ESCI 485 – Air/sea Interaction
Lesson 2 – Turbulence
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References: *Air-sea Interaction: Laws and Mechanisms*, Csanady
An Introduction to Dynamic Meteorology (3rd edition), J.R. Holton
An Introduction to Boundary Layer Meteorology, R.B. Stull
Structure of the Atmospheric Boundary Layer, Sorbjan

TURBULENT KINETIC ENERGY

- **Turbulent kinetic energy (TKE) is the kinetic energy (per unit mass) associated with turbulent eddies.**
- **Mathematically it is defined as**

$$e = \frac{1}{2}\left(\overline{u'^2 + v'^2 + w'^2}\right). \tag{1}$$

- **Turbulent kinetic energy is primarily transferred from larger scales to smaller scales.**
  - o **At the large scales the turbulence is generated via mechanical means (through sheared flows) or via buoyancy.**
  - o **Energy is *dissipated* at the very smallest scales due to molecular friction. This flow of turbulent energy from large scales to small scales is known as the *energy cascade*.**
  - o **The energy cascade is summed up very aptly in the verse,**

Big whorls have little whorls,
Which feed on their velocity,
And little whorls have lesser whorls,
And so on to viscosity.

- L. F. Richardson

- **An equation for the budget of turbulent kinetic energy (TKE) is given symbolically as**

$$\frac{\overline{D}e}{Dt} = MP + BP + TR - \varepsilon. \tag{2}$$

- **MP represents the mechanical production of turbulence.**
- **BP represents buoyant production.**
- **TR represents turbulent transport and redistribution by pressure perturbations.**
- **ε represents molecular dissipation.**

- ***Mechanical production (MP)* – The mechanical production term has the form**

$$MP = -\overline{u'w'}\frac{\partial \overline{u}}{\partial z}. \tag{3}$$

  - **Mechanical production occurs because of dynamic instabilities caused by sheared flow. The stronger the shear of the mean flow, the more turbulence will be produced.**
  - **This term can also be negative, and represent a loss of *TKE*.**

- ***Buoyant production* (*BP*) – The buoyant production term has the form**

$$BP = \overline{w'b'}, \tag{4}$$

where

$$b' = -g\frac{\rho'}{\rho_0} \tag{5}$$

and is the buoyancy perturbation.

  - **Buoyant production represents the production of turbulence due to thermals caused by heating of the surface (in the ocean, buoyant production would result from cooling of the surface).**
  - **Like MP, the BP term is usually largest near the surface. In a stable layer this term will be negative and represent a loss of *TKE*.**

- ***Transport and redistribution* (*TR*) – This term has two components, and is written as**

$$TR = -\left(\overline{u'\frac{\partial e}{\partial x}} + \overline{w'\frac{\partial e}{\partial z}}\right) - \frac{1}{\rho_0}\left(\overline{u'\frac{\partial p'}{\partial x}} + \overline{w'\frac{\partial p'}{\partial z}}\right). \tag{6}$$

  - **The first component represents the transport (or advection) of *TKE* by turbulent eddies.**
  - **The second component represents the effects of pressure perturbations.**

- ***Dissipation* – The dissipation term has the form**

$$\varepsilon = \upsilon \overline{\left[\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial z}\right)(u' + w')\right]^2} . \quad \textbf{(7)}$$

  - **Dissipation represents the loss of energy due to molecular friction (υ is the kinematic viscosity), and is always positive, so that in the *TKE* equation dissipation always represents a loss term.**

OTHER TURBULENCE CHARACTERIZATIONS

- **Velocity scale**
  - **One possible velocity scale is**

$$u_m^2 = \overline{u'^2} . \quad \textbf{(8)}$$

  - **If the turbulence is isotropic, then w' is of the same order as u', and Eq. (8) can be written as**

$$u_m^2 = \left|\overline{u'w'}\right| \quad \textbf{(9)}$$

    (the absolute value is needed to ensure that the velocity is real and not imaginary. This gives rise to the definition of *friction velocity*

$$u^* = \left(\left|\overline{u'w'}\right|\right)^{1/2} . \quad \textbf{(10)}$$

- **Eddy size**
  - **Eddy size is determined statistically**
  - **Use a "two-point" correlation function**

$$R_w(r) = \overline{w'(x)w'(x+r)} . \quad \textbf{(11)}$$

  - **The distance at which the velocity is uncorrelated (*R* = 0) is taken to be the characteristic eddy size or length scale, *l*.**
  - **The correlation function can also be defined in terms of horizontal velocities**

$$R_u(r) = \overline{u'(x)u'(x+r)} . \quad \textbf{(12)}$$

- **Fourier spectra**
  - **The Fourier transform pairs for a function of *x* are**

$$F(k) = \int_{-\infty}^{\infty} f(x)\exp(-ikx)dx$$
$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(k)\exp(ikx)dk \quad . \qquad \textbf{(13)}$$

  - **The Fourier transform of the correlation function $R_u(r)$ gives the *spectral energy density* of the turbulence**

$$\phi(k) = \int_{-\infty}^{\infty} R_u(r)\exp(-ikr)dr\,. \qquad \textbf{(14)}$$

  - **The wave number at which the energy density peaks (k_p) can be taken as a characteristic wave number of the turbulence, and from the relation**

$$\lambda_p = 2\pi/k_p \qquad \textbf{(15)}$$

characteristic wavelength of the turbulent eddies can be computed.

- **Dissipation rate**
  - **The rate at which turbulence dissipates kinetic energy is another means of characterizing turbulence.**
  - **Dissipation rate is given the symbol (ε) and has units of J s^{-1} kg^{-1}.**
  - **Dissipation converts kinetic energy into thermal energy, due to viscous effects.**
  - **Dissipation rate has been shown to be proportional to**

$$\varepsilon \propto u_m^3/l\,.$$

TAYLOR'S HYPOTHESIS

- **The correlation function $R(r)$ is a spatial correlation. However, data are usually taken at a single point as a time series.**
- **We make use of Taylor's hypothesis in order to use the time series data to infer spatial properties of the turbulence.**
- **Taylor's hypothesis states that if $u'/\overline{u} << 1$, then $x = Ut$, where U is the mean velocity.**

- **Using Taylor's hypothesis we can do spectral analysis in frequency (ω), and then convert it to wave number via**

$$k = \omega / U \text{ .}$$

TURBULENCE SPECTRA

- **The energy spectra of turbulence is theorized to have some general characteristics.**
- **The lowest wave numbers are the energy containing range.**
- **The highest wave numbers are the dissipation range**
  - **In the dissipation range, dimensional analysis predicts that the spectral density should be a function of dissipation rate and wave number only, via**

$$\phi(k) = a\varepsilon^{2/3}k^{-2/3} \text{ .} \qquad \textbf{(16)}$$

  - **If the proportionality constant is known, and if the spectral density is measured, then the dissipation rate can be inferred.**
- **In between the energy containing range and the dissipation range is the inertial subrange, where the energy is "just passing through" on its way to smaller scales.**
  - **In the inertial subrange, dimensional analysis predicts that the spectral density should be given by**

$$\phi(k) = b\varepsilon^{2/3}k^{-5/3} \text{ .} \qquad \textbf{(17)}$$

- **On a log-log plot, an idealized turbulence spectrum would look like**