5.6 The Richardson Number

5.6.1 Flux Richardson Number

In a statically stable environment, turbulent vertical motions are acting against the restoring force of gravity. Thus, buoyancy tends to suppress turbulence, while wind shears tend to generate turbulence mechanically. The buoyant production term (Term III) of the TKE budget equation (5.1b) is negative in this situation, while the mechanical production term (Term IV) is positive. Although the other terms in the TKE budget are certainly important, a simplified but nevertheless useful approximation to the physics is possible by examining the ratio of Term III to Term IV. This ratio, called the ***flux Richardson number***, R_f, is given by

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\overline{(w'\theta_v')}}{\overline{(u_i'u_j')}\frac{\partial \overline{U_i}}{\partial x_j}} \qquad (5.6.1a)$$

where the negative sign on Term IV is dropped by convention. The Richardson number is dimensionless. The denominator consists of 9 terms, as implied by the summation notation.

If we assume horizontal homogeneity and neglect subsidence, then the above equation reduces to the more common form of the flux Richardson number:

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\overline{(w'\theta_v')}}{\overline{(u'w')}\frac{\partial \overline{U}}{\partial z} + \overline{(v'w')}\frac{\partial \overline{V}}{\partial z}} \qquad (5.6.1b)$$

For statically unstable flows, R_f is usually negative (remember that the denominator is usually negative). For neutral flows, it is zero. For statically stable flows, R_f is positive.

Richardson proposed that $R_f = +1$ is a critical value, because the mechanical production rate balances the buoyant consumption of TKE. At any value of R_f less than +1, static stability is insufficiently strong to prevent the mechanical generation of turbulence. For negative values of R_f, the numerator even contributes to the generation of turbulence. Therefore, he expected that

Flow IS turbulent (dynamically unstable) when $R_f < +1$

Flow BECOMES laminar (dynamically stable) when $R_f > +1$

We recognize that statically unstable flow is, by definition, always dynamically unstable.

5.6.2 Gradient Richardson Number

A peculiar problem arises in the use of R_f ; namely, we can calculate its value only for turbulent flow because it contains factors involving turbulent correlations like $\overline{w'\theta_v'}$. In other words, we can use it to determine whether turbulent flow will become laminar, but not whether laminar flow will become turbulent.

Using the reasoning of section 2.7 and Fig 2.13, it is logical to suggest that the value of the turbulent correlation $-\overline{w'\theta_v'}$ might be proportional to the lapse rate $\partial\overline{\theta_v}/\partial z$. Similarly, we might suggest that $-\overline{u'w'}$ is proportional to $\partial\overline{U}/\partial z$, and that $-\overline{v'w'}$ is proportional to $\partial\overline{V}/\partial z$. These arguments form the basis of a theory known as K-theory or eddy diffusivity theory, which will be discussed in much more detail in chapter 6. For now, we will just assume that the proportionalities are possible, and substitute those in (5.6.1b) to give a new ratio called the ***gradient Richardson number***, Ri :

$$\mathrm{Ri} = \frac{\dfrac{g}{\overline{\theta_v}} \dfrac{\partial\overline{\theta_v}}{\partial z}}{\left[\left(\dfrac{\partial\overline{U}}{\partial z}\right)^2 + \left(\dfrac{\partial\overline{V}}{\partial z}\right)^2 \right]} \qquad (5.6.2)$$

When investigators refer to a "Richardson number" without specifying which one, they usually mean the gradient Richardson number.

Theoretical and laboratory research suggest that laminar flow becomes unstable to KH-wave formation and the ONSET of turbulence when Ri is smaller than the ***critical Richardson number***, R_c . Another value, R_T, indicates the termination of turbulence. The dynamic stability criteria can be stated as follows:

Laminar flow becomes turbulent when $\mathrm{Ri} < R_c$.
Turbulent flow becomes laminar when $\mathrm{Ri} > R_T$.

Although there is still some debate on the correct values of R_c and R_T, it appears that R_c = 0.21 to 0.25 and R_T = 1.0 work fairly well. Thus, there appears to be a ***hysteresis*** effect because R_T is greater than R_c.

One hypothesis for the apparent hysteresis is as follows. Two conditions are needed for turbulence: instability, and some trigger mechanism. Suppose that dynamic instability occurs whenever $\mathrm{Ri} < R_T$. If one trigger mechanism is existing turbulence in or adjacent to the unstable fluid, then turbulence can continue as long as $\mathrm{Ri} < R_T$ because of the presence of both the instability and the trigger. If KH waves are another trigger mechanism, then in the absence of existing turbulence one finds that Ri must get well below R_T before KH waves can form. Laboratory and theoretical work have shown that the criterion for KH wave formation is $Ri < R_c$. This leads to the apparent hysteresis, because the Richardson number of nonturbulent flow must be lowered to R_c before turbulence will start, but once turbulent, the turbulence can continue until the Richardson number is raised above R_T.

5.6.3 Bulk Richardson Number

The theoretical work yielding $R_c \cong 0.25$ is based on local measurements of the wind shear and temperature gradient. Meteorologists rarely know the actual local gradients, but can approximate the gradients using observations made at a series of discrete height intervals. If we approximate $\partial\overline{\theta_v}/\partial z$ by $\Delta\overline{\theta_v}/\Delta z$, and approximate $\partial\overline{U}/\partial z$ and $\partial\overline{V}/\partial z$ by $\Delta\overline{U}/\Delta z$ and $\Delta\overline{V}/\Delta z$ respectively, then we can define a new ratio known as the ***bulk Richardson number***, R_B :

$$R_B = \frac{g \, \Delta\overline{\theta_v} \, \Delta z}{\overline{\theta_v} \, [(\Delta\overline{U})^2 + (\Delta\overline{V})^2]} \qquad (5.6.3)$$

It is this form of the Richardson number that is used most frequently in meteorology, because rawinsonde data and numerical weather forecasts supply wind and temperature measurements at discrete points in space. The sign of the finite differences are defined, for example, by $\Delta\overline{U} = \overline{U}(z_{top}) - \overline{U}(z_{bottom})$.

Unfortunately, the critical value of 0.25 applies only for local gradients, not for finite differences across thick layers. In fact, the thicker the layer is, the more likely we are to average out large gradients that occur within small subregions of the layer of interest. The net result is (1) we introduce uncertainty into our prediction of the occurrence of turbulence, and (2) we must use an artificially large (theoretically unjustified) value of the critical Richardson number that gives reasonable results using our smoothed gradients. The thinner the layer, the closer the critical Richardson number will likely be to 0.25. Since data points in soundings are sometimes spaced far apart in the vertical, approximations such as shown in the graph and table in Fig 5.19 can be used to estimate the probability and intensity of turbulence (Lee, et al., 1979).

Table 5-1 shows a portion of a rawinsonde sounding, together with the corresponding values of bulk Richardson number. The resulting turbulence diagnosis is given in the rightmost column of Table 5-1. Note that the Richardson number itself says nothing about the intensity of turbulence, only about the yes/no presence of turbulence.