PHYS-2020: General Physics II
Course Lecture Notes
Section VIII

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Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 9th Edition* (2012) textbook by Serway and Vuille.

VIII. Sound

A. Characteristics of Sound Waves.

1. Sound waves are compression and rarefactions is some medium (*e.g.*, air or water) and propagate like longitudinal waves.

2. Categories of Sound Waves:

a) **Audible waves** are longitudinal waves which the human ear is sensitive $\Longrightarrow$ 20 – 20,000 Hz.

b) **Infrasonic waves** have $f < 20$ Hz.

c) **Ultrasonic waves** have $f > 20,000$ Hz $= 20$ kHz.

3. The method of transforming electrical energy to mechanical energy in crystals is called the **piezoelectric effect**.

4. The speed of sound in a medium follows

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \ . \qquad \text{(VIII-1)}$$

a) For liquid or gas, elastic properties are described by the Bulk Modulus (from General Physics I):

$$B \equiv -\frac{\Delta P}{\Delta V/V} \ , \qquad \text{(VIII-2)}$$

where P is pressure (in Pa) and V is volume (ΔV and V must have the same units).

b) Inertial properties are described by the mass-density ρ, hence

$$\boxed{v = \sqrt{\frac{B}{\rho}}} \quad \text{(for liquid or gas).} \qquad \text{(VIII-3)}$$

c) For a solid rod, elastic properties are given by Young's modulus Y, or

$$\boxed{v = \sqrt{\frac{Y}{\rho}}} \quad \text{(for solid rod).} \qquad \text{(VIII-4)}$$

d) For a gas, the Bulk Modulus is given by $B = \gamma P$, where γ is constant which depends upon the composition of the gas (it is determined by the ratio of the *specific heat at constant pressure* to *specific heat at constant volume*, which are concepts covered in the upper-level Thermodynamics course that we offer). As such, the speed of sound in a gas depends the pressure and density of the gas following

$$v = \sqrt{\frac{\gamma P}{\rho}} \; . \qquad \text{(VIII-5)}$$

i) If we make use of the ideal gas law, $P = \rho k_{\rm B} T/(\mu m_{\rm H})$, we can write the speed of sound in terms of temperature T (measured in K):

$$v = \sqrt{\frac{\gamma \rho k_{\rm B} T}{\mu m_{\rm H} \rho}} = \sqrt{\frac{\gamma k_{\rm B} T}{\mu m_{\rm H}}} \; . \qquad \text{(VIII-6)}$$

ii) We can determine sound speeds at any temperature if we can determine it at some reference temperature $T_\circ$ by setting up a ratio using Eq. (VIII-6):

$$\frac{v}{v_\circ} = \sqrt{\frac{\gamma k_{\rm B} T/(\mu m_{\rm H})}{\gamma k_{\rm B} T_\circ/(\mu m_{\rm H})}} = \sqrt{\frac{T}{T_\circ}} \; .$$

iii) At $T_\circ = 0°\text{C} = 273$ K, we can use the values of γ and μ (the mean molecular weight of the gas) in Eq. (VIII-6) to determine the speed of sound in <u>air</u> at this temperature to be $v_\circ = 331.3$ m/s.

iv) Using this value in the above equation, the sound speed in air depends upon air temperature via

$$v = (331.3 \text{ m/s}) \sqrt{\frac{T}{273 \text{ K}}} \,, \qquad \text{(VIII-7)}$$

where T is measured in Kelvins.

v) We also can express Eq. (VIII-7) as a function of temperature measured in degrees Celsius by making use of the relation $T_C = T - 273$, then

$$\begin{aligned} v &= (331.3 \text{ m/s}) \sqrt{\frac{273 + T_C}{273}} \,, \\ &= (331.3 \text{ m/s}) \sqrt{1 + \frac{T_C}{273^\circ\text{C}}} \,. \qquad \text{(VIII-8)} \end{aligned}$$

Example VIII–1. Problem 14.1 (Page 506) from the Serway & Vuille textbook: Suppose that you hear a clap of thunder 16.2 s after seeing the associated lightning stroke. The speed of sound waves in air is 343 m/s and the speed of light in air is 3.00×10^8 m/s. (a) How far are you from the lightning stroke? (b) Do you need to know the value of the speed of light to answer this question? Explain.

Solution (a):

Since $v_{\text{light}} \gg v_{\text{sound}}$, we ignore the time required for the lightning flash to reach the observer (it is virtually instantaneous) in comparison to the transit time for sound. Since the sound wave travels at a constant speed, we only need to use the definition of velocity, $v = \Delta x/\Delta t = d/\Delta t$, to determine the distance d:

$$d = v\,\Delta t = (343 \text{ m/s})(16.2 \text{ s}) = 5.56 \times 10^3 \text{ m} = \boxed{5.56 \text{ km}} \,.$$

Solution (b):

Since the time interval given is related to hearing the sound and not seeing the flash, and the fact that the speed of light is much greater than the speed of sound, the speed of light is not needed to solve this problem.

B. Energy and Intensity of Sound Waves.

1. The **intensity** I of a wave is the rate at which energy flows through a unit area A, $\perp$ to the direction of travel of the wave:

$$I \equiv \frac{1}{A}\frac{\Delta E}{\Delta t} \ . \qquad \text{(VIII-9)}$$

a) Since power is $\mathcal{P} = \Delta E/\Delta t$, we can rewrite Eq. (VIII-9) as

$$I = \frac{\mathcal{P}}{A} = \frac{\text{power}}{\text{area}} \ . \qquad \text{(VIII-10)}$$

b) Units of intensity are $\mathrm{W/m^2}$.

2. Levels of intensity:

a) **Threshold of hearing**: $I_{\rm th} = 1.00 \times 10^{-12}\ \mathrm{W/m^2}$.

b) **Threshold of pain**: $I_{\rm tp} = 1.00\ \mathrm{W/m^2}$.

c) The human ear can work *undamaged* at intensities between these two extremes.

d) The human ear is a *logarithmic* detector (so is the human eye) — a sound that is 1000 times louder than another sound is perceived as being only 30 times louder!

e) The relative intensity of a sound wave with respect to a different sound wave is called the **intensity level** or **decibel level** β:

$$\boxed{\beta \equiv 10 \log\left(\frac{I}{I_\circ}\right) \ .} \qquad \text{(VIII-11)}$$

i) $I_\circ$ is the reference sound level which is set to the threshold of hearing: $I_\circ = I_{\rm th}$.

ii) If the sound of interest also is equal to this threshold ($I = I_{\rm th}$), then

$$\beta = 10 \log \left(\frac{I_{\rm th}}{I_{\rm th}} \right) = 10 \log(1) = 10 \cdot 0 = 0 \text{ dB} ,$$

where dB ≡ **decibel**.

iii) If the sound of interest is equal to the threshold of pain ($I = I_{\rm tp}$), then

$$\begin{aligned} \beta &= 10 \log \left(\frac{I_{\rm tp}}{I_{\rm th}} \right) = 10 \log \left(\frac{1.00 \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) \\ &= 10 \log \left(10^{12}\right) = 10 \cdot 12 = 120 \text{ dB} . \end{aligned}$$

Example VIII–2. A microphone in the ocean is sensitive to sounds emitted by porpoises. To produce a usable signal, sound waves striking the microphone must have an intensity of 10 dB. If the porpoises emit sound waves with a power of 0.050 W, how far can the porpoise be from the microphone and still be heard? Disregard absorption of sound waves by the water.

Solution:

The intensity required for a sound level of 10 dB is found from Eq. (VIII-11). Using this equation and solving for I gives

$$\begin{aligned} \beta &= 10 \log \left(\frac{I}{I_\circ} \right) \\ \log \left(\frac{I}{I_\circ} \right) &= \frac{\beta}{10} \\ \frac{I}{I_\circ} &= 10^{\beta/10} \\ I &= I_\circ \, 10^{\beta/10} = (1.0 \times 10^{-12} \text{ W/m}^2) \cdot 10^{10/10} \end{aligned}$$

$$= (1.0 \times 10^{-12}\ \mathrm{W/m^2}) \cdot 10^1 = 1.0 \times 10^{-11}\ \mathrm{W/m^2}$$

Intensity is power per unit area (see Eq. VIII-10). If we assume the sound propagates in spherical wavefronts (see §VIII.C) under water (and it does), such a wave has a total surface area of $4\pi r^2$, where r is the distance that the wavefront is from the source. We can use this to solve for r (*i.e.*, the distance from which the porpoises can be heard):

$$\begin{aligned} I &= \frac{\mathcal{P}}{4\pi r^2} \\ r^2 &= \frac{\mathcal{P}}{4\pi I} \\ r &= \sqrt{\frac{\mathcal{P}}{4\pi I}} = \sqrt{\frac{0.050\ \mathrm{W}}{4\pi(1.0\times 10^{-11}\ \mathrm{W/m^2})}} \\ &= 2.0 \times 10^4\ \mathrm{m} = \boxed{20\ \mathrm{km}\ .} \end{aligned}$$

C. Spherical and Plane Waves.

1. Ideally, one can treat a sound emitter as a *point source* $\Longrightarrow$ the sound waves propagate out as a spherical wave as shown in Figure 14.4 of your textbook.

   a) The spherical surfaces of maximum intensity (*i.e.*, the crests of the wave) are called **wavefronts**. The separation between 2 wavefronts is called the **wavelength** (λ) of the spherical wave.

   b) The radial lines pointing outward from the source and cutting the wavefronts perpendicularly are called **rays**.

   c) Since the surface area of a sphere is $A = 4\pi r^2$, we can use Eq. (VIII-10) to write the intensity of a spherical wave (whether it be sound or light) as

$$I = \frac{\mathcal{P}}{4\pi r^2} \ , \qquad \text{(VIII-12)}$$

where r is the distance that the wavefront is from the source and $\mathcal{P}$ is the *average* power of the wave at the source's location.

d) As such, the intensity follows an inverse-square law, just as was the case with the gravitational force and the Coulomb force. Since $\mathcal{P}$ is the power emitted by the source, this is a constant term when comparing the intensity from the same source at 2 different positions (which we will label as '1' and '2' here):

$$\frac{I_1}{I_2} = \frac{\mathcal{P}/(4\pi r_1^2)}{\mathcal{P}/(4\pi r_2^2)} = \frac{r_2^2}{r_1^2} \ . \qquad \text{(VIII-13)}$$

2. If $r \gg \lambda$ (the wavelength of the wave), the wavefronts become parallel surfaces.

a) When examining a small sectional area of this spherical wavefront when $r \gg \lambda$, the rays going through this small area are effectively parallel lines! We will make use of this fact when we discuss optics.

b) When this is true, the wave is called a **plane wave**.

Example VIII–3. Problem 14.20 (Page 507) from the Serway & Vuille textbook: An outside loudspeaker (considered a small source) emits sound waves with a power output of 100 W. (a) Find the intensity 10.0 m from the source. (b) Find the intensity level, in decibels, at this distance. (c) At what distance would you experience the sound at the threshold of pain, 120 dB?

Solution (a):

To calculate the intensity, we only need to make use of Eq. (VIII-12):

$$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{100\ \text{W}}{4\pi (10.0\ \text{m})^2} = \boxed{7.96 \times 10^{-2}\ \text{W/m}^2\ .}$$

Solution (b):

To determine the decibel level, we make use of Eq. (VIII-11):

$$\begin{aligned} \beta &= 10 \log\left(\frac{I}{I_\circ}\right) = 10 \log\left(\frac{7.96 \times 10^{-2}\ \text{W/m}^2}{1.00 \times 10^{-12}\ \text{W/m}^2}\right) \\ &= 10 \log(7.96 \times 10^{10}) = \boxed{109\ \text{dB}\ .} \end{aligned}$$

Solution (c):

At the threshold of pain ($\beta = 120$ dB), the intensity is $I = 1.00\ \text{W/m}^2$. Thus, from Eq. (VIII-12), the distance from the speaker is

$$\begin{aligned} I &= \frac{\mathcal{P}}{4\pi r^2} \\ r^2 &= \frac{\mathcal{P}}{4\pi I} \\ r &= \sqrt{\frac{\mathcal{P}}{4\pi I}} = \sqrt{\frac{100\ \text{W}}{4\pi(1.00\ \text{W/m}^2)}} \\ &= \boxed{2.82\ \text{m}\ .} \end{aligned}$$

Think about this the next time you go to a concert or night club where the speakers are generally delivering anywhere from 500 W to 1 kW of power!

D. The Doppler Effect.

1. In the mid-1800s, Christian Doppler discovered that whenever there is relative motion between a source of waves and an observer, the observer hears a higher frequency emitted by the source if the two are moving towards each other as compared to when both are at rest with respect to each other. Also, a lower frequency is heard if the source and observer are moving apart from each other $\Longrightarrow$ the **Doppler effect**.

a) This effect is present for any type of wave phenomenon, whether is be sound or light (*i.e.*, electromagnetic radiation, see §IX of the notes).

b) For sound, the change in frequency is perceived as a change in **pitch**.

c) For light, the change in frequency is perceived as a change in **color** — **blueshifts** for higher-frequency shifts, **redshifts** for lower-frequency shifts.

2. If the source is at rest ($v_s = 0$) with respect to the observer and the observer is moving ($v_o \neq 0$), the Doppler effect, or frequency shift, takes the following form:

$$\boxed{f_o = f_s \left(\frac{v \pm v_o}{v} \right) .} \qquad \text{(VIII-14)}$$

a) $f_o \equiv$ frequency heard by the observer.

b) $f_s \equiv$ frequency emitted by the source.

c) $v_o \equiv$ velocity of the observer.

d) $v \equiv$ velocity of the wave.

e) The positive sign is used when the observer is moving towards the source, negative when moving away.

3. If the observer is at rest ($v_o = 0$) and the source is moving ($v_s \neq 0$), the Doppler effect formula becomes

$$\boxed{f_o = f_s \left(\frac{v}{v \mp v_s} \right)} \tag{VIII-15}$$

$\Longrightarrow$ negative sign when the source is moving toward observer, positive if away from observer.

4. Finally, if <u>both</u> source and observer are moving, then

$$\boxed{f_o = f_s \left(\frac{v \pm v_o}{v \mp v_s} \right) .} \tag{VIII-16}$$

a) The "upper" signs (*i.e.*, $+v_o$ and $-v_s$) refer to motion of one towards the other.

b) The "lower" signs (*i.e.*, $-v_o$ and $+v_s$) refer to motion of one away from the other.

Example VIII–4. **Problem 14.24 (Page 508) from the Serway & Vuille textbook**: An airplane traveling at half the speed of sound (v = 172 m/s) emits a sound of frequency 5.00 kHz. At what frequency does the stationary listener hear the sound (a) as the plane approaches? (b) after it passes?

Solution (a):

Here we could use either Eq. (VIII-15) or Eq. (VIII-16) with $v_o = 0$ since the observer is at rest. Using Eq. (VIII-15) with the negative sign in the denominator since the source is first approaching the listener, and noting that $v_s = v/2$ (*i.e.*, half the speed of sound) and $f_s = 5.00$ kHz, we get

$$f_o \;=\; f_s \left(\frac{v}{v - v/2} \right) = f_s \frac{v}{v/2} = f_s \cdot 2$$

$$= (5.00 \text{ kHz}) \cdot (2) = \boxed{10.0 \text{ kHz}\ .}$$

Solution (b):

Once again we use Eq. (VIII-15) but with a positive sign in the denominator since the source is receding from the stationary listener:

$$\begin{aligned} f_o &= f_s \left(\frac{v}{v + v/2} \right) = f_s \frac{v}{3v/2} = f_s \cdot \frac{2}{3} \\ &= (5.00 \text{ kHz}) \cdot \left(\frac{2}{3} \right) = \boxed{3.33 \text{ kHz}\ .} \end{aligned}$$

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5. If the source velocity exceeds the wave velocity, the sound waves can't escape the source before the next sound wave is emitted $\Longrightarrow$ the waves build into a **shock wave**!

a) The **Mach number** is defined as

$$M \equiv \frac{v_s}{v} \ , \qquad \text{(VIII-17)}$$

where v_s is the velocity of the source and v is the velocity of sound for the given temperature (or pressure and density) of the gas in which the source is traveling.

b) If $M < 1$, the object (*i.e.*, source) is said to be traveling **subsonically**.

c) If $M = 1$, the object (*i.e.*, source) is said to be traveling at the **sonic point** or (incorrectly) **sound barrier** (in reality it is not a barrier, the press actually came up with that name).

d) If $1 < M < 10$, the object (*i.e.*, source) is said to be traveling **supersonically**.

e) If $M \geq 10$, the object (*i.e.*, source) is said to be traveling **hypersonically**.

E. The Interference of Sound Waves.

1. Let's assume that we have two different speakers separated from each other delivering the same sound signal.

a) Let r_1 be the separation of the first speaker from the observer.

b) Let r_2 be the separation of the second speaker from the observer.

2. If the path difference $|r_2 - r_1|$ is zero or some integer multiple of wavelengths, **constructive interference** occurs:

$$|r_2 - r_1| = n\lambda \quad (n = 0, 1, 2, \ldots) \qquad \text{(VIII-18)}$$

$\Longrightarrow$ the sound intensity increases by a factor of two (assuming both speakers are delivering the same power).

3. If the path difference $|r_2 - r_1|$ is a half-integer multiple of the wavelengths, **destructive interference** occurs:

$$|r_2 - r_1| = \left(n + \frac{1}{2}\right)\lambda \quad (n = 0, 1, 2, \ldots) \qquad \text{(VIII-19)}$$

$\Longrightarrow$ the no sound is detected at the listening position.

4. These characteristics are important whenever you are setting up a sound system in your house.

a) Ideally, you want to place the listener such that they are not at a position that allows destructive interference for the audio range of frequencies.

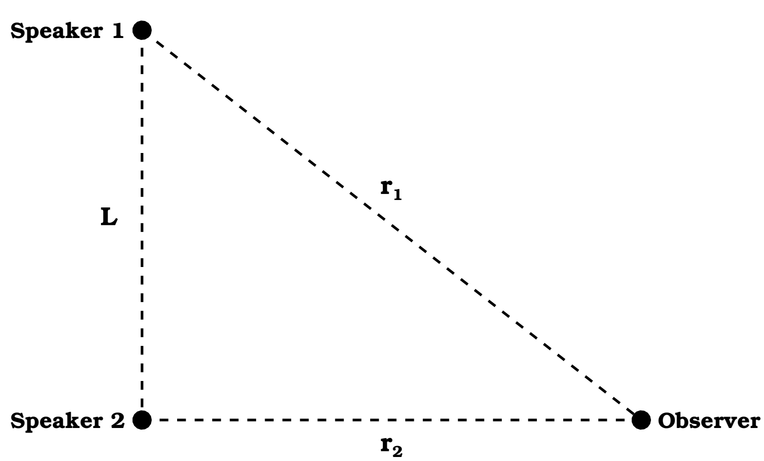
b) The *sweet spot* of a stereo system is the position such that maximum constructive interference occurs at a frequency around 1 kHz.

c) Destructive interference can occur at any position in the room if the speakers are not connected to the amplifier *in phase* (*i.e.*, same polarity) with each other.

Example VIII–5. Problem 14.37 (Page 509) from the Serway & Vuille textbook: A pair of speakers separated by 0.700 m are driven by the same oscillator at a frequency of 690 Hz. An observer originally positioned at one of the speakers begins to walk along a line perpendicular to the line joining the speakers as in Fig. P14.37. (a) How far must the observer walk before reaching a relative maximum in intensity? (b) How far will the observer be from the speaker when the first relative minimum is detected in the intensity?

Solution (a):

We will define the speaker that we are walking away from as *Speaker 2* and the second as *Speaker 1*. The figure below shows the geometry of the situation which will help us determine the distances from the speakers, where r_1 is the distance from Speaker 1, r_2 is the distance from Speaker 2, and L is the separation distance between the two speakers (= 0.700 m).



The first thing we need to do is to calculate the wavelength of the sound wave:

$$\lambda = \frac{v}{f} = \frac{345 \text{ m/s}}{690 \text{ Hz}} = 0.500 \text{ m} ,$$

where we have assumed that the speed of sound is $v = 345$ m/s. To hear the first relative maximum in sound intensity, we need to find the point from the speaker where the first constructive interference occurs from the waves of the two speakers. This occurs when $n = 1$ in Eq. (VIII-18), so with respect to Speaker 1, we add this extra wavelength length to the shorter of the two paths giving

$$r_1 = r_2 + \lambda .$$

Now we use the Pythagorean theorem to solve for r_2, the perdendicular distance from Speaker 2 with respect to the known quantities of λ and L, giving

$$\begin{aligned} r_1^2 &= r_2^2 + L^2 \\ (r_2 + \lambda)^2 &= r_2^2 + L^2 \\ r_2^2 + 2\lambda r_2 + \lambda^2 &= r_2^2 + L^2 \\ 2\lambda r_2 &= L^2 - \lambda^2 \\ r_2 &= \frac{L^2 - \lambda^2}{2\lambda} \\ &= \frac{(0.700 \text{ m})^2 - (0.500 \text{ m})^2}{2(0.500 \text{ m})} \\ &= \boxed{0.240 \text{ m}} \end{aligned}$$

to produce constructive interference.

Solution (b):

At the first relative minimum we have destructive interference. Setting $n = 1$ in Eq. (VIII-19), we get a path difference of

$$r_1 = r_2 + \lambda/2 .$$

Now we use the Pythagorean theorem once again to solve for r_2, giving a perpendicular distance of

$$\begin{aligned} r_1^2 &= r_2^2 + L^2 \\ (r_2 + \lambda/2)^2 &= r_2^2 + L^2 \\ r_2^2 + \lambda r_2 + \lambda^2/4 &= r_2^2 + L^2 \\ \lambda r_2 &= L^2 - \lambda^2/4 \\ r_2 &= \frac{L^2 - \lambda^2/4}{\lambda} \\ &= \frac{(0.700 \text{ m})^2 - (0.500 \text{ m})^2/4}{0.500 \text{ m}} \\ &= \boxed{0.855 \text{ m}} \end{aligned}$$

to produce destructive interference.

F. Standing Waves.

1. If a wave travels in a *closed* path (*i.e.*, one whose length does not change) such that the path length is a "half-integer" of the wave's wavelength,

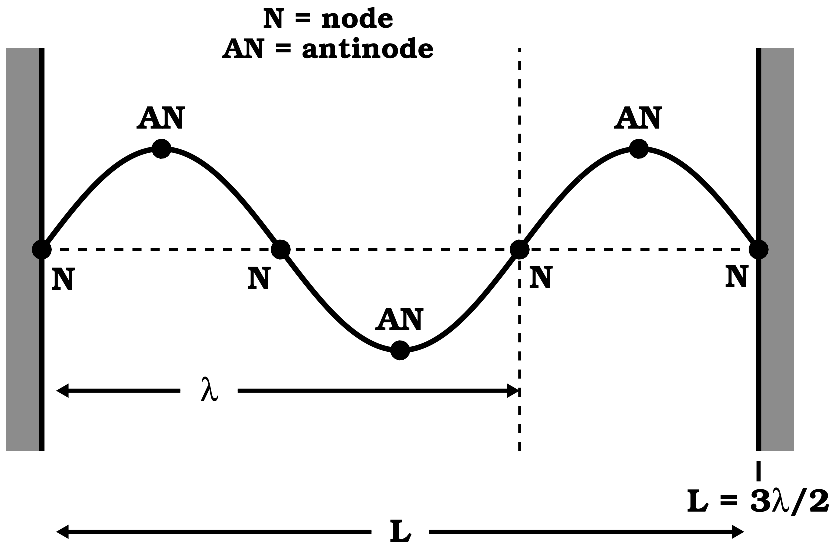
$$L = \frac{n}{2}\lambda \ , \quad (n = 1, 2, 3, \ldots), \qquad \text{(VIII-20)}$$

the wave is said to be a **standing wave** $\Longrightarrow$ the appears not to propagate.

a) Positions on the wave where $\Delta y = 0$ (zero amplitude points) at all times are called **nodes**.

b) Positions on the wave where $\Delta y = A$ (maximum amplitude) at all times are called **antinodes**.

c) All points on the wave oscillates with the same frequency except for the node points (which do not oscillate).



2. If a string is attached between 2 immovable walls of separation L as in the diagram above, we can use Eq. (VII-22) in Eq. (VIII-20) with

$$v = f\lambda$$

to get

$$f_n = \frac{v}{\lambda_n} = \frac{\sqrt{F/\mu}}{(2/n)L} = \frac{n}{2L}\sqrt{\frac{F}{\mu}}\ , \quad (n = 1, 2, 3, \ldots), \qquad \text{(VIII-21)}$$

where $F = T$ is the tension of the string.

a) When $n = 1$, half of a full wave sits in the cavity. This is lowest frequency of vibration is called the **fundamental frequency**:

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}\ . \qquad \text{(VIII-22)}$$

b) This fundamental frequency also is called the **first harmonic**.

c) $n = 2$ is the **second harmonic**: $f_2 = 2f_1$.

d) Note that all higher harmonics are integer multiples of the fundamental frequency:

$$f_n = n\,f_1\ . \qquad \text{(VIII-23)}$$

Example VIII–6. Problem 14.38 (Page 509) from the Serway & Vuille textbook: A steel wire in a piano has a length of 0.7000 m and a mass of 4.300×10^{-3} kg. To what tension must this wire be stretched in order that the fundamental vibration corresponds to middle C (f_C = 261.6 Hz on the chromatic musical scale)?

Solution:

In the fundamental mode of vibration, the wavelength of waves in the wire is

$$\lambda = 2L = 2(0.7000 \text{ m}) = 1.400 \text{ m} ,$$

(as given by Eq. VIII-20 with $n = 1$). If the wire is to vibrate at $f = 261.6$ Hz, the speed of the wave must be

$$v = f\lambda = (261.6 \text{ Hz})(1.400 \text{ m}) = 366.2 \text{ m/s} .$$

With $\mu = m/L = 4.300\times10^{-3}$ kg/0.7000 m $= 6.143\times10^{-3}$ kg/m, the required tension is given by $v = \sqrt{F/\mu}$ as

$$F = T = v^2\mu = (366.2 \text{ m/s})^2(6.143\times10^{-3} \text{ kg/m}) = \boxed{824.0 \text{ N} .}$$

G. Forced Vibrations and Resonance.

1. One can apply an external force to a mass that is already oscillating $\Longrightarrow$ **forced vibration**.

   a) The amplitude of such a vibration reaches a maximum when the frequency of the driving force, f, equals the natural frequency, $f_\circ$ = **resonance frequency**.

   b) Under this condition, the system is in **resonance**.

2. Forced vibrations applied at the resonance frequency will caused a *runaway* oscillation (see Figure 14.22 in the textbook for a disastrous example of a runaway oscillation).

H. Standing Waves in Air Columns.

1. If a pipe is **open at both ends**, the natural frequencies of vibration form a series in which all harmonics are present. In such a pipe, harmonics are obtained when the wavelength of the wave follows

$$\lambda_n = \frac{2}{n} L \qquad (n = 1, 2, 3, \ldots), \tag{VIII-24}$$

where L is the length of the pipe. Hence, these harmonics are equal to integer multiples of the fundamental frequency:

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L} v = n f_1 \qquad (n = 1, 2, 3, \ldots). \tag{VIII-25}$$

2. If a pipe is **closed at one end and open at the other end**, only *odd* harmonics are present (see Figure 14.23 in the textbook). The wavelengths of these harmonics follows

$$\lambda_n = \frac{4}{n} L \qquad (n = 1, 3, 5, \ldots) \tag{VIII-26}$$

which gives the frequencies of these harmonics as

$$f_n = \frac{v}{\lambda_n} = \frac{n}{4L} v = n f_1 \qquad (n = 1, 3, 5, \ldots). \tag{VIII-27}$$

Example VIII–7. Problem 14.49 (Page 510) from the Serway & Vuille textbook: The windpipe of a typical whooping crane is about 5.0 feet long. What is the lowest resonance frequency of this pipe assuming it is a pipe closed at one end? Assume a temperature of 37°C.

Solution:

Assuming an air temperature of $T = 37°\text{C} = 310$ K, the speed of sound inside the pipe is

$$v = (331 \text{ m/s})\sqrt{\frac{T}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{\frac{310 \text{ K}}{273 \text{ K}}} = 353 \text{ m/s} .$$

In the fundamental resonance mode, the wavelength of the sound waves in a pipe closed at one end is $\lambda_1 = 4L$ (see Eq. VIII-26). Thus, for the whooping crane

$$\lambda_1 = 4\,(5.0 \text{ ft}) = 20. \text{ ft} \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 6.1 \text{ m} .$$

and

$$f_1 = \frac{v}{\lambda_1} = \frac{353 \text{ m/s}}{6.10 \text{ m}} = \boxed{58 \text{ Hz} .}$$
