1. **Measure of Unbounded**

(12.1) **Theorem**: Let and . Then is measurable, if is measurable .

(12.2) **Example:** If

does not bounded

is measurable

is measurable, and

(12.3) **Example:** If

is a bounded

is measurable

is measurable, and

(12.4) **Example:** If

does not bounded

is measurable

is measurable, and

(12.5) **Example**s:

1. If is a countable, then is measurable and , so are measurable and their measure equal zero.
2. is measurable and .

(12.6) **Theorem:** A set is measurable an open set and a closed set and .

(12.7) **Theorem**:

1. The interval is measurable and .
2. The interval is measurable and .
3. Every closed set in is measurable.
4. Every open set in is measurable.
5. If , then is measurable and .
6. If , then is measurable and .

**Measurable Functions**

(12.8) **Notes**: If , then

1. .
2. .

(12.9) **Notes**: If and a function, then we have

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. .
9. .
10. .
11. .
12. .
13. .

(12.10) **Definition:** Let is a set. We said that a function is measurable, if measurable in then is measurable in .

(12.11) **Theorem:** Let is a set and is a function, then the following statements are equivalent.

1. is measurable.
2. , then is measurable.
3. , then is measurable.
4. , then is measurable.
5. , then is measurable.

**Proof:** (1) (2) let

Since is measurable and is measurable, then

is measurable in , but , then

is measurable.

(2) (3) let and let , then

From (2), we get is measurable.

is measurable, but

is measurable.

(3) (4) let , since is measurable.

is measurable, but

is measurable.

(4) (5) let and let , then

From (4), we get is measurable.

is measurable, but

is measurable.

(5) (1) check

(12.12) **Corollary:** Let is a set and is measurable function, then

1. , then is measurable.
2. , then is measurable.
3. and are measurable.

(12.13) **Example**: Let is a function defined as , then is measurable.

**Solution:** since

Since , , , , are measurable, then is measurable.

is measurable.

(12.14) **Example**: Let represents a set of rational numbers in and is a function defined as , then is measurable.

(12.15) **Example**: Let represents a set of rational numbers in and is a function defined as , then is measurable.

(12.16) **Theorem:** Let is measurable in , and is a continuous function, then is measurable.

**Proof:** let , put

neighborhood of

Let then an open set is measurable.

Since , then is measurable, then is measurable.

(12.17) **Theorem:** Let is a set, is measurable function and continuous function, then is measurable.

**Proof:** since continuous, then is open

is a countable family of disjoint open intervals.

Since is measurable.

is measurable is measurable.

But is measurable.

(12.18) **Theorem:** Let is a set, let and measurable functions, then we have

1. .
2. , , are measurable.
3. .

**Proof:** (1)let

(12.19) **Theorem:** Let is a set, let and measurable functions, and let , then

are measurable functions.

**Proof:** let , then

Since is measurable function, then

is measurable function

But

is measurable function

is measurable function.