1. **Series of Functions**
	1. **Definition**: Let $\{f\_{n}\}$ is a sequence of real value and defined on $Ω$. $∀x\in Ω$ we define $S\_{1}\left(x\right)=f\_{1}\left(x\right),S\_{2}\left(x\right)=f\_{1}\left(x\right)+ f\_{2}\left(x\right), S\_{3}\left(x\right)=f\_{1}\left(x\right)+ f\_{2}\left(x\right)+f\_{3}\left(x\right),…, S\_{n}\left(x\right)=f\_{1}\left(x\right)+ f\_{2}\left(x\right)+…+f\_{n}\left(x\right)$. We said that an infinite series $\sum\_{n=1}^{\infty }f\_{n}$ converges to $x$, if $\{S\_{n}\left(x\right)\}$ converges. We said that $\sum\_{n=1}^{\infty }f\_{n}$ converges on $Ω$, if $\{S\_{n}\}$ converges to $S$ on $Ω$, (this means $\lim\_{n\to \infty }S\_{n}=S$) $\ni S=\sum\_{n=1}^{\infty }f\_{n}$. If $\{S\_{n}\}$ diverges (this means $\lim\_{n\to \infty }S\_{n}$ does not exist) we say that $\sum\_{n=1}^{\infty }f\_{n}$ diverges.
	2. **Theorem**: An infinite series $\sum\_{n=1}^{\infty }f\_{n}$ is uniformly converges on $Ω⟺∀v>0 ∃k\in Z^{+}\ni \left|\sum\_{n=n+1}^{n+m}f\_{n}\right|<v∀x\in Ω, ∀m>0, n>k$.
	3. **Note**: We said that $\sum\_{n=1}^{\infty }f\_{n}$ absolutely converges to $x$, if $\sum\_{n=1}^{\infty }\left|f\_{n}(x)\right|$ converges.
	4. **Example**: Let a function $f\_{n}:Ω\rightarrow R$ is defined as $f\_{n}\left(x\right)=x^{n-1} ∀x\in Ω, ∀n$, if each one of $f\_{n}$ continuous on $Ω$, then $\sum\_{n=1}^{\infty }f\_{n}$ uniformly converges to $f$ on $Ω$, then $f$ continuous.

**Solution:** $S\_{n}\left(x\right)=\sum\_{k=1}^{n}f\_{k}(x)=\sum\_{k=1}^{n}x^{k-1}=\frac{1-x^{k}}{1-x}$with $n\ne 1$, so $\{S\_{n}\left(x\right)\}$ converges and $S\_{n}\left(x\right)\rightarrow \frac{1}{1-x}$ , if $\left|x\right|<1$

$⟹\sum\_{n=1}^{\infty }f\_{n}$ converges, if $\left|x\right|\geq 1$.

* 1. **Theorem:** Let $\sum\_{n=1}^{\infty }f\_{n}$ is an infinite series. If every function of $f\_{n}$ is continuous on $Ω$ and $\sum\_{n=1}^{\infty }f\_{n}$ uniformly converges to $f$ on $Ω$, then $f$ is continuous.

**Power Series**

* 1. **Definition**: Let $\{a\_{n}\}$ is a sequence of constants, $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ is a power series in $x \ni x^{0}=1 ∀x$. $\sum\_{n=0}^{\infty }a\_{n}(x-b)^{n}$ is a power series in $x-b \ni b $is a constant.
	2. **Examples**:
1. A power series $\sum\_{n=1}^{\infty }\frac{x^{n-1}}{(n-1)!}$ converges $∀x\in R$.
2. A power series $\sum\_{n=1}^{\infty }\frac{x^{n-1}}{r^{n-1}}$ converges where $\left|x\right|<r$ and $\sum\_{n=1}^{\infty }\frac{x^{n-1}}{r^{n-1}}$ diverges where $\left|x\right|\geq r$.
3. $\lim\_{n\to \infty }\frac{x^{n}}{n!}=0$ $∀n\in R$.
	1. **Theorem:** If $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ converges to $x\_{0}\ne 0$, then $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ absolutely converges $∀x\in R \ni \left|x\right|<\left|x\_{0}\right|$.
	2. **Corollary:** If $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ diverges at $x\_{0}$, then $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ diverges $∀x\in R \ni \left|x\right|>\left|x\_{0}\right|$.
	3. **Theorem:** For any $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ be one of the following:
4. $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ converges only where $x=b$.
5. $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ absolutely converges where $x=b$.
6. There is $r\in R\ni \sum\_{n=0}^{\infty }a\_{n}x^{n}$ absolutely converges $∀x$ where $\left|x-b\right|<r$ and $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ diverges $∀x$ where $\left|x-b\right|>r$.
	1. **Theorem:**
7. If $\lim\_{n\to \infty }\sqrt[n]{\left|a\_{n}\right|}=r\ne 0$, then convergent half of $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ is $\frac{1}{r}$ .
8. If $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ absolutely converges at one side of interval, then $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ absolutely converges to other side.
9. If $r$ is a radius of convergent $\sum\_{n=0}^{\infty }a\_{n}x^{n}$, then radius of $\sum\_{n=0}^{\infty }a\_{n}x^{n}$ is $\sqrt{r}$ .