Lecture 8

Laser Beam Profile

The TEM_{00} mode and the longitudinal mode is mathematically described by a Gaussian profile

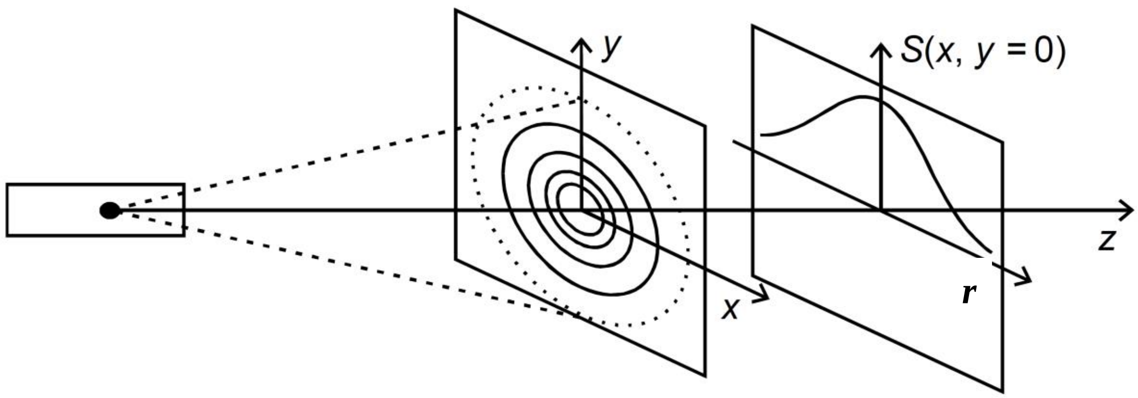


Figure 23: Intensity distribution in the plane perpendicular to the propagation direction

$$I(r) = I_0 \exp\left(\frac{-2r^2}{w^2}\right) \ldots \ldots (38)$$

Where w is beam waist, r is the distance from the center, where I_0 is the intensity on the beam axis at any location. The laser light propagation is shown in figure 24.

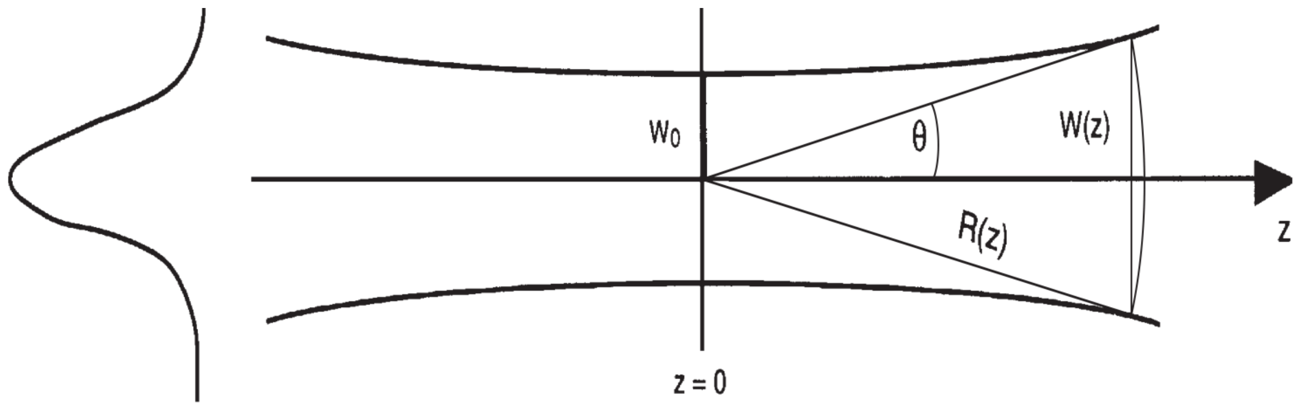


Figure 24: Schematic structure of a Gaussian light beam in the vicinity of a focal volume. The y-axis is perpendicular to the paper sheet. The z = 0 origin is at the minimum radius W_0 of the beam.

The beam waist, varying along the axis of the laser, the smallest waist is in the medal of the optical axes propagation of z-direction. Gaussian beams have a minimum beam waist w_0 that usually occurs somewhere between the laser mirrors. The beam then gradually expands from that location. If the laser mirrors have the same radius of curvature, the minimum beam waist occurs exactly halfway between the mirrors.

Types of spectral line shape broadening in the laser physics

We've treated the spectra of light emitted by atoms transitioning between states E_2 and E_1 as having a single frequency,

$$\nu = (E_2 - E_1)/h$$

The emission lines of atomic transitions have some finite spectral width, which is called line broadening; this broadening is caused by various broadening mechanisms due to light-matter interaction.

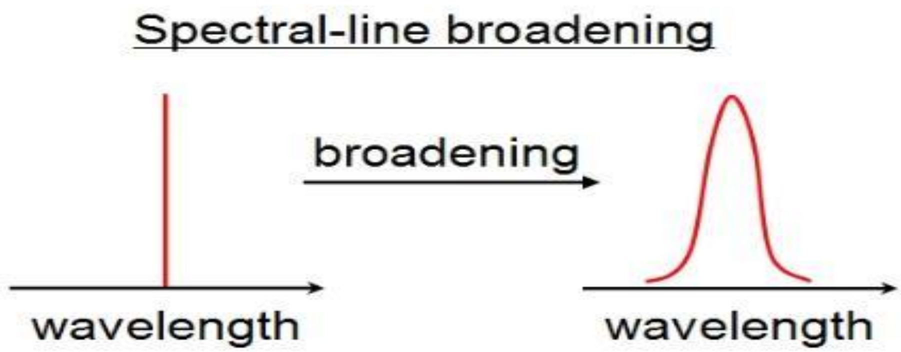


Figure 25: left side shows there isn't any kind of line shape broadening, the right side shows the broadening in the spectral line shape.

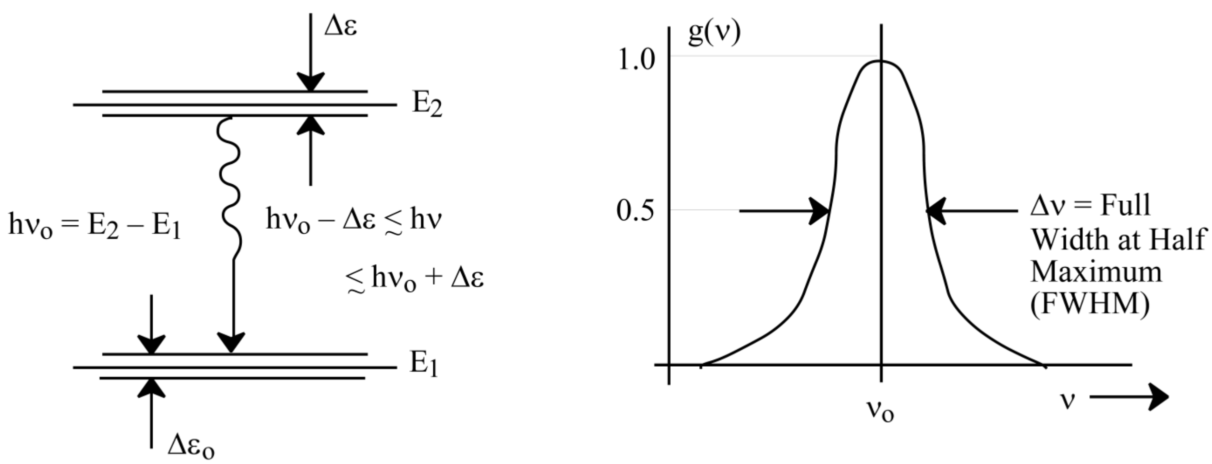


Figure 26: the right side shows the spectral line shape broadening. The left side of the figure shows the splitting of the energy levels due to present of EM field which causes the spectral line shape broadening.

The splitting of the energy levels $\Delta\varepsilon$ due to the presence of the electromagnetic field (EM) field causes the atom or molecules emits slightly different energy photons, which causes the broadening in the spectral line shape, see the left side of figure 26.

Materials can be classified into two basically different groups:

1. Homogeneous

All atoms, molecules behave similarly in the light-matter interaction, they have the same individual line shape function.

$$\int_0^\infty g(v)dv = 1$$

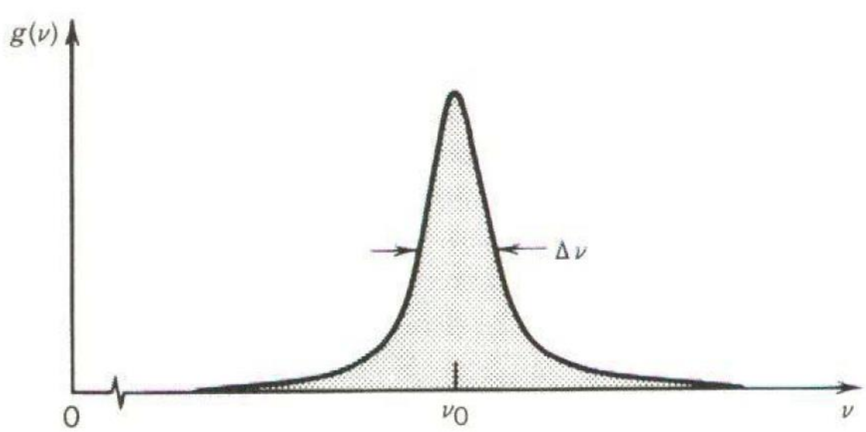
Where $g(\upsilon)d\upsilon$ is define the line shape function. The line centers, υ_0, for all atoms/molecules are the same position in the homogeneous broadening, see figure 26.

2. Inhomogeneous

Group of atoms and molecules behave differently in the light-matter interaction. The light-matter interaction, which often causes their line centers, υ_0, to be different position for inhomogeneous broadening, see figure 26.

1. Homogeneous broadening follows (Lorentzian distribution)

Homogenous broadening is following Lorentzian function. The homogeneous line shape is most often given by a "Lorentzian function"



Wavepacket emissions at random time and the Lorentz-function

There are two types of homogeneous broadening

a) Natural broadening

The energy levels of atom or molecule are split due to present of electromagnetic field (EM) field, therefor atoms can absorb photons with a slightly different energy, see the left side of figure 26. The spectral line shape of the natural broadening is shown on the right side of figure 26.

- Lifetime or natural broadening

From Heisenberg's uncertainty principle: The electron in an excited state is only there for a short time, so its energy cannot have a precise value.

$$\Delta E \Delta t \geq \hbar \ , \ \ \Delta E \Delta t = \frac{h}{2\pi},$$

Both selected energy levels are excited levels, both have lifetime broadenings

$$E_1 = \frac{h}{2\pi t_1}, \quad E_2 = \frac{h}{2\pi t_2},$$

$$\Delta E = E_1 + E_2 = \frac{h}{2\pi t_1} + \frac{h}{2\pi t_2} = h(\frac{1}{2\pi t_1} + \frac{1}{2\pi t_2})$$

Using the relation $E=h\upsilon$

$$h\Delta\nu = h\left(\frac{1}{2\pi t_1} + \frac{1}{2\pi t_2}\right)$$

$$\Delta\nu = \frac{1}{2\pi}\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$$

Where t_1 and t_2 is the total radiative lifetime of levels E_2 and E_2, $\Delta\upsilon$ is the FWHM (full width half maximum in frequency term) of the spectral line shape.

b) Collisional/pressure broadening

In gas and dye lasers, the collisions occur between atoms or molecules. These collisions lead to an exchange of energy (*elastic and inelastic collisions*), which causes effectively to a broadening of energy levels. If t_c is the mean time between collisions and each collision results in a transition between two states there is a line broadening $\Delta\upsilon_c$ of the transition, where

$$\Delta\nu_c = \frac{1}{2\pi}\left(\frac{1}{\mathrm{t}_{1\mathrm{collision}}} + \frac{1}{\mathrm{t}_{2\mathrm{collision}}}\right) = \frac{1}{\pi t_{\mathrm{collision}}}$$

Problem 19:

Calculate the FWHM of the natural broadening or the He-Ne laser red line (λ= 6328 A^o) obtained from the transition between the two energy levels $3S_2$ (τ_2 = 19.6 × 10^{-9} s) and $2P_4$ (τ_1 = 18.7 × 10^{-9} s).

Sol:-

$$\Delta\nu = \frac{1}{2\pi}\left(\frac{1}{\mathrm{t}_1} + \frac{1}{\mathrm{t}_2}\right) = 16.63 \text{ M Hz.}$$

2. Inhomogeneous broadening follows (Gaussian distribution)

Inhomogeneous distribution of doping materials in solids causes inhomogeneous broadening and its follow the Gaussian distribution, the inhomogeneous broadening is Doppler thermal broadening.

➢ Doppler thermal broadening

If an atom or molecule is traveling towards the detector with a velocity v, then the frequency υ_a at which a transition is observed to occur is related to the actual transition frequency υ (or the rest frequency) is:

$$\upsilon_a = \upsilon\left(1 \pm \frac{\mathrm{v}}{c}\right)$$

A particular case is the thermal Doppler broadening due to the thermal motion of the particles. The characteristic line broadening is given by:

$$\Delta\upsilon = \frac{\upsilon}{c}\left[\frac{2kT\ ln\ 2}{m}\right]^{1/2}$$

Where m is the mass of the atom or molecule, k Boltzmann constant, T the temperature, υ the actual or the rest frequency.

This effect causes for the same reason that an observer hears the whistle of a train traveling towards him or her as having a frequency apparently higher than it really is, and lower when it is traveling away from him.

Problem 20:

Calculate the Doppler broadening line width for the 488-nm transition of an argon ion laser, given that the temperature of the discharge is 6000 K and the atomic mass of argon is 39.95 amu. Repeat the previous calculation for the 632.8-nm line of a He-Ne laser, where the temperature of the discharge is about 400 K. The atomic mass of neon is M= 20.18amu.

Sol:

The Doppler linewidth is:

$$\Delta\upsilon = 2\upsilon\left(\frac{2kTln2}{mc^2}\right)^{1/2}$$

In case of argon, where $\upsilon=c/\lambda$, 1 amu=1.66×10^{-27} kg is an atomic mass unit, and Boltzmann constant k=1.38×10^{-23} m^2 kg/s^2K

$$\Delta\upsilon = 5.39\ GHz$$

In the case of He-Ne laser

$$\Delta\upsilon = 1.5\ GHz$$

3. Homogenous & Inhomogeneous broadening (Voigt distribution)

The two effects Homogenous & Inhomogeneous broadening may appear together in the spectral line shape and its follow Voigt profile see figure 27.

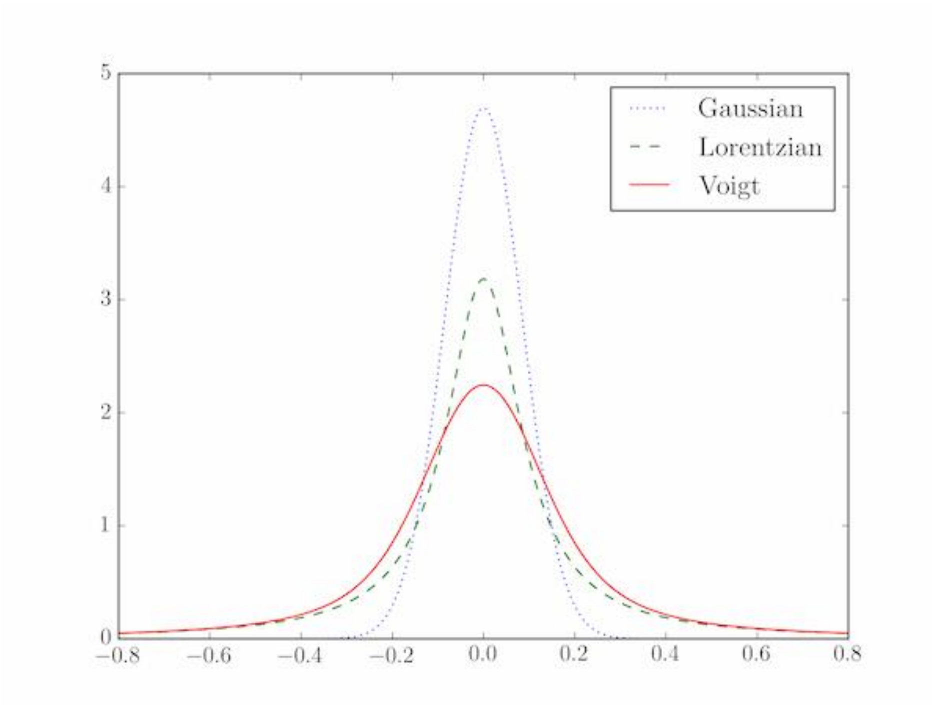


Figure 27: shows the Gaussian, Lorentzian and Voigt distributions

The Voigt profile is a probability distribution given by a convolution of a Lorentz distribution and a Gaussian distribution

Problem 21:-

The gas pressure in He-Ne laser equal (0.67 mbar) , The collision time of the between He-Ne atoms ($\tau_c = 0.5 \times 10^{-6}$ s). Find the width of spectral line at half maximum intensity.

Sol :-

$$\Delta\upsilon = \frac{1}{\pi t_{\text{collision}}} = 1/\ 3.14 \times 0.5 \times 10^{-6}\ \text{s} = 0.637 \times 10^{6}\ \text{Hz}$$