Some Basic Relationships between Pixels

4.1 Neighbors of a Pixel

A pixel *p* at coordinates (x,y) has four horizontal and vertical neighbors whose coordinates are given by:

(x+1,y),(x-1,y), (x,y+1), (x,y-1)

This set is called the 4-neighbors of *p,* is denoted by $N_4(p)$. Each pixel is a unit distance from (x,y), and some of the neighbor locations of p lie outside the digital image if (x,y) is on the border of the image.

The four diagonal neighbor of p have coordinates:

(x+1,y+1),(x+1,y-1), (x-1,y+1), (x-1,y-1)

And are denoted by $N_D(p)$. These points together with the 4-neighbors, are called the 8-neighbors of p, denoted by $N_8(p)$. As before , some of the neighbor locations in $N_D(p)$and $N_8(p)$ fall outside the image if (x,y) is on the border of the image.

x-1,y-1	X-1,y	X-1,y+1
X,y-1	X,y	X,y+1
X+1,y-1	X+1,y	X+1,y-1

Fig. (4.1): Sub-image of size 3x3of 8-neighbor

4-2 Adjacency, Connectivity, Regions, and Boundaries

Let V be the set of intensity values used to define adjacency. In binary image V={1} if we referring to adjacency of pixels with value **1.** In a gray-scale image, the idea is the same, but set V typically contain more elements. For example, in the adjacency of pixels with range of possible intensity values 0 to 255, set V could be any subset of these 256 values. we consider three types of adjacency:

1. 4-adjacency,two pixels p and q with values from are 4-adjacency if q is in the set $N_4(p)$.
2. 8-adjacency, two pixels p and q with values from are 8-adjacency if q is in the set $N_8(p)$.

3. m-adjacency (mixed adjacency), two pixels p and q with values from are m-adjacency if q :

- ❖ q is in $N_4(p)$, or
- ❖ q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V

mixed adjacency is a modification of 8-adjacency.

<u>Example:</u>

Consider the pixel arrangement shown in fig.(a) for V={1}. The three pixels at the top fig.(b) show multiple 8-adjacency as indicated by the lines. This ambiguity is removed by using m-adjacency, as shown in fig.(c).

0	1	1
0	1	0
0	0	1

Fig.(a)

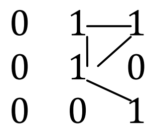


Fig.(b)
8-adjacency

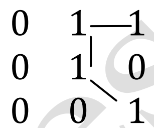


Fig.(c)
m-adjacency

A (digital) path (or curve) from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels with coordinates:

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where

$$(x_0, y_0) = (x, y), (x_n, y_n) = (s,t),$$

and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \leq i \leq n$.

In this case, n is the length of the path. If $(x_0, y_0) = (x_n, y_n)$, the path is a closed path. We define 4-, 8-, or m-paths depending on the type of adjacency specified. For example, the path shown in fig.(b) between the top right and bottom rights are 8-paths, and the path in fig.(c) is an m-path.

Let S represent a subset of pixels in an image. Two pixels p and q are said to be ***connected*** in S if there exists a path between them consisting entirely of pixels in S. For any pixel p in S, the set of pixels that are connected to it in S is called a ***connected component*** of S. If it only has one connected component, then set S is called a ***connected set***.

Let R be a subset of pixels in an image. We call R a region of the image if R is a connected set. Two regions, R_i and R_j are said to be adjacent if their union forms a connected set. Regions that are not adjacent are said to be disjoint.

Example:

The two regions are adjacent only if 8-adjacent is used. A 4-path between the two regions does not exist, so there union is not a connected set.

$$
\begin{array}{ccccc}
1 & 1 & 1 & & \\
1 & 0 & 1 & \} & R_i \\
0 & 1 & 0 & & \\
0 & 0 & 1 & & \\
1 & 1 & 1 & \} & R_j \\
1 & 1 & 1 & &
\end{array}
$$

Suppose that an image contains K disjoint regions, $R_{k,}, k = 1,2, \ldots, K$, none of which touches the image border. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement. We call all the points in R_u the ***foreground*** And all the points in $(R_u)^c$ the ***background*** of the image.

The boundary (also called the border or counter) of a region R is the set of points that are adjacent to points in the complement of R. said another way, the border of a region is the set of pixels in the region that have at least one background neighbor.

Example:

The point circled in figure below is not a member of the border of the 1-valued region if 4-connectivity is used between the region and its background. As a rule, adjacency between points in a regions and its background is defined in terms of 8-connectivity to handle situation like this.

$$
\begin{array}{ccccc}
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 1 & \textcircled{1} & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}
$$

4-3 Distance Measures

For pixels p, q, and z, with coordinates (x,y),(s,t),and(v,w), respectively, D is a distance function or metric if:

1. D(p,q)≥ 0, D(p,q)=0 if p=q
2. D(p,q)= D(q,p) ,and
3. D(p,z)≤ D(p,q)+ D(q,z).

1) **The Euclidean distance** between p and q is define as

$$D_e(p,q) = [(x-s)^2 + (y-t)^2]^{\frac{1}{2}}$$

For this distance measure, the pixels having a distance less than or equal to some value r from (x,y) are the points contained in the disk of radius r centered at (x,y).

2) **The D_4 distance** (called the city-block distance)between p and q is defined as:

$$D_4(p,q) = |x-s| + |y-t|$$

In this case, the pixels having a D_4 distance from (x,y) less than or equal to some value r from a diamond centered at (x,y).

<u>Example</u>

The pixels with D_4 distance ≤ 2 from (x,y) (the center point) from the following contours of constant distance:

		2		
	2	1	2	
2	1	0	2	1
	2	1	2	
		2		

The pixels with D_4=1 are the 4-nieghbors of (x,y).

3) **The D_8 distance** (called the chessboard distance)between p and q is defined as:

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

In this case, the pixels having a D_8 distance from (x,y) less than or equal to some value r from a square centered at (x,y).

<u>Example</u>

The pixels with D_8 distance ≤ 2 from (x,y) (the center point) from the following contours of constant distance:

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

The pixels with D_8=1 are the 8-nieghbors of (x,y).

Note

D_4 and D_8 distances between p and q are independent of any paths that might exist between the points because these distances involve only the coordinates of the points.

4) **D_m -distance** between two points is defined as the shortest m-path between the points.

In this case the distance two pixels will depend on the values of the pixels along the path as well as the values of their neighbors.

Example

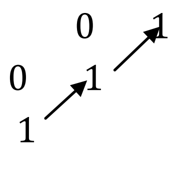
Consider the following arrangement of pixels and assume that p, p_2, and p_4 Have value 1 and that $p_1$and p_3 can have a value of 0 or 1:

	p_3	p_4
p_1	p_2	
p		

Suppose that we consider adjacency of pixels valued 1 (i.e., v={1})

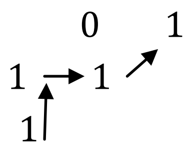
1) if $p_1$and p_3 are 0,

The m-path($D_m$distance between p and p_4 is 2



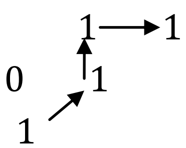
2) if p_1 is 1

The $D_m$distance between p $p_1p_2p_4$ is 3

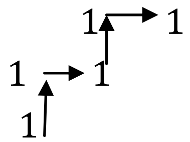


3) if p_3=1 and p_1 = 0

The $D_m$distance between p $p_2p_3p_4$ is 3



4) if p_1 and p_3 are 1



The $\boldsymbol{D_m}$ distance between p $p_1p_2p_3p_4$ is <u>4</u>