PHYS-2020: General Physics II
Course Lecture Notes
Section XI

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Edition 3.3

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 9th Edition* (2012) textbook by Serway and Vuille.

XI. Reflection and Refraction of Light

A. Huygens' Principle.

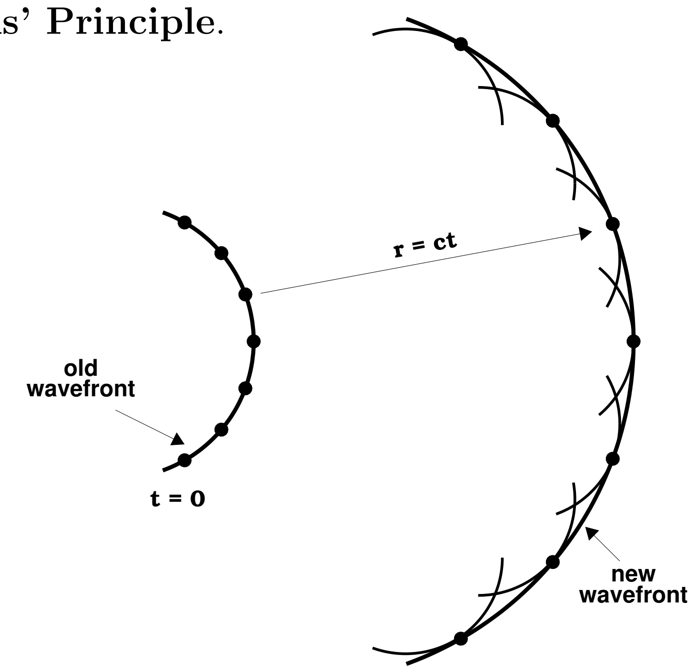
1. In 1678, Huygens proposed the wave theory of light.

a) At that same time, Newton maintained that light acted like a particle.

b) As we saw in the last section, Planck derived in 1900 that light has the characterists of both a wave and a particle $\Longrightarrow$ a *wavicle*, which he called a **photon**.

c) In the last sections of the notes, we discussed the particle characteristics of a photon. In this section, we will concentrate on the wave-like characteristics.

2. All points on a **wavefront** can be considered point sources for the production of **spherical** secondary wavelets. After time t, the new position of the wavefront will be the surface of *tangency* to these secondary wavelets. This *geometrical argument* is called **Huygens' Principle**.



B. Reflection of Light.

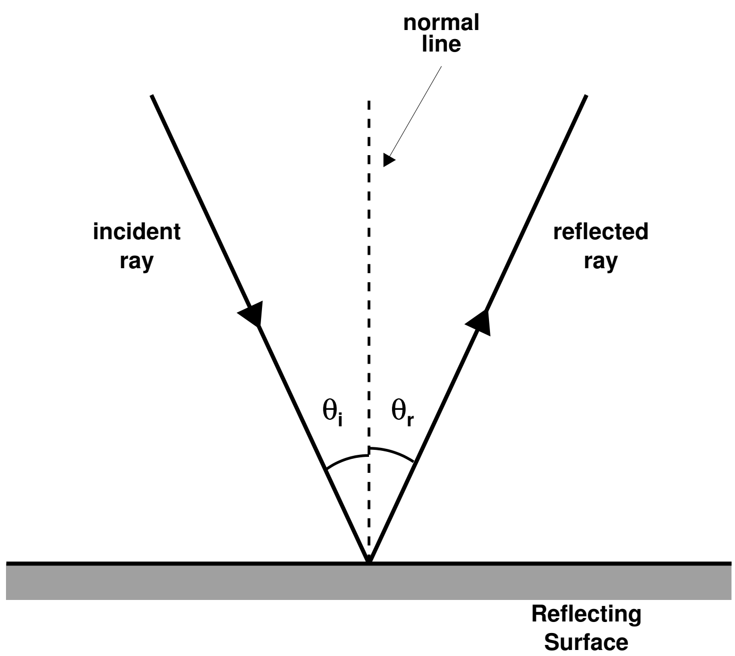
1. When light travels from one medium to another, part of the light can be **reflected** at the media interface.

a) Reflection off of a *smooth* surface is called **specular reflection** (which we will assume from this point forward).

b) Reflection off of a *rough* surface is called **diffuse reflection**.

2. **Law of Reflection**: The angle of incidence with respect to the normal of the reflecting surface, θ_i, equals the angle of reflection, θ_r:

$$\boxed{\theta_i = \theta_r\ .} \tag{XI-1}$$



3. The amount of energy that is reflected compared to the amount incident is called the **reflectivity** of the surface.

a) This also is called **albedo**.

b) The reflectivity of a mirror is about 96% (albedo = 0.96).

C. Refraction of Light.

1. When light travels from one medium to another, part of the light can be **transmitted** across the media surface and **refracted**.

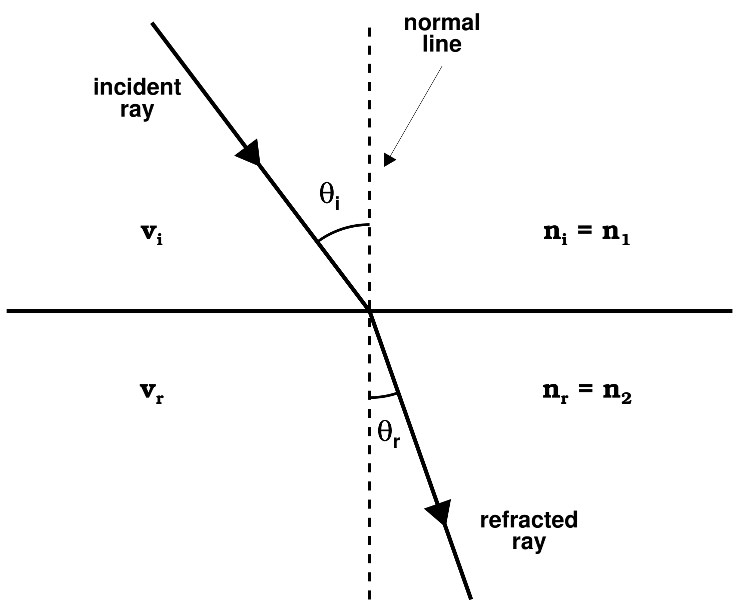
a) **Refraction** means that the light beam bends.

b) This bending takes place because the light beam's (*i.e.*, photon's) velocity changes as it goes from one medium to the next, following the relation:

$$\boxed{\frac{\sin\theta_r}{\sin\theta_i} = \frac{v_r}{v_i} = \text{constant}\ .} \qquad \text{(XI-2)}$$

i) v_r and θ_r are the velocity and the angle of the refracted beam with respect to the normal line of the surface.

ii) v_i and θ_i are the velocity and the angle of the incident beam with respect to the normal line of the surface.



2. The **index of refraction** of a material is

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}\ . \qquad \text{(XI-3)}$$

a) n for some common substances:

i) Vacuum: 1.000000

ii) Air (0°C, 1 atm): 1.000293

iii) Ice (H_2O, 0°C): 1.309

iv) Glass (crown): 1.52

b) n also is a function of wavelength. For the index of refraction in air we have

$$n_{\rm air} = 1+6.4328\times10^{-5}+\frac{2.949810\times10^{6}}{1.46\times10^{10}-\overline{\nu}^2}+\frac{2.5540\times10^{4}}{4.1\times10^{9}-\overline{\nu}^2}\ , \qquad \text{(XI-4)}$$

where $\overline{\nu} = 1/\lambda_{\rm vac}$ is called the wavenumber and is measured in cm^{-1} in this equation (just include the value of the wavenumber without its units in the equation above, remember, n is unitless).

3. Eq. (XI-2) can be re-expressed as a function of $n \Longrightarrow$ **Law of Refraction** better known as **Snell's Law**:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2\ ,} \qquad \text{(XI-5)}$$

where the '1' label indicates the first medium the light is in and the '2' label indicates the second medium.

4. Besides a change in light beam direction, the wavelength of light also changes when light goes from one medium to the next:

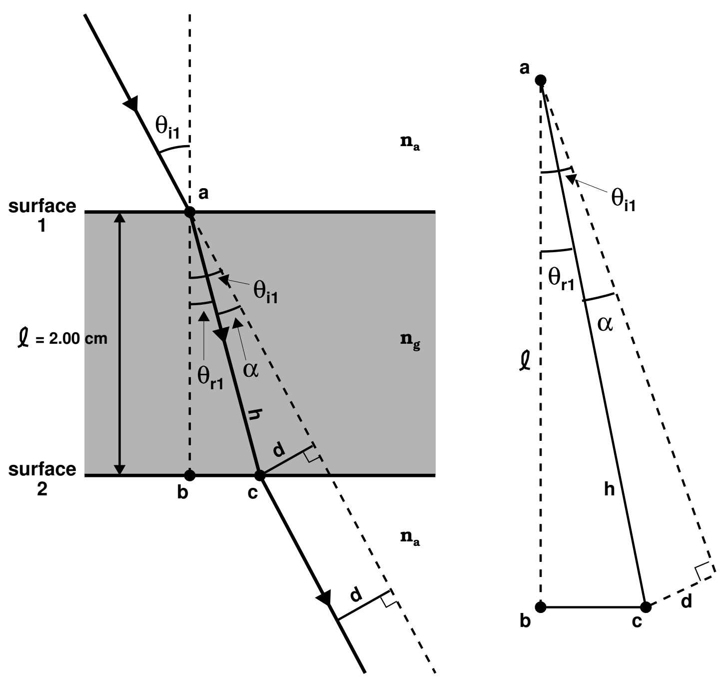
$$\boxed{\lambda_1 n_1 = \lambda_2 n_2\ .} \qquad \text{(XI-6)}$$

a) For starlight coming in from outer space,

$$\lambda_{\rm air} = \frac{\lambda_{\rm vac}}{n_{\rm air}}\ . \qquad \text{(XI-7)}$$

b) Spectral lines are shifted in wavelength as they pass through the Earth's atmosphere which needs to be taken into account when planetary, stellar, or galactic spectra are being analyzed.

Example XI–1. Problem 22.18 (Page 785) from the Serway & Vuille textbook: A ray of light strikes a flat, 2.00-cm thick block of glass ($n = 1.50$) at an angle of $30°$ with respect to the normal (see Fig. P22.18 in the text and the figure below). (a) Find the angle of refraction at the top surface. (b) Find the angle of incidence at the bottom surface and the refracted angle at this surface. (c) Find the lateral distance d by which the light beam is shifted. (d) Calculate the speed of light in the glass and (e) the time required for the light to pass through the glass block. (f) Is the travel time through the block affected by the angle of incidence? Explain.



Solution (a):

At the top surface (labeled 1), the angle of incidence is given to us as $\theta_{i1} = 30.0°$. We will assume that the beam is hitting the glass from air, hence $n_a = 1.00$. Snell's law gives the refracted angle in the glass (with $n_g = 1.50$) at surface 1 as

$$\begin{aligned} n_a \sin\theta_{i1} &= n_g \sin\theta_{r1} \\ \sin\theta_{r1} &= \frac{n_a \sin\theta_{i1}}{n_g} \\ \theta_{r1} &= \sin^{-1}\left[\frac{n_a \sin\theta_{i1}}{n_g}\right] \\ &= \sin^{-1}\left[\frac{(1.00)\sin 30.0^\circ}{1.50}\right] = \boxed{19.5^\circ\ .} \end{aligned}$$

Solution (b):

Since the second surface (labeled 2) is parallel to the first, the angle of incidence at the bottom surface is exactly the same as the refracted angle at the top surface following the theorem of geometry, so $\boxed{\theta_{i2} = \theta_{r1} = 19.5^\circ.}$ The angle of refraction at this bottom surface (back into the air) is then

$$\begin{aligned} n_g \sin\theta_{i2} &= n_a \sin\theta_{r2} \\ \sin\theta_{r2} &= \frac{n_g \sin\theta_{i2}}{n_a} \\ \theta_{r2} &= \sin^{-1}\left[\frac{n_g \sin\theta_{i2}}{n_a}\right] \\ &= \sin^{-1}\left[\frac{(1.50)\sin 19.5^\circ}{1.00}\right] = \boxed{30.0^\circ\ .} \end{aligned}$$

Thus, the light emerges traveling parallel to the incident beam.

Solution (c):

Let $\ell = 2.00$ cm be the thickness of the glass. The angle of refraction at the first surface from Part (a) is $\theta_{r1} = 19.5^\circ$. Let h represent the distance from point 'a' to 'c' (*i.e.*, the hypotenuse of triangle abc), then

$$h = \frac{\ell}{\cos\theta_{r1}} = \frac{2.00\ \text{cm}}{\cos 19.5^\circ} = 2.12\ \text{cm}\ .$$

From the drawing above, note that angle $\alpha = \theta_{i1} - \theta_{r1} = 30.0^\circ - 19.5^\circ = 10.5^\circ$ and also that d represents the opposite side of the right-angle triangle defined by angle α with h being the hypotenuse of this triangle. Then

$$d = h \sin\alpha = (2.12 \text{ cm}) \sin 10.5^\circ = \boxed{0.386 \text{ cm}} \ .$$

Solution (d):

The speed of light in the glass is given by Eq. (XI-3):

$$v = \frac{c}{n_g} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = \boxed{2.00 \times 10^8 \text{ m/s}} \ .$$

Solution (e):

For this question, we only need to use the definition of velocity $v = h/t$, where h is the light beam path in the glass as calculated in Part (c) and v is the velocity of light in the glass calculated in Part (d). The time it will take the light beam to travel through the glass is therefore

$$t = \frac{h}{v} = \frac{2.12 \times 10^{-2} \text{ m}}{2.00 \times 10^8 \text{ m/s}} = \boxed{1.06 \times 10^{-10} \text{ s} = 0.106 \text{ ns}} \ .$$

Solution (f):

If the angle of incidence θ_{i1} on the top surface is changed, the angle of refraction for that surface θ_{r1} will change and the path length h that the light beam takes will be modified base on the equation for h on the previous page. Since the value for h changes the time it takes for the light beam to travel through the glass will change based upon the equation above.

Example XI–2. A star emits a spectral line at 567.27 nm. At what wavelength will it be seen on the ground?

Solution:

First, calculate the wavenumber (note that 1 cm = 10^{-7} nm):

$$\overline{\nu} = \frac{1}{\lambda_{\rm vac}} = \frac{1}{567.27 \times 10^{-7}\ {\rm cm}} = 1.76283 \times 10^4\ {\rm cm}^{-1}\ .$$

Now use Eq. (XI-4) to calculate the index of refraction of the air at this wavelength:

$$\begin{aligned} n_{\rm air} &= 1 + 6.4328 \times 10^{-5} + \frac{2.949810 \times 10^6}{1.46 \times 10^{10} - \overline{\nu}^2} + \frac{2.5540 \times 10^4}{4.1 \times 10^9 - \overline{\nu}^2} \\ &= 1 + 6.4328 \times 10^{-5} + \frac{2.949810 \times 10^6}{1.46 \times 10^{10} - (1.76283 \times 10^4)^2} + \\ &\qquad \frac{2.5540 \times 10^4}{4.1 \times 10^9 - (1.76283 \times 10^4)^2} \\ &= 1 + 6.4328 \times 10^{-5} + 2.0644 \times 10^{-4} + 6.74 \times 10^{-6} \\ &= 1.000278 \end{aligned}$$

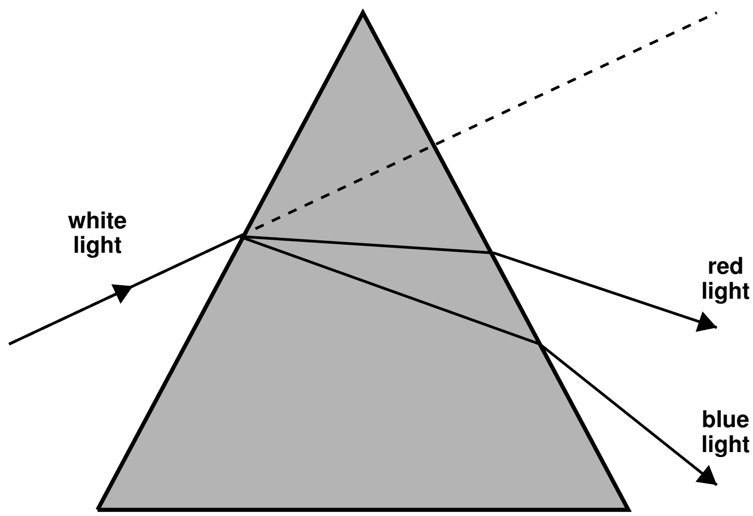
Finally use Eq. (XI-7) to determine the wavelength of this photon in air (*i.e.*, on the ground):

$$\lambda_{\rm air} = \frac{\lambda_{\rm vac}}{n_{\rm air}} = \frac{567.27\ {\rm nm}}{1.000278} = \boxed{567.11\ {\rm nm}\ .}$$

D. Dispersion and Prisms.

1. Different λs of light are refracted at different angles $\Longrightarrow$ called **dispersion**. As we have seen, this results from the wavelength dependence of the index of refraction.

2. The dispersion of light through an air-glass interface is the principle behind a **prism**.



a) White light is composed of the rainbow of colors (*i.e.*, the continuum).

b) Prisms are therefore used in spectrographs (also called spectroscopes and spectrometers).

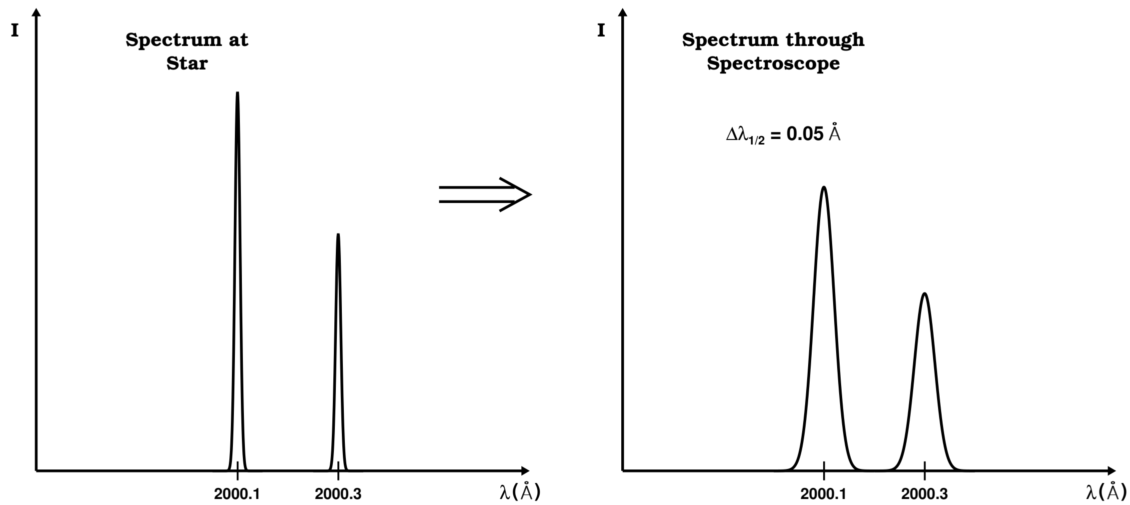
c) A similar dispersion effect can be produced by having light pass through a plate with 'fine' parallel lines etched on its surface $\Longrightarrow$ called a **grating**.

i) Most professional spectrographs are actually grating spectrographs.

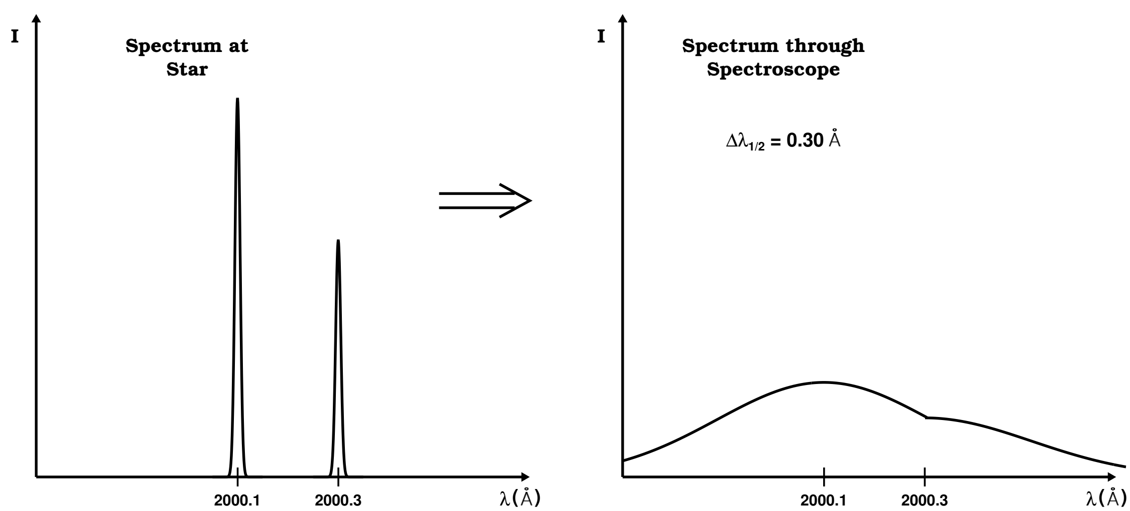
ii) Grating spectrographs work on the principle of diffraction of light and <u>not</u> refraction of light as is the case with dispersion in prisms.

d) The **spectral resolution** or **dispersion** of a spectrograph is measured by how narrow an infinitely narrow emission line appears in an *observed* spectrum. The thickness of a spectral line is measured half-way up to the peak flux of the line, $\Delta\lambda_{1/2}$, called the *full-width-at-half-maximum* (FWHM) of the spectrograph (also called the instrument profile).

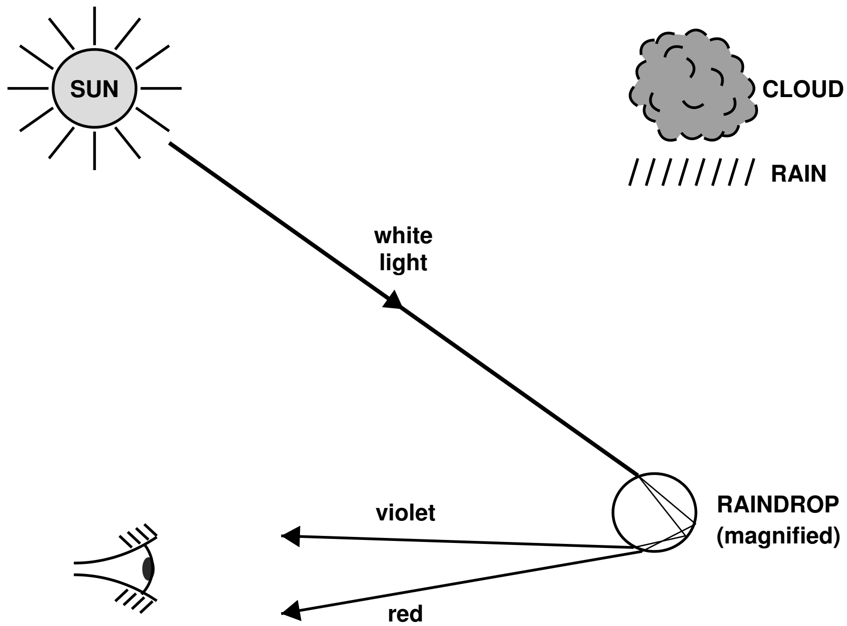
i) A *high-resolution* spectrograph has $\Delta\lambda_{1/2} < 0.1$ Å (= 0.01 nm). An example of a high-resolution spectrum is shown below.



ii) A *low-resolution* spectrograph has $\Delta\lambda_{1/2} > 0.1$ Å. An example of a low-resolution spectrum is shown below.



3. The dispersion of light through raindrops is the cause of **rainbows**:



a) Red always appears on the top of the rainbow, violet and blue on the inner arcs.

b) If the raindrop is big enough, some of the photons being refracted in the drop can suffer a secondary reflection inside the drop producing a **secondary rainbow** above the primary bow with the order of the colors reversed.

c) Each person looking at a rainbow sees a different rainbow!

Example XI–3. **Problems 22.29 (Page 786) from the Serway & Vuille textbook**: The index of refraction for red light in water is 1.331, and that for blue light is 1.340. If a ray of white light enters the water at an angle of incidence of 83.00°, what are the underwater angles of refraction for the (a) blue and (b) red components of the light?

Solutions (a) & (b):

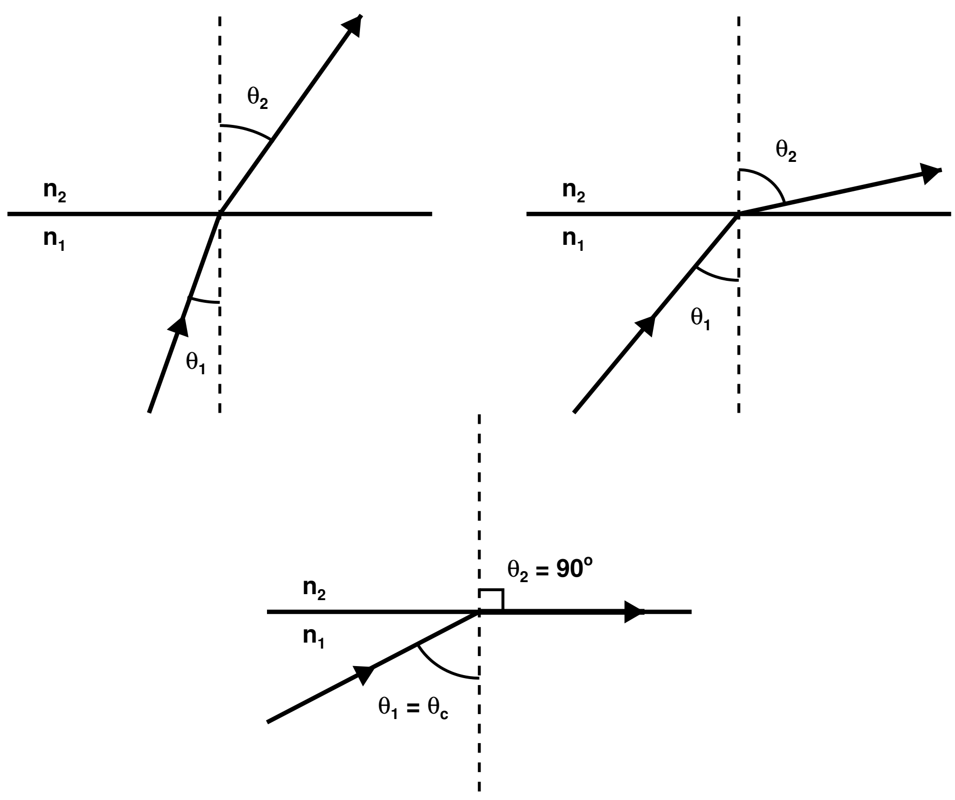
For this we only have to use Snell's law:

$$\theta_{\rm red} = \sin^{-1}\left[\frac{n_{\rm air}\sin\theta_i}{n_{\rm red}}\right] = \sin^{-1}\left[\frac{(1.000)\sin 83.00^\circ}{1.331}\right] = \boxed{48.22^\circ\ .}$$

$$\theta_{\rm blue} = \sin^{-1}\left[\frac{n_{\rm air}\sin\theta_i}{n_{\rm blue}}\right] = \sin^{-1}\left[\frac{(1.000)\sin 83.00^\circ}{1.340}\right] = \boxed{47.79^\circ\ .}$$

E. Total Internal Reflection.

1. Total internal reflection occurs only when light attempts to move from a medium of high index of refraction to a medium of lower index of refraction.



a) The critical angle is defined as that incident angle inside a medium produces a refracted angle that follows the surface. It is determined by

$$n_1 \sin\theta_c = n_2 \sin 90^\circ = n_2$$

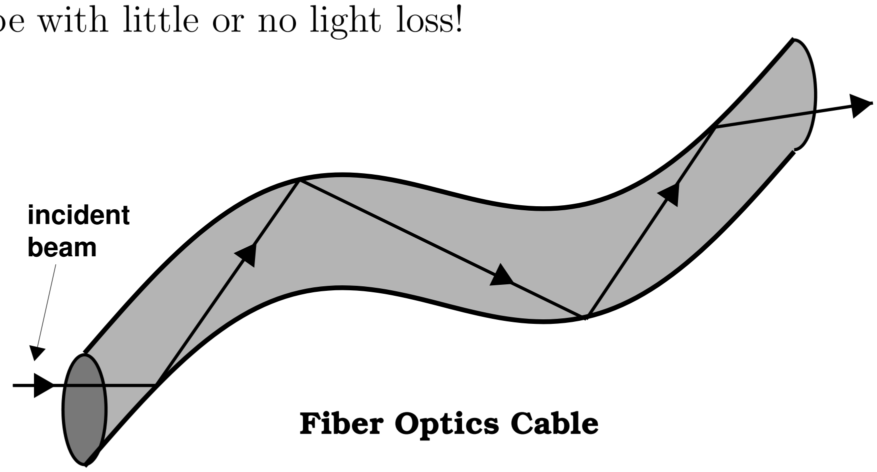
or

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}\ .} \qquad \text{(XI-8)}$$

b) As can be seen, this only occurs when $n_2 < n_1$, since $\sin\theta_c \leq 1$.

2. This is the physical principle which allows **fiber optics** to work.

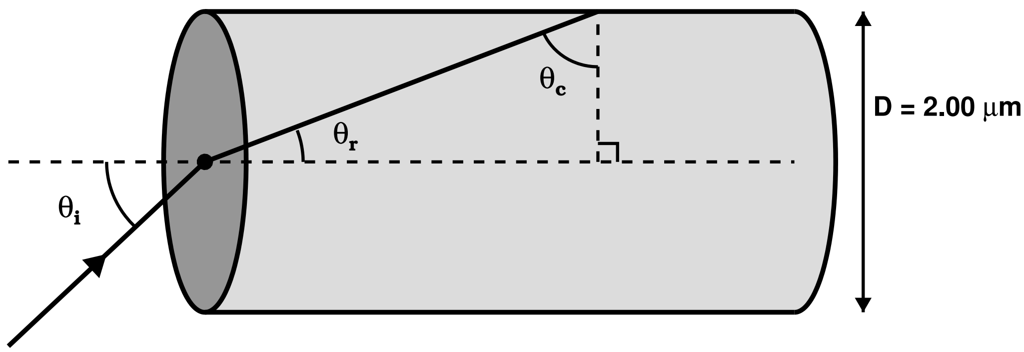
a) Light sent down a fiber optics tube will continue down the tube with little or no light loss!



b) Due to its much shorter wavelengths, visible light can carry a lot more information than electric currents. In time, all electrical wires for communications will be replaced by fiber optics cable for those communications that require cabling.

Example XI–4. **Problems 22.38 (Page 786) from the Serway & Vuille textbook**: Determine the maximum angle θ for which the light rays incident on the end of the light pipe in Figure P22.38 (see below) are subject to total internal reflection along the walls of the pipe. Assume the light pipe has an index of refraction of 1.36

and that the outside medium is air.



Note: $\theta_r + \theta_c = 90^\circ$

Solution:

The critical angle for this material in air is

$$\theta_c = \sin^{-1}\left(\frac{n_{\rm air}}{n_{\rm pipe}}\right) = \sin^{-1}\left(\frac{1.00}{1.36}\right) = 47.3^\circ \ .$$

Thus, the refracted angle of the light beam entering the cross-sectional area of the pipe (see diagram above) is $\theta_r = 90.0^\circ - \theta_c = 42.7^\circ$ and from Snell's law, the angle of incidence at this surface is

$$\theta_i = \sin^{-1}\left(\frac{n_{\rm pipe}\sin\theta_r}{n_{\rm air}}\right) = \sin^{-1}\left[\frac{(1.36)\sin 42.7^\circ}{1.00}\right) = \boxed{62.7^\circ \ .}$$