Lecture 6

The Centripetal Acceleration, Centrifugal Force and Gravity

6.1 Balanced Forces

A ball of mass m is attached to a string and whirled through a circle of radius r at a constant angular velocity ω. From the point of view of an observer in inertial space, the speed of the ball is constant, but its direction of travel is continuously changing so that its velocity is not constant. To compute the acceleration we consider the change in velocity $\delta\vec{V}$ that occurs for a time increment δt during which the ball rotates through an angle $\delta\theta$ as shown in Fig. 6.1. Because $\delta\theta$ is also the angle between the vectors $\vec{V}$ and $\vec{V} + \delta\vec{V}$, the magnitude of $\delta\vec{V}$ is just $|\delta\vec{V}| = |\vec{V}|\,\delta\theta$. If we divide by δt and note that in the limit $\delta t \to 0$, δV is directed toward the axis of rotation, we obtain:

$$\frac{\delta\vec{V}}{\delta t} = |\vec{V}|\frac{\delta\theta}{\delta t}\left(-\frac{\vec{r}}{r}\right) \qquad (6.1)$$

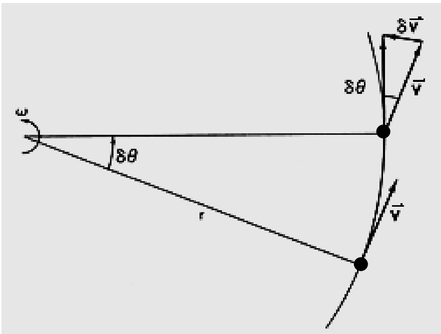

Fig. 6.1 Centripetal acceleration is given by the rate of change of the direction of the velocity vector, which is directed toward the axis of rotation, as illustrated here by $\delta\vec{V}$

However, $|\vec{V}| = \omega r$ and $\delta\theta/\delta t = \omega$, so that:

$$\frac{\delta\vec{V}}{\delta t} = -\omega^2 r \qquad (6.2)$$

Therefore, viewed from fixed coordinates the motion is one of uniform acceleration directed toward the axis of rotation and equal to the square of the angular velocity times the distance from the axis of rotation. This acceleration is called centripetal acceleration. It is caused by the force of the string pulling the ball.

Now suppose that we observe the motion in a coordinate system rotating with the ball. In this rotating system, the ball is stationary, but there is still a force acting on the ball, namely the pull of the string. Therefore, in order to apply Newton's second law to describe the motion relative to this rotating coordinate system, we must include an additional apparent force, the centrifugal force, which just balances the force of the string on the ball. Thus, the centrifugal force is equivalent to the inertial reaction of the ball on the string and just equal and opposite to the centripetal acceleration.

To summarize, observed from a fixed system the rotating ball undergoes a uniform centripetal acceleration in response to the force exerted by the string. Observed from a system rotating along with it, the ball is stationary and the force exerted by the string is balanced by a centrifugal force.

6.2 Gravity Force

An object at rest on the surface of the earth is not at rest or in uniform motion relative to an inertial reference frame except at the poles. Rather, an object of unit mass at rest on the surface of the earth is subject to a centripetal acceleration directed toward the axis of rotation of the earth given by $-\Omega^2\vec{R}$, where $\vec{R}$ is the position vector from the axis of rotation to the object and $\Omega = 7.292 \times 10^{-5}\ rad\ s^{-1}$ is the angular speed of rotation of the earth.

Viewed from a frame of reference rotating with the earth, however, a geopotential surface is everywhere normal to the sum of the true force of gravity, g^*, and the centrifugal force $\Omega^2\vec{R}$ (which is just the reaction force of the centripetal acceleration). A geopotential surface is thus experienced as a level surface by an object at rest on the rotating earth. Except at the poles, the weight of an object of mass m at rest on such a surface, which is just the reaction force of the earth on the object, will be slightly less than the gravitational force mg^* because, as illustrated in Fig. 6.2, the centrifugal force partly balances the gravitational force. It is, therefore, convenient to combine the effects of the gravitational force and centrifugal force by defining gravity g such that:

$$\vec{g} \equiv -g\,k \equiv \vec{g}^* + \Omega^2\vec{R}$$

where k designates a unit vector parallel to the local vertical. Gravity, g, sometimes referred to as “apparent gravity,” will here be taken as a constant ($g = 9.81\ ms^{-2}$). Except at the poles and the equator, g is not directed toward the center of the earth, but is perpendicular to a geopotential surface as indicated by Fig. 6.2.

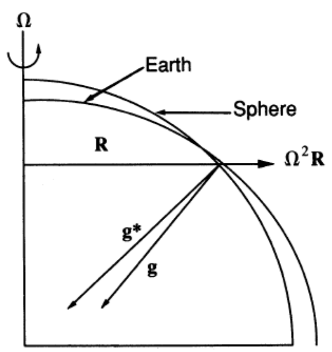

Fig. 6.2