Table 5-1. Example of a nighttime rawinsonde sounding analyzed to give stability, shear, Richardson number, and the probability and intensity of turbulence. Probabilities are expressed as a percent, and intensities are abbreviated by:

N = no turbulence, L = light (0.5 G), M = moderate (1 G), S=severe (2 G)

These intensity levels correspond to the turbulence reporting recommendations used in aviation, where the vertical acceleration measured in Gs (number of times the pull of gravity) is relative to the center of gravity of the aircraft. For practical purposes, a probability greater than 50% AND an intensity greater than L were required before a CAT forecast would be issued.

z (m)	Wind Dir (°)	Speed (m/s)	T (K)	θ (K)	Lapse (K/m)	Shear (s^{-1})	R_B	CAT Prob(%)	CAT Inten.
1591	154	9.8	281	294.4	0.0021	0.0034	6.19	41	N
1219	150	10.7	-	-	0.0021	0.0045	3.43	68	N
914	144	9.7	-	-	0.0021	0.0091	0.86	94	N-L
702	-	-	287.8	292.5	0.0020	0.0091	0.81	94	N-L
610	134	7.4	-	-	0.0020	0.0170	0.23	100	L-M
393	-	-	290.2	291.9	0.0204	0.0170	2.37	79	L-M
305	95	3.5	-	-	0.0204	0.0137	3.64	66	N
222	79	2.7	288.4	288.4	0.0133	0.0071	8.92	13	N
4	45	2.5	287.6	285.5	-	-	-	-	-

5.6.4 Examples

Problem A: Given the same data from problem 5.2.8, calculate the flux Richardson number and comment on the dynamic stability.

Solution. Since the flux Richardson number is defined as the ratio of the buoyancy term to the negative of the shear term, we can use the values for these terms already calculated in example 5.2.8:

$$R_f = \frac{\text{buoyancy term}}{-\text{ shear term}} = \frac{0.00493}{-0.0003} = -16.4$$

Discussion. A negative Richardson number is without question less than +1, and thus indicates dynamic instability and turbulence. This is a trivial conclusion, because any flow that is statically unstable is also dynamically unstable by definition.

Problem B: Given a fictitious SBL where $(g/\overline{\theta_v}) = 0.033\ \text{m s}^{-2}\ \text{K}^{-1}$, $\partial\overline{U}/\partial z = [u_* / (0.4 \cdot z)]\ \text{s}^{-1}$, $u_* = 0.4$ m/s, and where the lapse rate, c_1, is constant with height such that there is 6°C $\overline{\theta_v}$ increase with each 200 m of altitude gained. How deep is the turbulence?

Solution. We can use the gradient Richardson number as an indicator of dynamic stability and turbulence. Using the prescribed gradients, we find that:

$$\text{Ri} = \frac{\dfrac{g}{\overline{\theta_v}} \dfrac{\partial\overline{\theta_v}}{\partial z}}{\left(\dfrac{\partial\overline{U}}{\partial z}\right)^2} = \frac{\dfrac{g}{\overline{\theta_v}} c_1}{\left(\dfrac{u_*}{0.4z}\right)^2} = \frac{(0.033)\cdot(0.03)}{(0.4\,/\,0.4)^2} z^2 = (0.00099\ \text{m}^{-2})\ z^2$$

If we use $R_c = 0.25$, then we can use this critical value in place of Ri above and solve for z at the critical height above which there is no turbulence:

$$z = \sqrt{(1010\ \text{m}^2)\ R_c} = \sqrt{252.5\ \text{m}^2} = 15.9\ \text{m}$$

Discussion. If we has used a critical termination value of $R_T = 1.0$, then we would have found a critical height of 31.8 m. Thus, below 15.9 m we expect turbulence, while above 31.8 m we expect laminar flow. Between these heights the turbulent state depends on the past history of the flow at that height. If previously turbulent, it is turbulent now.

5.7 The Obukhov Length

The Obukhov length (L) is a scaling parameter that is useful in the surface layer. To show how this parameter is related to the TKE equation, first recall that one definition of the surface layer is that region where turbulent fluxes vary by less that 10% of their magnitude with height. By making the constant flux (with height) approximation, one can use surface values of heat and momentum flux to define turbulence scales and nondimensionalize the TKE equation.

Start with the TKE equation (5.1a), multiply the whole equation by $(-k\,z/u_*^3)$, assume all turbulent fluxes equal their respective surface values, and focus on just terms III, IV, and VII:

$$\ldots = -\frac{k\,z\,g\,(\overline{w'\theta_v'})_s}{\overline{\theta_v}\,u_*^3} + \frac{k\,z\,(\overline{u_i'u_j'})_s}{u_*^3}\frac{\partial\overline{U_i}}{\partial x_j} + \ldots - \frac{k\,z\,\varepsilon|_s}{u_*^3} \tag{5.7a}$$

III IV VII

Each of these terms is now dimensionless. The last term, a dimensionless dissipation rate, will not be pursued further here.

The ***von Karman constant***, k, is a dimensionless number included by tradition. Its importance in the log wind profile in the surface layer is discussed in the next section. Investigators have yet to pin down its precise value, although preliminary experiments suggest that it is between about 0.35 and 0.42 . We will use a value of 0.4 in most of this book, although some of the figures adopted from the literature are based on k=0.35.

Term III is usually assigned the symbol, ζ, and is further defined as $\zeta \equiv z/L$, where L is the ***Obukhov length***. Thus,

$$\zeta = \frac{z}{L} = \frac{-k\, z\, g\, (\overline{w'\theta_v'})_s}{\overline{\theta_v}\, u_*^3} \tag{5.7b}$$

The Obukhov length is given by:

$$L = \frac{-\overline{\theta_v}\, u_*^3}{k\, g\, (\overline{w'\theta_v'})_s} \tag{5.7c}$$

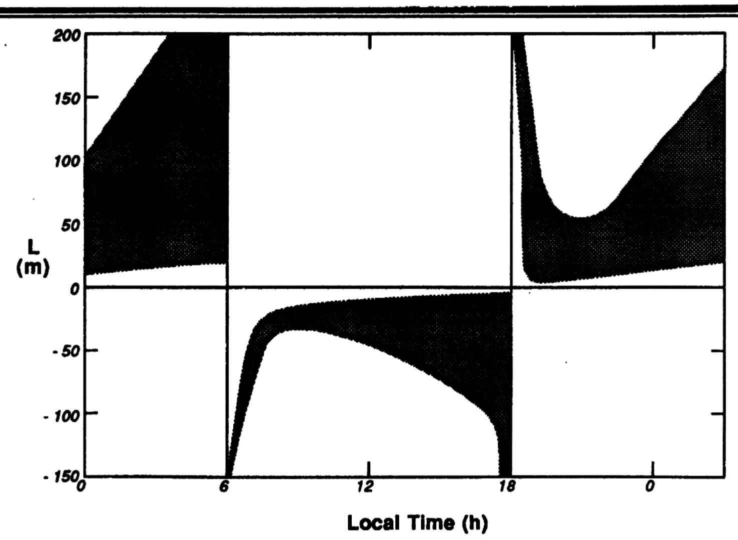


Fig. 5.21 Typical ranges of Obukhov length (L) evolution over a diurnal cycle.

One physical interpretation of the Obukhov length is that it is proportional to the height above the surface at which buoyant factors first dominate over mechanical (shear) production of turbulence. For convective situations, buoyant and shear production terms are approximately equal at $z = -0.5\,L$. Fig 5.21 shows the typical range of variations of the Obukhov length in fair weather conditions over land.

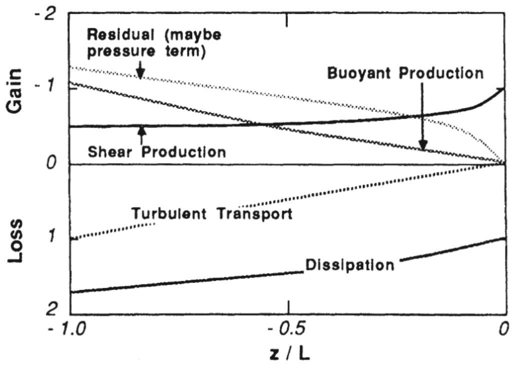
The parameter ζ turns out to be very important for scaling and similarity arguments of the surface layer, as will be discussed in more detail in a later chapter. It is sometimes called a stability parameter, although its magnitude is not directly related to static nor dynamic stability. Only its sign relates to static stability: negative implies unstable, positive implies statically stable. A better description of ζ is "a surface-layer scaling parameter".

We can write an alternative form for ζ by employing the definition of w_*:

$$\zeta = \frac{z}{L} = -\frac{k\,z\,w_*^3}{z_i\,u_*^3} \qquad (5.7d)$$

Fig. 5.22 shows the variation of TKE budget terms with ζ, as ζ varies between 0 (statically neutral) and -1 (slightly unstable). The decrease in importance of shear and increase of buoyancy as ζ decreases from 0 to -1 is particularly obvious.

Fig. 5.22
Behavior of the terms (made dimensionless with kz/u_*^3) in the unstable surface-layer turbulence kinetic energy budget. After Wyngaard (1973).



Figs. 5.23 shows the variation of Ri with ζ from slightly unstable to slightly stable conditions. For unstable situations, Ri $\cong \zeta$. One must keep in mind that ζ can be calculated only for turbulent flow, thus this figure shows only the subset of all data that

was turbulent. Nonturbulent flow can occur in stable situations, but it does not appear in this figure.

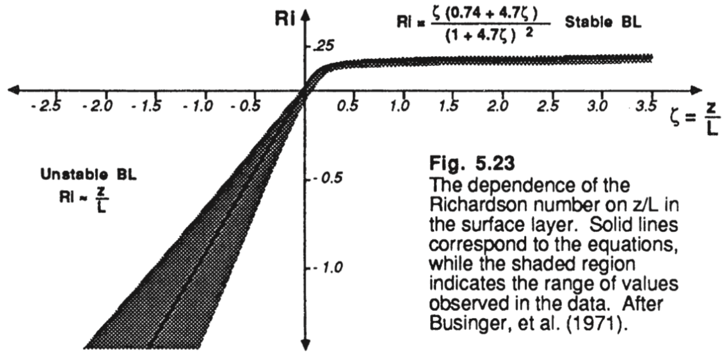


Fig. 5.23
The dependence of the Richardson number on z/L in the surface layer. Solid lines correspond to the equations, while the shaded region indicates the range of values observed in the data. After Businger, et al. (1971).

5.8 Dimensionless Gradients

We can simplify term IV of the dimensionless TKE equation (5.7a) by choosing a coordinate system aligned with the mean wind, assuming horizontal homogeneity, neglecting subsidence, and using the definition that $u_*^2 = -(\overline{u'w'})_s$:

$$\text{Term IV} = \frac{-k\,z}{u_*}\frac{\partial \overline{U}}{\partial z}$$

Based on this dimensionless term, we can define a ***dimensionless wind shear***, ϕ_M, by

$$\phi_M = \frac{k\,z}{u_*}\frac{\partial \overline{U}}{\partial z} \qquad (5.8a)$$

This parameter is primarily useful for studies of surface-layer wind profiles and momentum fluxes. In chapter 9 we will use ϕ_M in similarity theory to estimate momentum flux (as given by u_*) from the local mean wind shear. This is particularly valuable because it is easy to measure mean wind speeds at a variety of heights in the surface layer, but much more difficult and expensive to measure the eddy correlations such as $\overline{u'w'}$.

By analogy, a ***dimensionless lapse rate***, ϕ_H, and a ***dimensionless humidity gradient***, ϕ_E , can be defined:

$$\phi_H = \frac{k\,z}{\theta_*^{SL}} \frac{\partial \overline{\theta}}{\partial z} \tag{5.8b}$$

$$\phi_E = \frac{k\,z}{q_*^{SL}} \frac{\partial \overline{q}}{\partial z} \tag{5.8c}$$

These dimensionless gradients are equally as valuable as the dimensionless shear, because using similarity theory we can estimate the surface layer heat flux and moisture flux from simple measurements of lapse rate and moisture gradient, respectively.

5.9 Miscellaneous Scaling Parameters

5.9.1 Definitions

A few additional dimensionless scaling groups have been suggested in the literature to help explain boundary layer characteristics. Again, these are often inappropriately called stability parameters. One parameter that is useful in the surface layer is:

$$\mu^{SL} = \frac{k\,u_*}{f_c\,L} \tag{5.9.1a}$$

$$= \frac{g\,k^2 \overline{(w'\theta_v')}_s}{\overline{\theta_v}\; f_c \overline{(u'w')}_s} \tag{5.9.1b}$$

$$= \frac{g\,k^2\,\theta_*^{SL}}{\overline{\theta_v}\; f_c\, u_*} \tag{5.9.1c}$$

Another scaling parameter that is useful in the ML is

$$\mu^{ML} = k \frac{z_i}{L} \tag{5.9.1d}$$

$$= \frac{-k^2 \left(\frac{g}{\overline{\theta_v}}\right) \overline{(w'\theta_v')}_s}{u_*^3} \tag{5.9.1e}$$

$$= -k^2 \frac{w_*^3}{u_*^3} \qquad (5.9.1f)$$

It's important not to confuse either of these two parameters with the dynamic viscosity, which traditionally uses the same symbol.

Another parameter occasionally used is:

$$s_G = \frac{\overline{\theta}_s - \overline{\theta}_{air}}{\overline{M}^2 \left[1 + \log\left(\frac{10}{z} \right) \right]^2} \qquad (5.9.1g)$$

which looks like a modified Richardson number. Additional scaling parameters and dimensionless groups will be introduced in later chapters where appropriate.

5.9.2 Example

Problem: Given surface measurements: $u_* = 0.2$ m·s^{-1}, $g/\overline{\theta_v} = 0.0333$ m·s^{-1}K^{-1}, and $\overline{w'\theta_v'} = -0.05$ K·m·s^{-1}; and at 10 m: $\partial\overline{U}/\partial z = 20$ m·s^{-1}/100m, and $\partial\overline{\theta_v}/\partial z =$ 20 °C/100m. Find scaling parameters L, ζ, θ_*^{SL}, ϕ_M, ϕ_H, and μ^{SL} at z = 10 m, at a latitude where $f_c = 10^{-4}$ s^{-1}.

Solution:

$$L = \frac{-u_*^3}{k\,(g/\overline{\theta_v})\,\overline{w'\theta_v'}} = \frac{(0.2)^3}{(0.4)\,(0.0333)\,(0.05)} = 12.0 \text{ m}$$

$$\zeta = \frac{z}{L} = \frac{10}{12} = 0.83$$

$$\phi_M = \frac{k\,z}{u_*}\frac{\partial\overline{U}}{\partial z} = \frac{(0.4)\,(10)}{(0.2)}(0.2) = 4.0$$

$$\theta_*^{SL} = \frac{-\overline{w'\theta_v'}_s}{u_*} = \frac{0.05}{0.2} = 0.25 \text{ K}$$

$$\phi_H = \frac{k\,z}{\theta_*^{SL}} \frac{\partial \overline{\theta_v}}{\partial z} = \frac{(0.4)\,(10)}{(0.25)}\,(0.2) = 3.2$$

$$\mu^{SL} = \frac{k\,u_*}{f_c\,L} = \frac{(0.4)\,(0.2)}{(10^{-4})\,(12)} = 66.7$$

5.10 Combined Stability Tables

Static and dynamic stability concepts are intertwined, as sketched in Fig 5.24a. Negative Richardson numbers always correspond to statically and dynamically unstable flow. This flow will definitely become turbulent. Positive Richardson numbers are always statically stable, but there is the small range of 0 < Ri < 1 where positive Richardson numbers are dynamically unstable, and may be turbulent depending on the past history of the flow. Namely, nonturbulent flow will become turbulent at about Ri = 0.25, while flow that is presently turbulent will stay turbulent if Ri < 1.

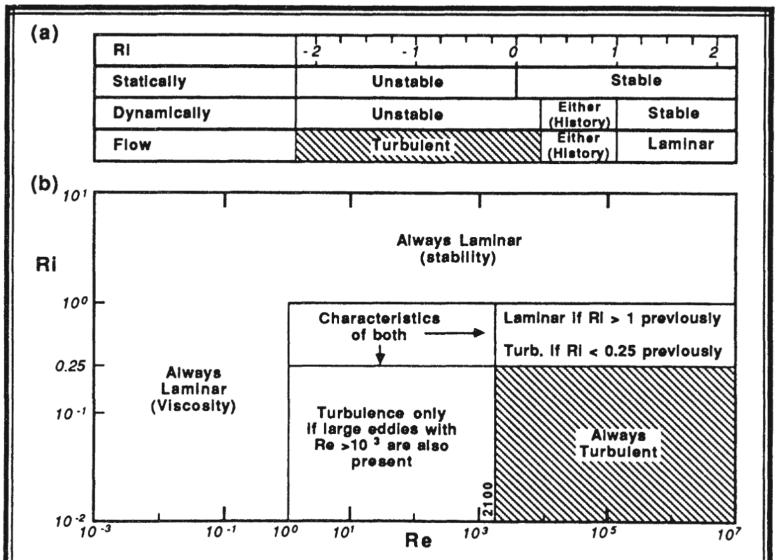


Fig. 5.24 Stability parameter relationships (see text). (After Woods, 1969).