \big(f(x) \wedge \sim P(x)\big) \equiv \sim \big[\forall x \,\big(f(x) \longrightarrow Q(x)\big)\big]$.
- English Logic: r: Not all even integers are odd.

Remark 1.8.15.
Sometimes the English sentences are **unclear** with respect to quantification, or in another wards, quantified statements are often misused in **casual conversation**.

For example:
(i) "If you can solve any problem we come up with, then you get an A for the course"
The phrase **"you can solve any problem we can come up with"** could reasonably be interpreted as either a universal or existential quantification:

(a) "you can solve every problem we come up with",

(b) "you can solve at least one problem we come up with".

(ii) r**:** All students do not pay full tuition.
Here " r " could reasonably be interpreted as
(a) Not all students pay full tuition.
(b) There exist some students pay full tuition.
Mathematical context: Not all students pay full tuition.

(iii) r: All integers are not even."
The logical proposition means**: " there are no even integers".**
Mathematical context: Not all integers are even.

Combined Quantifiers 1.8.16. There are six ways in which the quantifiers can be combined when two variables are present:

(1) $\forall\, x\, \forall\, y\; f(x,y) \equiv \forall\, y\, \forall\, x\; f(x,y)$=For every x, for every y, $f(x,y)$.

(2) $\forall\, x\, \exists\, y\; f(x,y) \equiv$ For every x, there exists a y such that $f(x,y)$.

(3) $\forall\, y\, \exists\, x\; f(x,y) \equiv$ For every y, there exists an x such that $f(x,y)$.

(4) $\exists\, x\, \forall\, y\; f(x,y) \equiv$There exists an x such that for every y $f(x,y)$.

(5) $\exists\ y\ \forall\ x\ f(x,y) \equiv$ There exists a y such that for every $y\ \ f(x,y)$.

(6) $\exists\ x\ \exists\ y\ f(x,y) \equiv\ \exists\ y\ \exists\ x\ f(x,y)$= There exists an x such that there exists a y $f(x,y)$.

Example 1.8.17.

(i) r: $\exists\ x \in \mathbb{R}\ \exists\ y \in \mathbb{R} : P(x,y) = (x^2 + y^2 = 2xy)$. The proposition " r " is true since $x = y = 1$ is one of many solutions.

(ii) s: $\forall\ x \in \mathbb{R}\ \ \exists\ y \in \mathbb{R} : P(x,y) = (y^3 = x)$. The proposition " s " is true since $y = \sqrt[3]{x}$ is solution for $P(x,y)$.

(iii) s: $\exists\ x \in \mathbb{R}\ \ \forall y \in \mathbb{R} : P(x,y) = (y^3 = x)$. Here "s" mean there is an "x" real such that for every "y" real such $P(x,y)$ is true. The proposition " s " is not true since no real numbers have this property.

(iv) The following are equivalents.

(a) $\sim[\forall\ x\ \forall\ y\ f(x,y)] \equiv \exists x\ \ \exists y \sim f(x,y)$.

(b) $\sim[\exists\ x\ \forall\ \exists\ f(x,y)] \equiv \forall x\ \ \forall y \sim f(x,y)$.

(c) $\sim[\forall\ x\ \exists\ y\ f(x,y)] \equiv \exists x\ \ \forall y \sim f(x,y)$.

(d) $\sim[\exists\ x\ \forall\ y\ f(x,y)] \equiv \forall x\ \ \exists y \sim f(x,y)$.

Solution. Exercise.

1.9. Logical Reasoning

Definition 1.9.1. (Arguments)
An **argument** is a series of statements starting from hypothesis (premises/ assumptions) and ending with the conclusion.

From the definition, an argument might be valid or invalid.
Definition 1.9.2. (Valid Arguments)(Proofs)
An argument is said to be **valid** if the hypothesis implies the conclusion; that is, if s is a statement implies from the statements $s_1, s_2, \ldots, s_n$, then write as

$$s_1, s_2, \ldots, s_n \mapsto s.$$

Note 1.9.3. In mathematics, the word ***proof*** is used to mean simply a valid argument. Many proofs involve more than two premises and a conclusion.

Example 1.9.4.
(i) Let s_1: Some mathematicians are engineering
s_2 : Ali is mathematician
s : Ali is engineering
Show that the argument is valid.
Solution.
The argument $s_1, s_2 \mapsto s$ is not valid, since not all mathematicians are engineering.

(ii) Let s_1: There is no lazy student
s_2 : Ali is artist
s_3 : All artist are lazy
Find a conclusion s for the above premises making the argument $s_1, s_2, s_3 \mapsto s$ is valid.
Solution.
Ali is----------------------.

Remark 1.9.5.
(i) An argument $\boxed{s_1, s_2, \ldots, s_n \mapsto s}$
is valid if and only if $\boxed{(s_1 \wedge s_2 \wedge \ldots \wedge s_n) \to s}$
is tautology; that is, $\boxed{(s_1 \wedge s_2 \wedge \ldots \wedge s_n) \Longrightarrow s.}$
(ii) An argument does not depend on the truth of the premises or the conclusion but it just interested only in the question

“**Is the conclusion implied by the conjunction of the premises?**”

Example 1.9.6. Show that the following argument is valid using truth table.

A_1: $p \wedge q$

A_2: $p \to \sim (q \wedge r)$

A_3: $s \to r$

C: $\therefore \sim s$

				A_1			A_2	A_3
p	**q**	**r**	**s**	**p ∧ q**	**(q ∧ r)**	**~(q ∧ r)=I**	**p→ I**	**s→ r**
T	T	T	T	T	T	F	F	T
T	T	T	F	T	T	F	F	T
T	T	F	T	T	F	T	T	F
T	T	F	F	T	F	T	T	T
T	F	T	T	F	F	T	T	T
T	F	T	F	F	F	T	T	T
T	F	F	T	F	F	T	T	F
T	F	F	F	F	F	T	T	T
F	T	T	T	F	T	F	T	T
F	T	T	F	F	T	F	T	T
F	T	F	T	F	F	T	T	F
F	T	F	F	F	F	T	T	T
F	F	T	T	F	F	T	T	T
F	F	T	F	F	F	T	T	T
F	F	F	T	F	F	T	T	F
F	F	F	F	F	F	T	T	T

	A_1	A_2	A_3	$A_1 \wedge A_2 \wedge A_3$	C	$(A_1 \wedge A_2 \wedge A_3) \to$ **C**	
s	$p \wedge q$	$p \to I$	$s \to r$		$\sim s$		
T	T	F	T	F	F	T	
F	T	F	T	F	T	T	
T	T	T	F	F	F	T	
F	**T**	**T**	**T**	**T**	**T**	**T**	⟵
T	F	T	T	F	F	T	
F	F	T	T	F	T	T	
T	F	T	F	F	F	T	
F	F	T	T	F	T	T	
T	F	T	T	F	F	T	
F	F	T	T	F	T	T	
T	F	T	F	F	F	T	
F	F	T	T	F	T	T	
T	F	T	T	F	F	T	
F	F	T	T	F	T	T	
T	F	T	F	F	F	T	
F	F	T	T	F	T	T	

1.10. Mathematical Proof

In this section some common procedures of proofs in mathematics are given with examples.

1.10.1 To Prove Statement of Type (p → q).
(1) Rule of conditional proof.
Let p is true statement and $s_1, s_2, \dots, s_n$ all previous axioms and theorems. To prove p → q it is enough to prove

$$s_1, s_2, \dots, s_n, p \mapsto q$$

is valid argument.

Example 1.10.2. Prove that, a is an even number → a^2 is an even number.
Proof.
Suppose a is an even number.
(1) $a = 2k$, k is an integer (definition of even number).
(2) $a^2 = 4k^2$, square both sides of (1)
(3) $a^2 = 2(2k^2)$, Common factor
(4) a^2 is even number, since $2k^2$ is an integer and definition of even number.

Note that in the above proof, we proved the tautology

$$(s_1 \wedge s_2 \wedge p) \to q$$

where
p: a is an even number
s_1: $a = 2k$,
s_2: $a^2 = 4k^2$,
q: a^2 is even number.

(2) Contrapositive
To prove p → q we can proof that (∼ q → ∼p) since (p → q) ≡ (∼ q → ∼p).

Example 1.10.3. Prove that, $a^2$is an even number → a is an even number.
Proof.
Let p: $a^2$is an even number,
q: a is an even number.
Then
∼p: $a^2$is an odd number,
∼q: a is an odd number.
Therefore, the contrpositive statement is

a is an odd number $\rightarrow$ a^2 is an odd number.
(1) $a = 2k + 1$ k is an integer (Definition of odd number)
(2) $a^2 = 4k^2 + 4k + 1$ Square both sides of (1)
(3) $a^2 = 2(2k^2 + 2k) + 1$
(4) a^2 is odd number since $2k^2 + 2k$ is an integer and definition of odd number.

Prove Statement of Type (p ⟷ q). 1.10.4.
(i) Since (p ⟶ q) ∧ (q ⟶ p) ≡ (p ⟷ q), so we can proved first p ⟶ q and then proved q ⟶ p.
(ii) Moved from p into q through series of logical equivalent statements s_i as follows:

$$p \leftrightarrow s_1$$
$$s_1 \leftrightarrow s_2$$
$$\vdots$$
$$s_{n-1} \leftrightarrow s_n$$
$$s_n \leftrightarrow q$$

This is exactly the tautology

$$((p \leftrightarrow s_1) \wedge (s_1 \leftrightarrow s_2) \wedge \ldots \wedge (s_n \leftrightarrow q)) \rightarrow (p \leftrightarrow q).$$

Prove Statement of Type $\forall x\, P(x)$ or $\exists x\, P(x)$. 1.10.5.

(i) To prove a sentence of type $\forall\, x\, P(x)$, we suppose x is an arbitrary element and then prove that $P(x)$ is true.
(ii) To prove a sentence of type $\exists\, x\, P(x)$, we have to prove there exist at least one element x such that $P(x)$ is true.

Prove Statement of Type (p ∨ r) ⟶ q. 1.10.6.

Depending on the tautology

$$[(p\rightarrow q)\wedge(r\rightarrow q)] \rightarrow[(p\vee r)\rightarrow q]$$

We must prove that p⟶q and r⟶q.

Example 1.10.7. Prove that

$$(a = 0 \vee b = 0) \rightarrow (ab = 0)$$

where a, b are real numbers.
Proof.
Firstly, we prove that $(a = 0) \rightarrow (ab = 0)$.
Suppose that $a = 0$, then $ab = 0.b = 0$.
Secondly, we prove that $(b = 0) \rightarrow (ab = 0)$.
Suppose that $b = 0$, then $ab = a.0 = 0$.

Therefore, the statement $(a = 0 \vee b = 0) \rightarrow (ab = 0)$ is tautology.

Proof by Contradiction 1.10.8.

The contradiction is always false statement whatever the truth values of its components. Proof by contradiction is type of indirect proof.
The way of proof logical proposition **p** by contradiction start by supposing that ~p and then try to find sentence of type

$$R \wedge \sim R$$

where R is any sentence contain **p** or any pervious theorem or any axioms or any logical propositions.
By this way we can also prove sentences of type $\forall x\, P(x)$ or $\exists x\, P(x)$ or (p → q) or (p ⟹q).

Example 1.10.9. Prove that $(x \neq 0) \Longrightarrow (x^{-1} \neq 0)$, x is real number.
Proof.
Let p: $x \neq 0$,
q: $x^{-1} \neq 0$.
We must prove p ⟹q.
Suppose ~(p ⟹q) is true.
(1) ~ (p → q) is tautology — Def. of logical implication.
(2) ~ (p → q)≡ ~ (~ p∨q) — Implication Law
(3) p ∧ ~q is tautology, — De Morgan's Law
(4) $x \neq 0 \wedge x^{-1} = 0$.
(5) $x \cdot x^{-1} = 1 \neq 0$.
(6) $x \cdot x^{-1} = x \cdot 0 = 0$.
(7) $1 = 0$, — Inf. (5), (6).
(8) This is contradiction, since $(1 \neq 0) \wedge (1 = 0)$ — Contradiction Law
Thus, the statement ~(p ⟹q) is not true. Therefore, p ⟹q.

Application Example:
Cryptography (التشفير)

A	1 0 1 0 0	Q	1 0 0 1 1
B	0 0 0 1 0	R	1 0 0 1 0
C	1 0 1 0 1	S	1 0 0 0 0
D	0 0 1 1 0	T	0 1 1 1 0
E	1 0 1 1 0	U	0 0 0 1 1
F	1 0 1 1 1	V	0 1 1 0 1
G	1 1 0 0 0	W	0 1 1 1 1
H	1 1 0 1 0	X	0 0 1 0 0
I	0 0 0 0 1	Y	0 1 1 0 0
J	1 1 0 0 1	Z	1 0 0 0 1
K	0 0 1 1 1	Space	1 1 1 1 1
L	0 1 0 1 1	0	1 1 0 1 1
M	0 1 0 1 0	1	1 1 1 0 0
N	0 1 0 0 1	2	1 1 1 0 1
O	0 1 0 0 0	3	1 1 1 1 0
P	0 0 1 0 1	4	0 0 0 0 0

Key: 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0 1 0 1 1 1 1 0 0 0

Plaintext: GO HOME

Plaintext	G	O		H	O	M	E
Code	11000	01000	11111	11010	01000	01010	10110
Key	00101	01100	11010	10111	10000	01010	11001
XOR							
Encryption	11101	00100	00101	01101	11000	00000	01111
Ciphertext	2	X	P	V	G	4	W

Ciphertext	2	X	P	V	G	4	W
Code	11101	00100	00101	01101	11000	00000	01111
Key	00101	01100	11010	10111	10000	01010	11001
XOR							
Decryption	11000	01000	11111	11010	01000	01010	10110
Plaintext	G	O		H	O	M	E