Chapter three

Hydrostatic define: the science that deal with the force on static fluids.

Hydrodynamic define: the science that deal with the force on fluids during movement.

1- **Pressure** is defined as force divided by area (force/area)

• One can also assume that pressure = weight/area (Show figure (1))

$$P = \frac{F}{A}$$

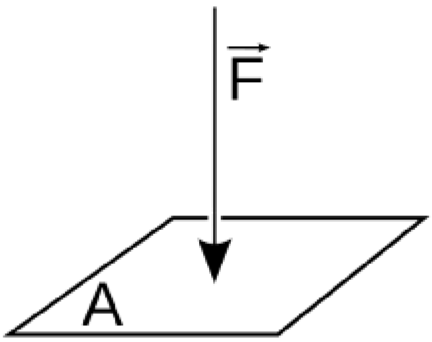


Figure 1: Show the pressure

$$P = \frac{Force}{Area} = \frac{Force \times distance}{Area \times distance} = \frac{Work}{Valume} = \frac{Energy\ (J)}{volumem^3}$$

Can consider the pressure as energy density.

The SI unit for pressure is the pascal (Pa), equal to one newton per square metre (N/m^2, or kg·m^{-1}·s^{-2}). This name for the unit was added in 1971; before that, pressure in SI was expressed simply in newtons per square metre. Other unit atmosphere called torr

1 torr=1.33*10^5 (N/m^3)*10^{-3} 133 N/m^2

Calibrated atmospheric pressure =760 torr =1.01*10^5 N/m^2

1 atmosphere = 10^5 N m^{-2}

2-Find the pressure on static fluid.

If we have tank of high (H) and suppose a point inside the tank on depth (h) see figure (2). Put a cylinder in the tank and fill Put a cylinder in the tank and fill with fluid suppose that (ρ) is density, (m) mass and (V) volume of fluid. The normal force on the surface cylinder is atmosphere (P_0), and all side force will vanish (equal in values and opposite in direct).
The normal force on the surface cylinder upward equal.

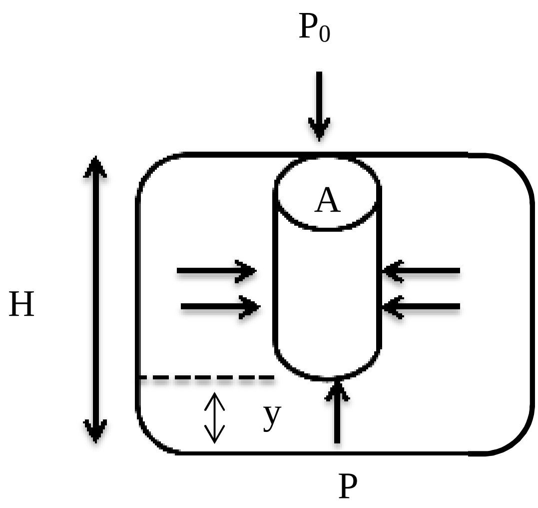


Figure :2

F=PA

P = pressure and (A) cross section area of cylinder. Force up equal force down.

$PA=P_0A$

This force equal to the weight of the fluid in the cylinder.

Density is defined as mass divided by volume (mass/volume).

$\rho= m/V$

$m =\rho V$ and $V= h A$

$m = \rho hA$

$W = mg$

$W = \rho gh$

$(P-P_0)A = \rho\, g\, h\, A$

$(P-P_0) = \rho gh$ …….This equation of Hydrostatic.

The pressure differences (P-P0) represent the pressure at the depth (h). And called pressure gauge at this point. And depend only on the **(depth)** and not depend on the shape.

Example: A sub can dive to a depth equal (1000m) under the sea. Calculate maximum pressure can the body of sub hold. If the atmosphere (75cm.Hg) and the sea density (1.o3gm /cm^3) and the mercury density (13.6gm /cm^3).

Solution

Maximum pressure = atmosphere +pressure of water

$P = P_0 + \rho gh$

$P_0 = \rho gh = 13.6 * 10^3 *9.8 *75 *10^2 = 99.96 *10^3$ N/m^2

$P = P_0 + \rho gh = 99.96 *10^3 +1.03 *10^3 *9.8 *1000 = 10.2 *106$ N/m^2

3- Pressure gauges

The device which is used in measuring the pressure in liquid or gases. There is many kind of pressure gauges.

1-Monometer

2-Electromanometer

3-Barometer

Monometer the device of tube (U shape) contain a liquid (water or Mercury) one of the ends tube opened and other end is connected to container we want to measure pressure. See figure 3. Points (A&B) are equal pressure because they are same level. The high of liquid in opened end (h) from Hydrostatic equation

$(P-P_0) = \rho gh$

Mercury density (13.6 gm/cm^3)

$\rho gh = 13.6*10^3 \ Kg\ m^{-3} * 9.8* h$

$= (133.3 *10^3)\ h$

$= (1.33* 10^5 h)\ Nm^{-3}$

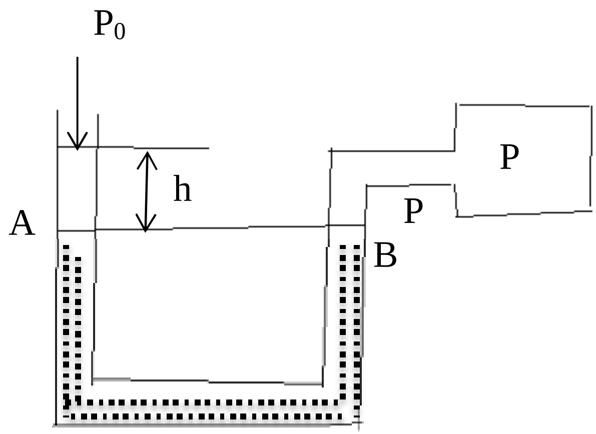


figure 3: Monometer device

4-The flow of ideal fluid

To begin with, let us define a fluid as “a substance as a liquid, gas or powder, that is capable of flowing and that changes its shape at steady rate when acted upon by a force”. We can distinguish four main types of fluid flow.

1- **Stationary flow:** velocity doesn’t change in time.

2- **Nonstationary:** velocity vectors components of fluid elements are not the functions of the time.

3- **Streamline:** the flow is smooth, such that neighboring layers of the fluid slide by each other smoothly

Ideal fluid can be described uncompressible, have no inner force of fraction effect no it and on viscosity. These forces caused also in mechanical energy.

5- Bernoulli's equation

The basic equation of hydrodynamic is Bernoulli's equation (Mathematical relationship between pressure, velocity and the high of the lines of flow fluid (uncompressible ignored viscosity). suppose that (ρ) is density, (m) mass and (V) velocity of fluid. (A_1) Cross section area has the distance high (Y_1) the (V_1) velocity, pressure (P_1) Let and (Δx_1)during (Δt_1) and the second (A_2) cross section area has high (Y_2) the (V_2) velocity and pressure (P_2). Let the distance (Δx_2) during (Δt_2) show figure (4). Conditions

1-Regular flow

2-Conlinous flow

3-Uncompressable

4-Unvisscuse

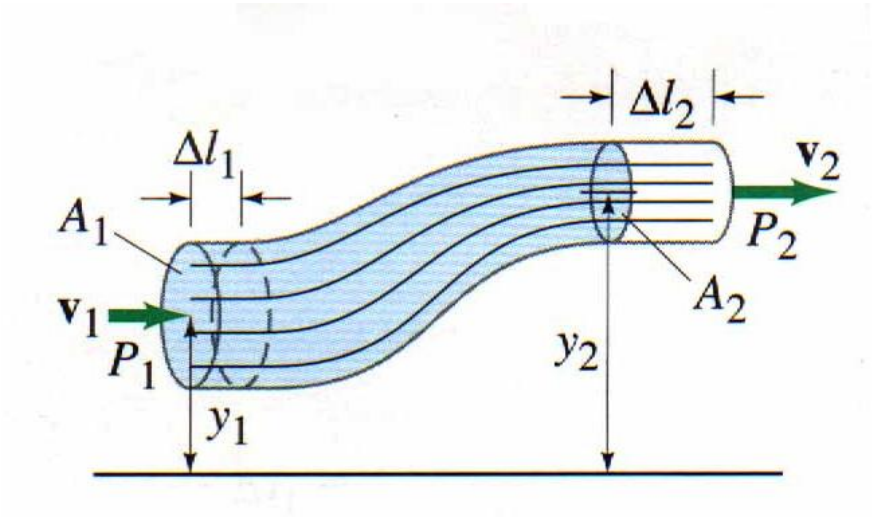


Figure 4. Fluid flow: for derivation of Bernoulli's equation.

Work done on system =FX = $P_1A_1\Delta x_1$

V=X/t

P= F/A

F=PA

ρ= m/V

Volume = A Δx

Net Work done on system = $P_1A_1\Delta x_1$ - $P_2A_2\Delta x_{21}$

(m) is mass

V = m/ ρ

$A_1\Delta x_1 = A_2\Delta x_2$ = m / ρ =

Network = (P_1-P_2) m/ ρ

Kinetic energy of system equal = $1/2mV^2_2$ $-1/2\ mV^2_1$

Potential energy = $mgy_2 - mgy_1$

According to the conservation of energy the network equal to the change in its kinetic and potential energy

$(P_1 - P_2)$ m/ ρ = $(½\ mV^2_2 - ½\ mV^2_1) + (mgy_2 - mgy_1)\}*\ \rho/m$

$(P_1 - P_2) = (½\ \rho\ V^2{}_2 - ½\ \rho\ V^2{}_1) + (\rho g y_2 - \rho g y_1)$

$P_1 + ½\ \rho\ V^2{}_1 + \rho g y_1 = P_2 + ½\ \rho\ V^2{}_2 + \rho g y_2$

Where $(1/2\ \rho\ V^2)$ represent the density of kinetic energy.

The **Bernoulli Equation state** (That the sum of the kinetic, potential and pressure energies of a fluid particle is constant along streamline during steady flow).

Limitation on the Use of the Bernoulli Equation

1-Steady flow

2. Frictionless flow
3. No shaft work
4. Incompressible flow
5. No heat transfer
6. Flow along a streamline.

Problem 1

Water is flowing in a fire hose with a velocity of 1.0 m/s and a pressure of 200000 Pa. At the nozzle the pressure decreases to atmospheric pressure (101300 Pa), there is no change in height. Use the Bernoulli equation to calculate the velocity of the water exiting the nozzle. (Hint: The density of water is 1000 kg/m3 and gravity g is 9.8 m/s2. Pay attention to units!)]

Answer:

$\frac{1}{2}\rho v_1^2 + \rho g h1 + P1 = \frac{1}{2}\rho v_2^2 + \rho g h2 + P2$

Since the height does not change (*h1=h2*), the height term can be subtracted from both sides.

$\frac{1}{2}\rho v_1^2 + P1 = \frac{1}{2}\rho v_2^2 + P2$

Algebraically rearrange the equation to solve for v_2, and insert the numbers.

$$\sqrt{\frac{2}{\rho}\left(\frac{1}{2}\rho V_1^2 + P_1 - P_2\right)} = V_2 = 14m/s$$

6- Continuity equation

Continuously equation Suppose the figure (5) below which referents a tube where the fluid is flow from left to right side. Let (A) cross section area and (v) velocity during a time (t) The volume of fluid (V)

$X_1 = v_1 t$ and $X_2 = v_2 t$

V= the volume of fluid pass throw (A)

$V_1 = A_1 v_1 t$ and $V_2 = A_2 v_2 t$

The volume per unit time pass throw point (1) to point (2) is

$A_1v_1 = A_2v_2$

Av = constant

Condition

1-The fluid flow regular

2-containous

3-Uncompressable

4-unviscous

AV is the valium of the flow fluid throw the tube per unit of time.

When the tube gets narrow the flow of fluid will be faster.

For ideal fluid the velocity on the cross section of the tube is steady at any point. The rate of velocity of fluid is 0

Average velocity of fluid =$\bar{\upsilon}$ = volume rate of flow /cross section area

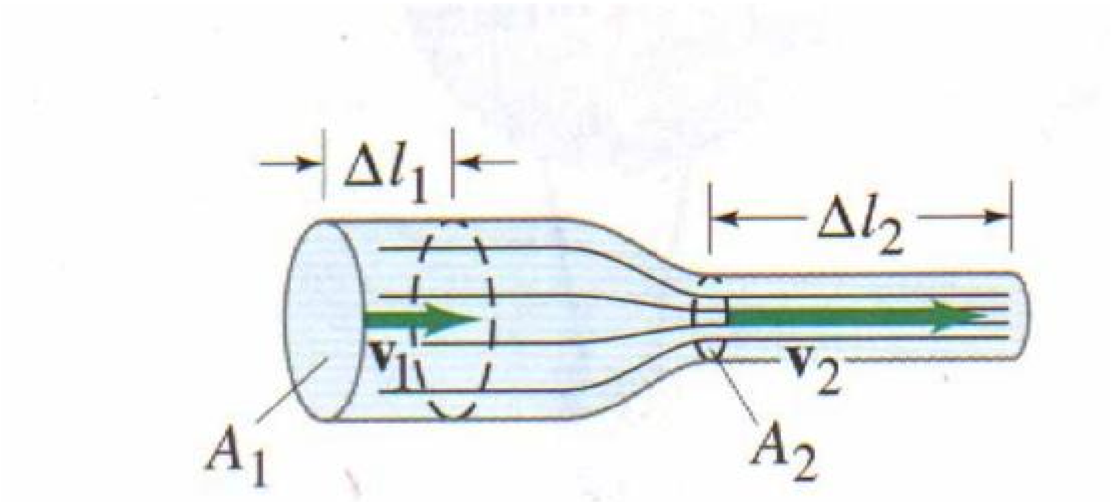


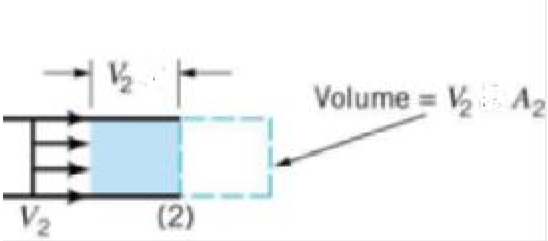
figure 5: below which referents a tube where the fluid is flow from left to right side

The equation of continuity and the Bernoulli's equation are used into conjunction to analyze many flow situations.

Example

Given the water velocity at (2) is 8.0 m/s and the pipe diameter is 0.10 m , what are the volume and mass flow rates?

$Q = vA = v\,\pi\,(d/2)^2$
$= 8.0\ \times \pi(\ 0.050)^2 = 0.06283\ m^3/s$



The mass flow is just

$M = Q\ \times \rho = 1000\ \times\ 0.06283 = 62.83\ kg/s$

Example

A fluid flow in a tube its lower diameter (40 cm) then it's get narrow and its diameter become (25 cm) at high (6 m) if the pressure at the lower cross section is ($1.5*10^3\ N/m^2$) and the density of fluid ($0.8\ gm/\ cm^3$) and the rate of flow is (0.1 m3/ sec) calculate the pressure at the upper cross section.

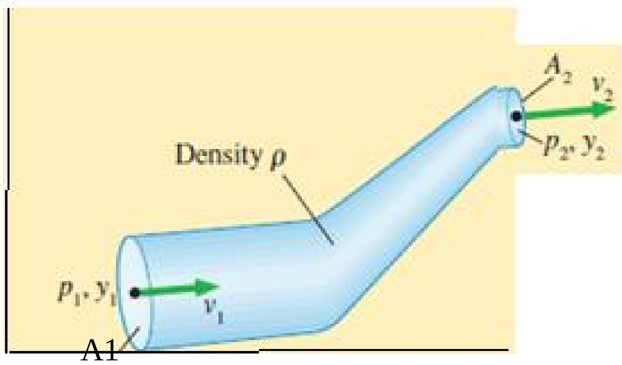


Figure 6: A fluid flow in a tube

Solution

$Q = A_1V_1 = A_2V_2$

$V_1 = Q/\ A_1 = 0.1/\ (40/2\ *10^{-2})^2\ \pi = 0.8\ m/s$

$V_2 = Q/ A_2 = 0.1/ (25/2 * 10^{-2})^2 \quad \pi = 2.04 m/s$

$½ \rho v_1^2 + \rho g y_1 + P_1 = ½ \rho v_2^2 + \rho g y_2 + P_2$

$1.5*10^3 + 1/2 * 0.8 * 1000 * (0.8)^2 + 0 = P_2 + 1/2 * 0.8 * 1000 * (2.04)^2 + 0.8 * (1000*9.8*6)$

$P_2 = 1.0155*10^5$ N/m2

7- Application of Bernoulli's equation

The equation of Hydrostatic are **special cases** of Bernoulli's eq. For example when velocity approach to zero.

$P_2 = P_1$

$V_1 = V_2 = 0$

And calculate the high from ground

$P_1 + \rho g y_1 = P_2 + \rho g y_2$

$P_1 - P_2 = (y_2 - y_1)$

$P_1 - P_2 = h$

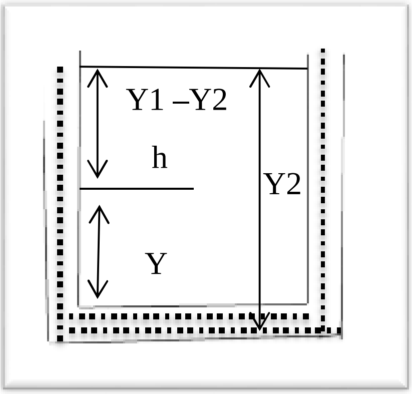


Figure (7)

8-Torricellis theorem

Torricelli's law was first explained by D. Bernoulli, via an energy argument. Torricelli's law is typically presented as an example of the steady-state Bernoulli equation, which does lead to the prediction that the fluid emerges from a hole with velocity (v). Figure (8) show a container from upper side and inside it liquid of density (ρ) at point (1) where the depth is (h) then the pressure at point (1) to (2) is atmospheric (P_0) because both point are open to atmosphere if the gap at point small then the fluid will decrease slowly so (V_2) is very small can be considered zero.

$½ \rho v_1^2 + \rho g y_1 + P_0 = P_0 + \rho g y_2 + 0$

$P_2 = P_1 = P_0$

$V_1^2 = 2g\,(y_2 - y_1)$

$V1 = \sqrt{2gh}$

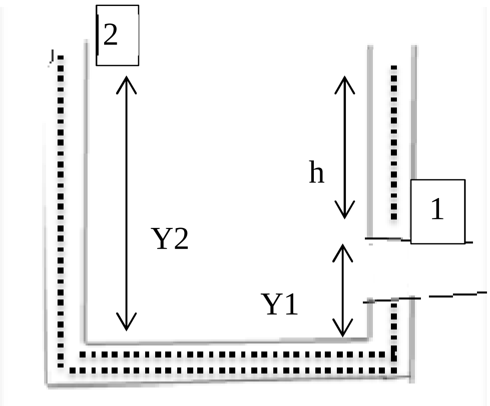


Figure 9

Which mean that the velocity of fluid proportion with the square root of the high (h) and called Torricelli theorem.

Example

A circular hole of diameter (2.5) cm has been cut in the side of a large stand pipe at a high (610) cm under the level of water inside the pipe find.
1-Velocity of flow

2- Volume of flow per unit of time

$$V1 = \sqrt{2gh} = \sqrt{2 * 9.8 * 610 * 10^{-2}} = 10.934 \text{ m/s}$$

$Q = AV\ (2.54/2 * 10^{-2})^2\ \pi * 10.934 = 5540.34 * 10^{-6}\ \text{m}^3/\text{s}$

9- Venturi Meter

A venturi meter is a measuring or also considered as a meter device that is usually used to measure the flow of a fluid in the pipe. A Venturi meter may also be used to increase the velocity of any type fluid in a pipe at any particular point. It basically works on the principle of Bernoulli's Theorem. The pressure in a fluid moving through a small cross section drops suddenly leading to an increase in velocity of the flow. The fluid of the characteristics of high pressure and low velocity gets converted to the low pressure and high velocity at a particular point and again reaches to high pressure and low velocity. The point where the characteristics become low pressure and high velocity is the place where the venturi flow meter is used.

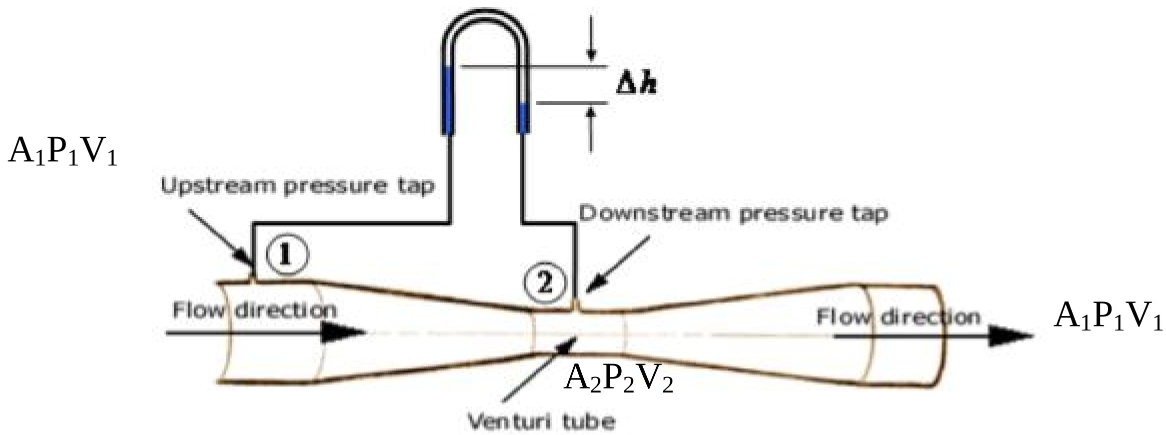


Figure 10. Venturi meter

Venturi tube is used to measure the volume of fluid pass throw the tube per unit of time.

$P_1 + ½\, \rho v_1^2 = P_2 + ½\, \rho v_2^2$

$V_2 > V_1$

$P_2 < P_1$

The difference in pressure ($P1 – P_2$)

$A_1V1 = A_2V_2$

$V_2 = A_1/A_2\, V_1$ ……………. (1)

$P_1 – P_2 = ½\, \rho v_1^2 – ½\, \rho v_2^2$

$P_1 – P_2 = ½\ (v_1^2 - v_2^2\ )$ ………….. (2)

Put (1) in (2)

$P_1 – P_2 = ½\ (A_1 / A_2\, (v_1^2 – v_1^2\ )$

$P_1 – P_2 = ½\ v_1^2\, [\, (A_1 / A_2)^2 - 1]$

$v_1{}^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$

If (h) is the difference of fluid high between the manometer.

$P_1 - P_2 = \rho g h$

$v_1{}^2 = \frac{2\rho_1 g h}{\rho_2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$

Where K is constant for any Venturian tube

This method is used to measure the velocity of blood flow.

Example

1- The diameter of venturiau tube is (4cm) and the diameter of the nik of the tube is (2cm) this tube is used to measure the rate of water flow and its found that the difference in pressure measured by manometer (22 torr)

Find (a) the velocity of the liquid at the basic end (b)the rate of volume flow.

Solution

The area of cross section of the tube is proportional the square of the diameter

$(A_1/A_2)^2 = (4/2)^2 = 4$

$(A_1/A_2)^2 - 1 = 19 - 1 = 15$

$v_1{}^2 = \frac{2\rho_1 g h}{\rho_2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$

$= 2 * 1.36 * 10^4 \, Kgm^{-3} * 9.8 * 22 * 10^{-3} m \,/\, 103 \, Kgm^{-3} * 15$

$= \quad 0.39 m^2 S^{-2}$

$V_1 = 0.62 mS^{-1}$

$A1V1 = \pi\,(0.04/2)^2 * 0.62 mS^{-1}$

Stokes law

The motion of small balls in a relatively viscous fluid produces a laminar flow at the surface of the ball. Given this prerequisite, Stokes' law can be used to calculate the frictional force acting on the ball:

$F_R = 6\,\pi\,\eta\,r\,v$

r - radius of ball
v - rate of fall of ball (= flow velocity)

Different fluids possess different amounts of viscosity: syrup is more viscous than water; grease is more viscous than the engine oil; liquids in general are much more viscous than gases. The viscosity of different fluids can be expressed quantitatively by the coefficient of viscosity, η (the Greek lowercase letter eta), which could be defined using the following experiment.
coefficient of viscosity, η: In the CGS system, the unit is dyne · s / cm^2 and the unit is called a poise (P). Viscosities are often given in centipoise* (1cP $=10^{-2}$P).

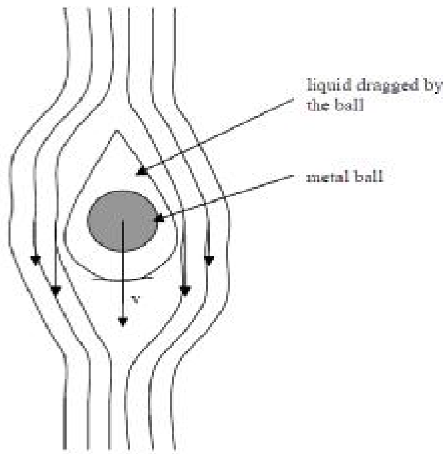


Figure 11: Scheme of gravitationally falling ball in viscous liquid.

If the fall of this ball under the gravity effect in the fluid then its final velocity will increase very fast when the fraction force equal to the ball weight (w). If ( ) is density of the ball and (ρ_0) is density of fluid

W= mg

ρ = m/V

$M = \rho V$

The weight equal to ($4/3\ \pi\ \rho\ r^3 g$) and the rising force toward the surface (B) equal to the weight of the removed fluid ($4/3\ \pi\ \rho_0\ r^3 g$).

The acceleration of the ball will vanish when its reach its final velocity.

The net of force upward = the net of force downward

$[4/3\ \pi\ \rho\ r^3 g = 4/3\ \pi\ \rho_0\ r^3 g + 6\ \pi\ \eta\ r\ v]\ /\ \pi\ r$

$[4/3\ \rho\ r^2 g = 4/3\ \rho_0\ r^2 g + 6\ \eta\ v]\ *3$

$[4\rho\ r^2 g = 4\rho_0\ r^2 g + 18\ \eta\ v]/2$

$2\rho\ r^2 g = 2\rho\ r^2 g + 9\ \eta\ v$

$2\ r^2 g\ (\rho - \rho_0) = 9\ \eta\ v$

$V = 2\ r^2 g\ (\rho - \rho_0)\ /\ 9\ \eta$

Where V is the final velocity.

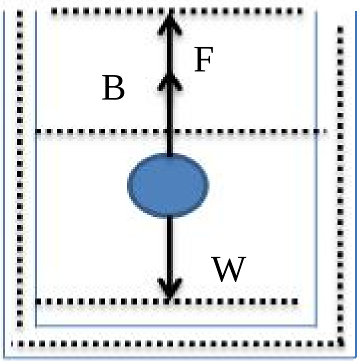


Figure 12:

Example

A bubble of radius (5mm) raise throw syrup with constant velocity (2mm/s) calculate the viscosity of the system.

$\rho_0 > \rho$

The air density inside the bubble can be ignored.

$V = \ = 2\ r^2 g\ (\rho - \rho_0)\ /\ 9\ \eta$

r = 0.005m

$\rho - \rho_0 = 1.4*10^3$ Khm^{-3}

V = 0002 m/s

$\eta = 2 r^2 g (\rho - \rho_0) / 9V$

$= 38 Kgm^{-1}s^{-1}$

Good Luck