1. **Functions of Bounded Variation**

(14.1) **Definition**: Let and is a function, we said that a number is a limit of at , if , if , then. Such that .

(14.2) **Definition**: Let . We say that a function is continuous at , if .

(14.3) **Definition**: Let . We say that a function is

1. Decreasing or non-decreasing, if .
2. Strictly increasing, if .
3. Decreasing or non-increasing, if .
4. Strictly decreasing, if .

(14.4) **Theorem:** Ifa function is an increasing, then and are exist and .

(14.5) **Theorem**: Let . If a function is a strictly increasing, then exists.

(14.6) **Corollary:** If a function is a strictly increasing, then strictly increasing and continuous on .

(14.7) **Definition**: Let , we said that a function is a bounded, if .

(14.8) **Theorem**:Let is a function and let partitions of , put , then

1. .
2. If is an increasing, then .
3. If is a decreasing, then .
4. If is an increasing, then .

**Proof:** (1) .

(14.9)**Theorem**:Let is a monotone function and discontinuous at, then is a countable.

(14.10)**Definition**:Let is a function and partitions of . We say that is a bounded variation, if . If a function is a bounded variation, define partitions of. If a function is a bounded variation, then is a finite, but since partitions of. is continuous on .

(14.11)**Example**:Let a function defined as , then does not bounded variation.

**Solution:** , let partitions of .

Since diverges, then does not bounded variation.

(14.12)**Theorem**:If a function is a continuous and exists and bounded on , then is bounded variation.

(14.13)**Example**:Let a function defined as , then is bounded variation.

**Solution:** since is continuous on , also exists and bounded on .

, also .

(14.14)**Theorem**:If a function is a monotone, then is bounded variation.

(14.15)**Theorem**:If a function is a bounded variation, then is a bounded.

(14.16)**Example**:Let , then a function defined as is bounded, but does not bounded variation.

**Solution:** since , so is a bounded.

If partitions of .

(14.17)**Theorem**:If the functions are bounded variation, , then

1. is bounded variation on and .
2. is bounded variation on and .
3. is bounded variation on and .

(14.18)**Theorem**:Let a function is bounded variation and let . Then is bounded variation on and .

(14.19)**Theorem**:Let a function is bounded variation and let , then is bounded variation on and .

(14.20)**Example**:If a function defined as .

**Solution:**

Let partitions of

By same way calculate

(14.21)**Theorem**:Let a function is bounded variation, define a function as , then are increasing on .

(14.22)**Theorem**:Let a function, then is bounded variation are increasing functions.

(14.23)**Theorem**:Let is bounded variation function on , let , then is continuous at is continuous at .

(14.24)**Theorem**:Let is continuous, then is bounded variation function on are continuous.

(14.25)**Theorem**:Let is bounded function and Lebesgue or Riemann integrable on . Define a function as , then is bounded variation on .

(14.26)**Theorem**:If a function is differentiable and is a bounded and Riemann integrable on , then is bounded variation on and .