is a measure of the intensity of turbulence. Thus, Taylor's hypothesis should be satisfactory when the turbulence intensity is small relative to the mean wind speed.

1.5 Virtual Potential Temperature

Buoyancy is one of the driving forces for turbulence in the BL. Thermals of warm air rise because they are less dense than the surrounding air, and hence ***positively buoyant***. ***Virtual temperature*** is a popular variable for these studies because it is the temperature that dry air must have to equal the density of moist air at the same pressure. Thus, variations of virtual temperature can be studied in place of variations in density.

Water vapor is less dense than dry air; thus, moist unsaturated air is more buoyant than dry air of the same temperature. The virtual temperature of unsaturated moist air is therefore always greater than the absolute air temperature, T. Liquid water, however, is more dense than dry air, making cloudy air heavier or less buoyant than the corresponding cloud-free air. The suspension of cloud droplets in an air parcel is called ***liquid water loading***, and it always reduces the virtual temperature.

Virtual potential temperatures are analogous to potential temperatures in that they remove the temperature variation caused by changes in pressure altitude of an air parcel. Turbulence includes vertical movement of air, making a variable such as virtual potential temperature not just attractive but almost necessary.

1.5.1 Definitions

For saturated (cloudy) air, the virtual potential temperature, θ_v, is defined by:

$$\theta_v = \theta \cdot (1 + 0.61 \cdot r_{sat} - r_L) \quad (1.5.1a)$$

where r_{sat} is the water-vapor saturation mixing ratio of the air parcel, and r_L is the liquid-water mixing ratio. In (1.5.1a) the potential temperatures are in units of K, and the mixing ratios are in units of g/g. For unsaturated air with mixing ratio r, the virtual potential temperature is:

$$\theta_v = \theta \cdot (1 + 0.61 \cdot r) \quad (1.5.1b)$$

A derivation of the virtual temperature is given in Appendix D.

As usual, the potential temperature, θ, is defined as

$$\theta = T \left(\frac{P_o}{P} \right)^{0.286} \quad (1.5.1c)$$

where P is air pressure and P_o is a reference pressure. Usually, P_o is set to 100 kPa

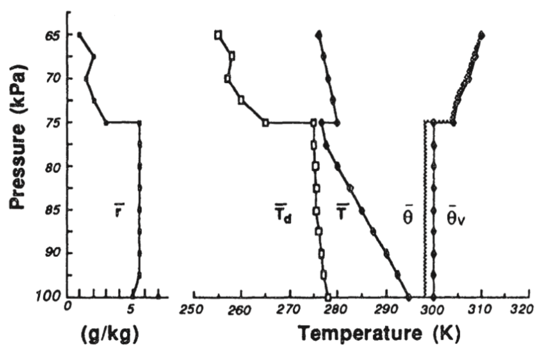
(1000 mb), but sometimes for boundary layer work the surface pressure is used instead.

To first order, we can approximate the potential temperature by

$$\theta \cong T + (g/C_p)\cdot z \tag{1.5.1d}$$

where z is the height above the 100 kPa (1000 mb) level, although sometimes height above ground level (agl) is used instead. The quantity $g/C_p = 0.0098$ K/m is just the negative of the dry adiabatic lapse rate (9.8 °C per kilometer), where g is the gravitational acceleration and C_p is the specific heat at constant pressure for air. Sometimes the quantity $C_p\cdot\theta$ is called the ***dry static energy***.

Fig. 1.5 Example of the difference between mean potential temperature, $\bar{\theta}$, and mean virtual potential temperature, $\bar{\theta}_v$, given observations of mixing ratio, $\bar{r}$, and absolute temperature, $\bar{T}$. Dew point, $\bar{T}_d$, is also shown.



An example of the difference between potential temperature and virtual potential temperature is shown in Fig 1.5 for a case of moist unsaturated air. The difference is small, but not negligible. Only when the air is very dry can we neglect the difference.

1.5.2 Example

Problem. Given a temperature of 25°C and a mixing ratio of 20 g/kg measured at a pressure of 90 kPa (900 mb), find the virtual potential temperature.

Solution. First, we must find the potential temperature:

$$\theta = T\,(P_o/P)^{0.286} = 298.16\cdot(100/90)^{0.286} = 307.28 \text{ K}$$

The air is unsaturated. allowing us to find the virtual potential temperature from:

$$\theta_v = \theta\cdot(1 + 0.61\cdot r) = 307.28\cdot[1 + 0.61\cdot(0.020)] = 311.03 \text{ K}$$