**Definition:-**

1. An monomorphism is $∝$:A$\rightarrow $B called split if Im($∝$) is direct summand of B.
2. An epimorphism $β$:B$\rightarrow $C is called split if ker ($β$) is direct summand of B.

**Proposition**:-

1. For $∝$:A$\rightarrow $B be an R-homomorphism. The following statement are equivalent:
2. $∝$ is split monomorphism.
3. $There $exists a homomrphism $β$:B$\rightarrow $A such that $β∝$=$1\_{A}$.
4. For $β$:B$\rightarrow $C be a R-homomorphism, the following are equivalent:
5. For $β$ is split epimorphism.
6. There exists a homomorphism $φ$:C$\rightarrow $B such that $β φ$=$1\_{C}$.

Proof: - (1) (a)$\rightarrow \left(b\right)$

Since $∝is split monomorphism .Then $Im($∝$) is direct summand of B i.e. $∃$ $B\_{1}$ $⊆$B such that B=$Im(∝) ⊕B\_{1}$ .

Let $π$: B$\rightarrow Im(∝)$ be a projection homomorphism $∝$(a)$\in $ Im($∝$), $b\_{1}\in B\_{1}$.

Let $∝\_{°}$:A$\rightarrow $Im($∝$) such that $∝\_{°}$(a)=$ ∝$(a).

(i.e.) $∝\_{°}$ be the restraction of $∝$

$∴ ∝\_{°}$ is isomorphism.

$∃∝\_{°}^{-1}$: Im($∝$) $⟶$ A ,then we have

 Define $β$=$∝\_{°}^{-1}π$:B$⟶$A such that $β∝$(a)=($ ∝\_{°}^{-1}$ $π∝$)(a)=$ ∝\_{°}^{-1}π$($∝$(a))=.$ ∝\_{°}^{-1}∝$(a)=a=$1\_{a}$.$∀$a$\in $A.

(b)$\rightarrow $(a)
proof:- Since $β∝$=$1\_{A}$. Thus $1\_{A}$ is isomorphism, where $β∝$ is monomorphism

$∴$ $∝$ is monomorphism.

**Corollary(\*):-** We have Im($∝$)$⊕$ker($β$)=B (i.e.) Im($∝$) is direct summand of B .Thus $∝$ is split monomorphism.

**Corollary(\*):-** The diagram A$∝\_{\rightarrow }$B is com. i.e. $⋋$=$ β∝$.

Then if $⋋$ is isomorphism$⟹$ B= Im($∝$)$⊕$ker($β$).