Chapter One
Review of Atmospheric Thermodynamics
(Part II)

Lapse Rate of a Saturated Parcel

- If an air parcel is saturated, then as it rises and cools the relative humidity will exceed 100%.
  - Any excess vapor will condense and release latent heat back into the parcel.
  - The lapse rate in saturated ascent will be less than the dry-adiabatic lapse rate.
- The value of the lapse rate in saturated ascent will depend on what assumptions are made as to:
  - whether or not the liquid water remains in the parcel for falls out immediately.
  - whether or not the correction to the specific heat, c_{pv}, for the water vapor is included or ignored.
- The three different lapse rates for saturated ascent are:
  - ***Reversible moist-adiabatic lapse rate***: This assumes that any condensed liquid remains with the air parcel. The specific heats of the dry air, water vapor, and liquid water are all included.
  - ***Pseudoadiabatic lapse rate***: This assumes that any condensed liquid immediately leaves the parcel. The specific heat of water vapor is still explicitly included.
    - ✓ Since the liquid water leaves the parcel, this process is *irreversible*.
  - ***Saturation-adiabatic lapse rate***: This is the same as the pseudoadiabatic lapse rate except is does not include the specific heat of the water vapor is ignored. This process is also irreversible.
- The lapse rate for an ascending saturated air parcel is given by

$$\Gamma_s = \Gamma_d \frac{1 + \dfrac{L_v r_s}{R_d T}}{1 + \dfrac{\varepsilon L_v^2 r_s}{R_d c_p T^2}} \qquad (1.28)$$

where L_v is the latent heat of vaporization, r_s is the saturation mixing ratio, and $\varepsilon = R_d/R_v$.

- At colder temperatures, such as those found in the upper troposphere, the moisture content of the air is very small, and there is little latent heating added to a rising air parcel. Thus, in the upper troposphere the moist adiabatic lapse rate approaches the dry adiabatic lapse rate.

Adiabatic Mixing of Air Parcels

- If two air parcels are adiabatically mixed together, many thermodynamics properties of the mixture are a *mass-weighted mean* of their properties before mixing.

  - A mass-weighted mean of some property s of two air parcels of masses m_1 and m_2 is given by the formula

$$\bar{s}_m = \frac{m_1}{m_1 + m_2} s_1 + \frac{m_2}{m_1 + m_2} s_2 \tag{1.29}$$

- Formula (1.29) applies exactly if s is specific humidity q, and approximately for mixing ratio r and potential temperature θ.

- If the air parcels being mixed are also at the same pressure (isobaric mixing), then temperature and vapor pressure also mix as mass-weighted means, and (1.29) also applies.

- Adiabatic mixing of two initially unsaturated air parcels may actually result in a saturated air parcel

  - This is why we can sometimes 'see our breath' on cold days.

- The concept of mass-weighted mean can be applied to a continuous layer of air as follows:

  - We imagine the layer consisting of a series of N very thin air parcels, each having a horizontal area A and thickness Δz_i.
  - The mass-weighted mean is given by the sum

$$\bar{s}_m = \frac{\sum_i m_i s_i}{\sum_i m_i} \tag{1.30}$$

  - Each parcel has a mass given by $\rho_i A \Delta z_i$, so that (1.30) becomes

$$\bar{s}_m = \frac{\sum_i \rho_i A \Delta z_i s_i}{\sum_i \rho_i A \Delta z_i} = \frac{\sum_i \rho_i \Delta z_i s_i}{\sum_i \rho_i \Delta z_i} \tag{1.31}$$

  - In the limit as the thicknesses of the air parcels go to zero the summation turns into an integral, and the formula for the mass-weighted mean of a layer becomes

$$\bar{s}_m = \frac{\int_{z_1}^{z_2} \rho s\,dz}{\int_{z_1}^{z_2} \rho\,dz} \tag{1.32}$$

- Formula (1.32) applies only to those parameters *s* that do not change as the air parcel moves up or down.

  - Thus, it can be applied to specific humidity, mixing ratio, and potential temperature.
  - Formula (1.32) *cannot* be applied to temperature.

- Formula (1.32) can be written as a derivative with respect to pressure. The hydrostatic equation allows us to write $\rho dz = -g dp$, so that (1.32) becomes

$$\bar{s}_m = \frac{\int_{p_2}^{p_1} g s\,dp}{\int_{p_2}^{p_1} g\,dp} = \frac{1}{\Delta p}\int_{p_2}^{p_1} s\,dp \tag{1.33}$$

where $\Delta p = p_1 - p_2$.

- During adiabatic mixing the specific humidity and potential temperature mix as mass-weighted means, and take on constant values in the mixed layer. The mixing ratio also mixes to a close approximation as a mass-weighted mean from (1.33) these values are

$$\bar{q}_m = \frac{1}{\Delta p}\int_{p_2}^{p_1} q\,dp \tag{1.34}$$

$$\bar{r}_m = \frac{1}{\Delta p}\int_{p_2}^{p_1} r\,dp \tag{1.35}$$

$$\bar{\theta}_m = \frac{1}{\Delta p}\int_{p_2}^{p_1} \theta\,dp \tag{1.36}$$

- On a diagram using a logarithmic pressure axis the averages for (1.34), (1.35), and (1.36) can be approximated graphically, by using the *method of equal areas*.

- The mass-weighted mean mixing ratio, r_m, is the isohume that splits the dewpoint profile into two equal areas, as shown in Figure 1.
- The mass-weighted mean potential temperature, θ_m, is the adiabatic that splits the temperature profile into two equal areas, as shown in Figure 1

- Adiabatic mixing is why surface temperatures on a windy night will be warmer than on a calm night.

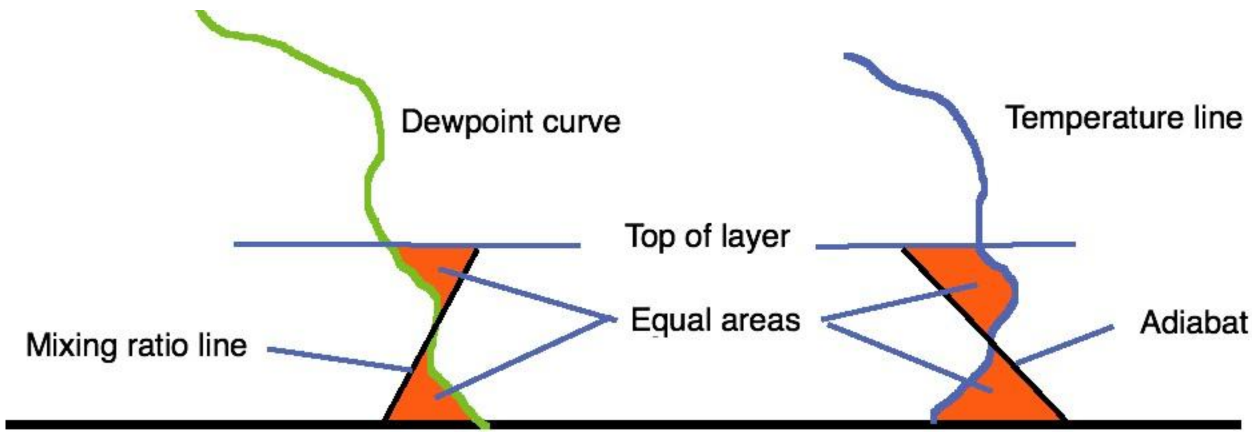

Figure 1: Equal-area method for determining mass-weighted mean of mixing ratio and potential temperature.

Convective Mixing and the Convective Condensation Level

- Another important mixing process for the atmosphere is *convective mixing*, which occurs when a layer of air is heated from below and the upward and downward overturning of the layer mixes the air parcels.
- Convective mixing is *not adiabatic*.
  - In a convectively mixed layer θ still takes on a constant value, but it is not given by (1.36).
  - Instead, θ in a convectively mixed layer will be the value of the adiabatic through the surface temperature.
  - Mixing ratio in a convectively mixed layer will still be given by (1.35).
- A morning sounding usually has a surface inversion

- The *convective temperature*, T_c, is the temperature that the surface must reach in order for an air parcel to ascend dry adiabatically to reach the CCL.

– The convective temperature is found by following a dry adiabatic downward from the CCL to the surface, and reading the temperature at that point.
– Figure 2 illustrates the determination of the convective temperature.

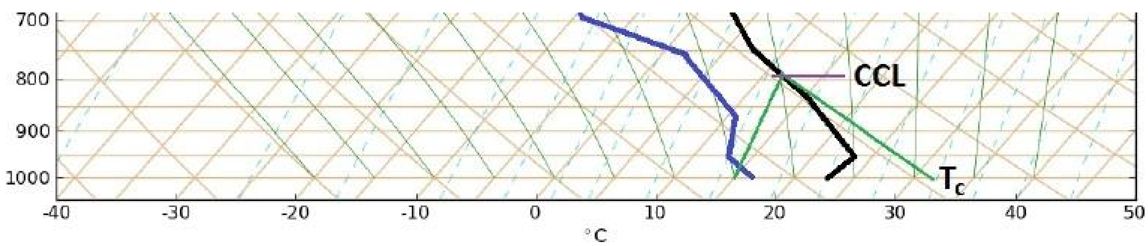

Figure 2: Example Skew-T showing how to find the convective temperature, T_c.