Properties of Electric Charges

A number of simple experiments demonstrate the existence of electric forces and charges. For example, after running a comb through your hair on a dry day, you will find that the comb attracts bits of paper. The attractive force is often strong enough to suspend the paper. The same effect occurs when certain materials are rubbed together, such as glass rubbed with silk or rubber with fur.

Another simple experiment is to rub an inflated balloon with wool. The balloon then adheres to a wall, often for hours. When materials behave in this way, they are said to be ***electrified,*** or to have become electrically charged. You can easily electrify your body by vigorously rubbing your shoes on a wool rug. Evidence of the electric charge on your body can be detected by lightly touching (and startling) a friend.

Under the right conditions, you will see a spark when you touch, and both of you will feel a slight tingle. (Experiments such as these work best on a dry day because an excessive amount of moisture in the air can cause any charge you build up to “leak” from your body to the Earth.) **there are two kinds of electric charges, which were given the names positive and negative**

We identify negative charge as that type possessed by electrons and positive charge as that possessed by protons. To verify that there are two types of charge, suppose a hard rubber rod that has been rubbed with fur is suspended by a sewing thread, as shown in Figure 23.1. When a glass rod that has been rubbed with silk is brought near the rubber rod, the two attract each other (Fig. 23.1a). On the other hand, if two charged rubber rods (or two charged glass rods) are brought near each other, as shown in Figure 23.1b, the two repel each other. This observation shows that the rubber and glass have two different types of charge on them. On the basis of these observations, we conclude that charges of the same sign repel one another and charges with opposite signs attract one another.

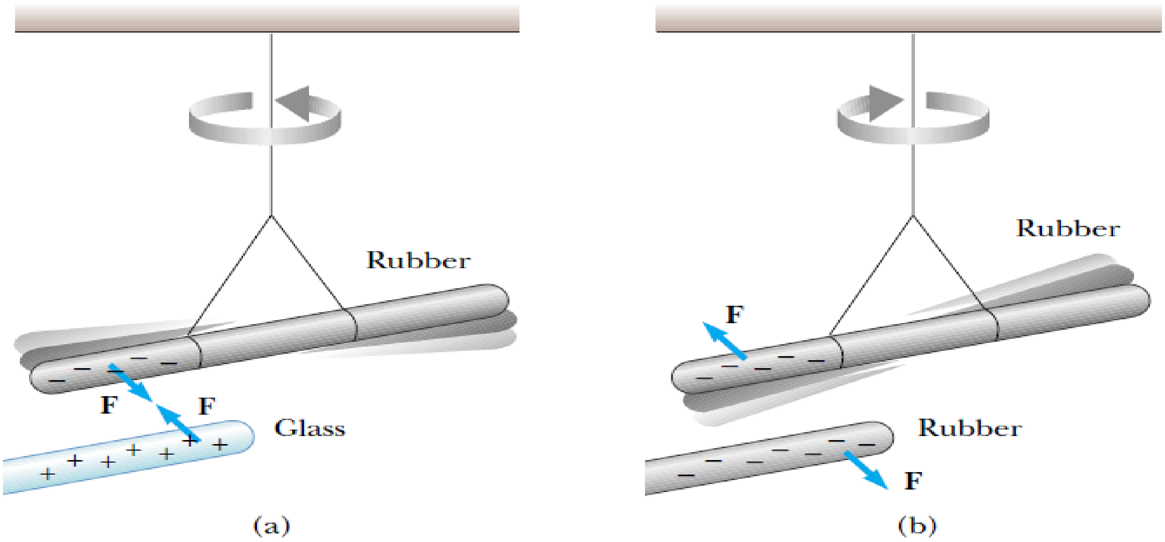


Figure 23.1 (a) A negatively charged rubber rod suspended by a thread is attracted to a positively charged glass rod. (b) A negatively charged rubber rod is repelled by another negatively charged rubber rod.

In addition to the existence of two types of charge, several other properties of charge have been discovered

• **Charge is quantized.** This means that electric charge comes in discrete amounts, and there is a smallest possible amount of charge that an object can have. In the SI system, this smallest amount is $e \equiv 1.602 \times 10{-}19$ C . No free particle can have less charge than this, and, therefore, the charge on any object—the charge on all objects—must be an integer multiple of this amount. All macroscopic, charged objects have charge because electrons have either been added or taken away from them, resulting in a net charge.

• **The magnitude of the charge is independent of the type.** Phrased another way, the smallest possible positive charge (to four significant figures) is $+1.602 \times 10{-}19$ C , and the smallest possible negative charge is $-1.602 \times 10{-}19$ C ; these values are exactly equal. This is simply how the laws of physics in our universe turned out.

• **Charge is conserved.** Charge can neither be created nor destroyed; it can only be transferred from place to place, from one object to another. Frequently, we speak of two charges "canceling"; this is verbal shorthand. It means that if two objects that have equal and opposite charges are physically close to each other, then the (oppositely directed) forces they apply on some other charged object cancel, for a net force of zero. It is important that you understand that the charges on the objects by no means disappear, however. The net charge of the universe is constant.

• **Charge is conserved in closed systems.** In principle, if a negative charge disappeared from your lab bench and reappeared on the Moon, conservation of charge would still hold. However, this never happens. If the total charge you have in your local system on your lab bench is changing, there will be a measurable flow of charge into or out of the system. Again, charges can and do move around, and their effects can and do cancel, but the net charge in your local environment (if closed) is conserved. The last two items are both referred to as the **law of conservation of charge**

In Summary

- There are two kinds of charges in nature; charges of opposite sign attract one another and charges of the same sign repel one another.
- Total charge in an isolated system is conserved.
- Charge is quantized.

H.W

Quick Quiz 23.1 If you rub an inflated balloon against your hair, the two materials attract each other, as shown in Figure 23.3. Is the amount of charge present in the system of the balloon and your hair after rubbing (a) less than, (b) the same as, or (c) more than the amount of charge present before rubbing?

Quick Quiz 23.2 Three objects are brought close to each other, two at a time. When objects A and B are brought together, they repel. When objects B and C are brought together, they also repel. Which of the following are true? (a) Objects A and C possess charges of the same sign. (b) Objects A and C possess charges of opposite sign. (c) All three of the objects possess charges of the same sign. (d) One of the objects is neutral. (e) We would need to perform additional experiments to determine the signs of the charges.

Figure 23.3 (Quick Quiz 23.1) Rubbing a balloon against your hair on a dry day causes the balloon and your hair to become charged.

Coulomb's Law

Experiments with electric charges have shown that if two objects each have electric charge, then they exert an electric force on each other.

From Coulomb's experiments, we can generalize the following properties of the **electric force** between two stationary charged particles. The electric force

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign;
- is a conservative force.

We will use the term **point charge** to mean a particle of zero size that carries an electric charge. The electrical behavior of electrons and protons is very well described by modeling them as point charges. From experimental observations on the electric force, we can express **Coulomb's law** as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) between two point charges:

$$F_e = k_e \frac{|q_1||q_2|}{r^2} \quad (23.1)$$

Coulomb's law

where k_e is a constant called the **Coulomb constant.** In his experiments, Coulomb was able to show that the value of the exponent of r was 2 to within an uncertainty of a few percent. Modern experiments have shown that the exponent is 2 to within an uncertainty of a few parts in 10^{16}.

The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the **coulomb** (C). The Coulomb constant k_e in SI units has the value

$$k_e = 8.9875 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \quad (23.2)$$

Coulomb constant

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0} \quad (23.3)$$

Table 23.1

Charge and Mass of the Electron, Proton, and Neutron		
Particle	**Charge (C)**	**Mass (kg)**
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

where the constant ϵ_0 (lowercase Greek epsilon) is known as the **permittivity of free space** and has the value

$$\epsilon_0 = 8.854\,2 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \tag{23.4}$$

The smallest unit of charge e known in nature[2] is the charge on an electron $(-e)$ or a proton $(+e)$ and has a magnitude

$$e = 1.602\,19 \times 10^{-19}\ \mathrm{C} \tag{23.5}$$

Therefore, 1 C of charge is approximately equal to the charge of 6.24×10^{18} electrons or protons. This number is very small when compared with the number of free electrons in 1 cm^3 of copper, which is on the order of 10^{23}. Still, 1 C is a substantial amount of charge. In typical experiments in which a rubber or glass rod is charged by friction, a net charge on the order of 10^{-6} C is obtained. In other words, only a very small fraction of the total available charge is transferred between the rod and the rubbing material.

The charges and masses of the electron, proton, and neutron are given in Table 23.1.

Quick Quiz 23.4 Object A has a charge of $+2\ \mu\mathrm{C}$, and object B has a charge of $+6\ \mu\mathrm{C}$. Which statement is true about the electric forces on the objects? (a) $F_{AB} = -3F_{BA}$ (b) $F_{AB} = -F_{BA}$ (c) $3F_{AB} = -F_{BA}$ (d) $F_{AB} = 3F_{BA}$ (e) $F_{AB} = F_{BA}$ (f) $3F_{AB} = F_{BA}$

Example 23.1 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the magnitude of the electric force is

$$F_e = k_e \frac{|e||-e|}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(1.60 \times 10^{-19}\ \mathrm{C})^2}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$

$$= 8.2 \times 10^{-8}\ \mathrm{N}$$

Using Newton's law of universal gravitation and Table 23.1 for the particle masses, we find that the magnitude of the gravitational force is

$$F_g = G\frac{m_e m_p}{r^2}$$

$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}) \times \frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.67 \times 10^{-27}\ \mathrm{kg})}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$

$$= 3.6 \times 10^{-47}\ \mathrm{N}$$

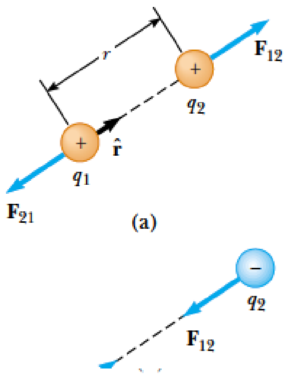
The ratio $F_e/F_g \approx 2 \times 10^{39}$. Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of universal gravitation and Coulomb's law of electric forces. Other than magnitude, what is a fundamental difference between the two forces?

When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. The law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written $\mathbf{F}_{12}$, is

Vector form of Coulomb's law

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} \tag{23.6}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q_1 toward q_2, as shown in Figure 23.7a. Because the electric force obeys Newton's third law, the electric force exerted by q_2 on q_1 is equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction; that is, $\mathbf{F}_{21} = -\mathbf{F}_{12}$. Finally, from Equation 23.6, we see that if q_1 and q_2 have the same sign, as in Figure 23.7a, the product q_1q_2 is positive. If q_1 and q_2 are of opposite sign, as shown in Figure 23.7b, the product q_1q_2 is negative. These signs describe the *relative* direction of the force but not the *absolute* direction. A negative product indicates an attractive force, so that the charges each experience a force toward the other—thus, the force on one charge is in a direction *relative* to the other. A positive product indicates a repulsive force such that each charge experiences a force away from the other. The *absolute* direction of the force in space is not determined solely by the sign of q_1q_2—whether the force on an individual charge is in the positive or negative direction on a coordinate axis depends on the location of the other charge. For



Active Figure 23.7 Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $\mathbf{F}_{21}$ exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{12}$ exerted by q_1 on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force is attractive.

Quick Quiz 23.5 Object A has a charge of $+2\ \mu$C, and object B has a charge of $+6\ \mu$C. Which statement is true about the electric forces on the objects? (a) $\mathbf{F}_{AB} = -3\mathbf{F}_{BA}$ (b) $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$ (c) $3\mathbf{F}_{AB} = -\mathbf{F}_{BA}$ (d) $\mathbf{F}_{AB} = 3\mathbf{F}_{BA}$ (e) $\mathbf{F}_{AB} = \mathbf{F}_{BA}$ (f) $3\mathbf{F}_{AB} = \mathbf{F}_{BA}$

When more than two charges are present, the force between any pair of them is given by Equation 23.6. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

Example 23.2 Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure 23.8, where $q_1 = q_3 = 5.0\ \mu$C, $q_2 = -2.0\ \mu$C, and $a = 0.10$ m. Find the resultant force exerted on q_3.

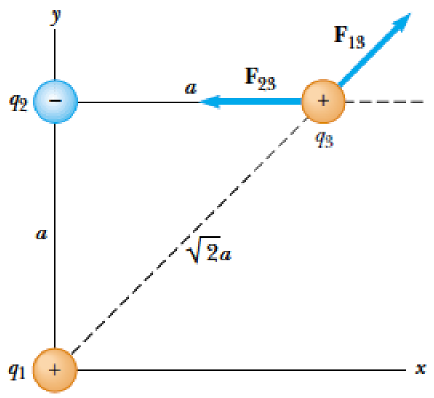


Figure 23.8 (Example 23.2) The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$. The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$. The resultant force $\mathbf{F}_3$ exerted on q_3 is the vector sum $\mathbf{F}_{13} + \mathbf{F}_{23}$.

Solution First, note the direction of the individual forces exerted by q_1 and q_2 on q_3. The force $\mathbf{F}_{23}$ exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs. The force $\mathbf{F}_{13}$ exerted by q_1 on q_3 is repulsive because both charges are positive.

The magnitude of $\mathbf{F}_{23}$ is

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ \frac{(2.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{(0.10\ \text{m})^2}$$

$$= 9.0\ \text{N}$$

In the coordinate system shown in Figure 23.8, the attractive force $\mathbf{F}_{23}$ is to the left (in the negative x direction).

The magnitude of the force $\mathbf{F}_{13}$ exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{2(0.10\ \mathrm{m})^2}$$

$$= 11\ \mathrm{N}$$

The repulsive force $\mathbf{F}_{13}$ makes an angle of 45° with the x axis. Therefore, the x and y components of $\mathbf{F}_{13}$ are equal, with magnitude given by $F_{13} \cos 45° = 7.9$ N.

Combining $\mathbf{F}_{13}$ with $\mathbf{F}_{23}$ by the rules of vector addition, we arrive at the x and y components of the resultant force acting on q_3:

$$F_{3x} = F_{13x} + F_{23x} = 7.9\ \mathrm{N} + (-9.0\ \mathrm{N}) = -1.1\ \mathrm{N}$$

$$F_{3y} = F_{13y} + F_{23y} = 7.9\ \mathrm{N} + 0 = 7.9\ \mathrm{N}$$

We can also express the resultant force acting on q_3 in unit-vector form as

$$\mathbf{F}_3 = (-1.1\hat{\mathbf{i}} + 7.9\hat{\mathbf{j}})\ \mathrm{N}$$

What If? **What if the signs of all three charges were changed to the opposite signs? How would this affect the result for $\mathbf{F}_3$?**

Answer The charge q_3 would still be attracted toward q_2 and repelled from q_1 with forces of the same magnitude. Thus, the final result for $\mathbf{F}_3$ would be exactly the same.

Example 23.3 Where Is the Resultant Force Zero?

Interactive

Three point charges lie along the x axis as shown in Figure 23.9. The positive charge $q_1 = 15.0\ \mu\mathrm{C}$ is at $x = 2.00$ m, the positive charge $q_2 = 6.00\ \mu\mathrm{C}$ is at the origin, and the resultant force acting on q_3 is zero. What is the x coordinate of q_3?

Solution Because q_3 is negative and q_1 and q_2 are positive, the forces $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are both attractive, as indicated in Figure 23.9. From Coulomb's law, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ have magnitudes

$$F_{13} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2} \qquad F_{23} = k_e \frac{|q_2||q_3|}{x^2}$$

For the resultant force on q_3 to be zero, $\mathbf{F}_{23}$ must be equal in magnitude and opposite in direction to $\mathbf{F}_{13}$. Setting the magnitudes of the two forces equal, we have

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$

Noting that k_e and $|q_3|$ are common to both sides and so can be dropped, we solve for x and find that

$$(2.00 - x)^2 |q_2| = x^2 |q_1|$$

$$(4.00 - 4.00x + x^2)(6.00 \times 10^{-6}\ \mathrm{C}) = x^2(15.0 \times 10^{-6}\ \mathrm{C})$$

This can be reduced to the following quadratic equation:

$$3.00x^2 + 8.00x - 8.00 = 0$$

Solving this quadratic equation for x, we find that the positive root is $x = 0.775$ m. There is also a second root, $x = -3.44$ m. This is another location at which the magnitudes of the forces on q_3 are equal, but both forces are in the same direction at this location.

What If? **Suppose charge q_3 is constrained to move only along the x axis. From its initial position at $x = 0.775$ m, it is pulled a very small distance along the x axis. When released, will it return to equilibrium or be pulled further from equilibrium? That is, is the equilibrium stable or unstable?**

Answer If the charge is moved to the right, $\mathbf{F}_{13}$ becomes larger and $\mathbf{F}_{23}$ becomes smaller. This results in a net force to the right, in the same direction as the displacement. Thus, the equilibrium is *unstable*.

Note that if the charge is constrained to stay at a *fixed* x coordinate but allowed to move up and down in Figure 23.9, the equilibrium is stable. In this case, if the charge is pulled upward (or downward) and released, it will move back toward the equilibrium position and undergo oscillation.

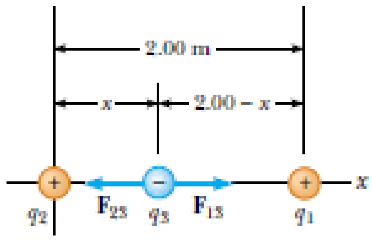


Figure 23.9 (Example 23.3) Three point charges are placed along the x axis. If the resultant force acting on q_3 is zero, then the force $\mathbf{F}_{13}$ exerted by q_1 on q_3 must be equal in magnitude and opposite in direction to the force $\mathbf{F}_{23}$ exerted by q_2 on q_3.