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Lecture (3) for PhD

EM Wave propagation in matter

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<u>Wave propagation in matter</u>

1. **Perfect dielectric** :

for these materials ($\sigma = 0$).wave propagation occurs without attenuation as in free space but with the propagation parameters governed by ϵ and μ instead of ϵ_o and μ_o respectively.

<u>2. Imperfect dielectrics</u>:

a material is classified as an imperfect dielectric for ($\sigma \ll \omega\epsilon$) that is conduction current density is small in magnitude compared to the displacement current .the only significant feature of wave propagation in an imperfect dielectric as compared to the in a perfect dielectric is the attenuation under gone by the wave.

<u>3. Good conductor</u>:

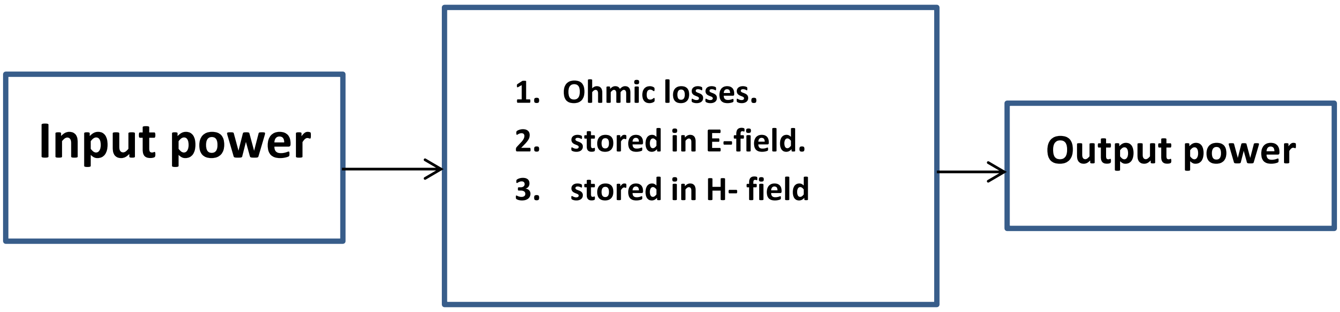
the material is classified as a good conductor for ($\sigma \gg \omega\epsilon$)that is ,conduction current density is a large in magnitude compare to the displacement current density. wave propagation in a good conductor medium is characterized by attenuation and phase constant both equal to

$$\sqrt{\pi f \mu \sigma}$$

Thus for large values of (f) and /or (σ) the fields do not penetrate very deeply in to conductor .this phenomenon is known as the skin effect from considerations of the frequency dependence of the attenuation and wave length for a fixed (σ),we learned that low frequencies are more suitable for communication with under water objects. we also learned that the intrinsic impedance of a good conductor is a very low in magnitude compare to that of a dielectric have the same ϵ and μ.

4 . Perfect conductor :

these are idealization of good conductors in the limit ($\sigma \approx \infty$).for $\sigma=\infty$ the skin depth ,that is the distance in which the fields inside a e^{-1} , is zero and hence there can be no penetration of fields into a perfect conductor.



Some definition

1. **Uniform (or homogeneous) medium**

ϵ is independent of (x).

2. Linear: ϵ is independent of E

3. Non dispersive: ϵ is independent of ω

4. Isotropic: $\epsilon 11=\epsilon 22=\epsilon 33$ i.e $\epsilon ij = 0$ for $i \neq j$

* ***Lossy medium*** $\sigma \neq 0$, $\epsilon = \epsilon_o \epsilon_r$, $\mu = \mu_o \mu_r$

* L***ossless medium*** $\sigma = 0$, $\epsilon=\epsilon_o\epsilon_r$, $\mu=\mu_o \mu_r$

* F***ree space*** $\sigma = 0$, $\epsilon=\epsilon_o$,$\mu=\mu_o$

* ***Good conductor*** $\sigma=\infty$, $\mu=\mu_o$, $\sigma >> \omega\epsilon$,

Electromagnetic wave propagation :

Let consider the Maxwell's eq.s in free space (*ρ=0 & J=0)*

$\nabla . E = 0$

$\nabla . H = 0$

$\nabla \times E$= - $\mu_{\circ}\frac{\partial H}{\partial t}$

$\nabla \times H = \epsilon_0 \frac{\partial E}{\partial t}$

$H = j\underset{\circ}{H} e^{j(\omega t - kz)}$, $E = i\underset{\circ}{E} e^{j(\omega t - kz)}$ $for\ plane\ wave$

We postulate the existence of uniform plane wave in which both fields E &H, lie in the transverse plane , that is the plane whose normal is the direction of propagation .a plane wave has no electric or magnetic field components along its direction of propagation.

1) wave propagation in lossy medium

$\vec{\nabla} . \vec{E} = 0$ $\vec{\nabla} . \vec{H} = 0$

$\vec{\nabla} \times \vec{E}$= -$\mu_{\circ}\frac{\partial \vec{H}}{\partial t}$, $\vec{\nabla} \times \vec{H} = \frac{\epsilon_{\circ}\ \partial \vec{E}}{\partial t}$ Type equation here.

$\vec{H} = \hat{j}\underset{\circ}{H} e^{j(\omega t - kz)}$, $\vec{E} = \hat{\imath}\underset{\circ}{E} e^{j(\omega t - kz)}$

The Maxwell's eq.s in a linear ,isotropic ,homogenous , lossy dielectric medium that is charge free is become:

$\vec{\nabla} . \vec{E} = 0$……………………….(1) $\vec{j} = \sigma \vec{E}$

$\vec{\nabla} . \vec{H} = 0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)$

$\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H} \ldots \ldots \ldots \ldots \ldots \ldots (3)$

$\vec{\nabla} \times \vec{H} = \sigma E + j\omega\epsilon E = (\sigma + j\omega\epsilon)E \ldots \ldots \ldots (4)$

For eq(3) take curl on both sides

$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -jw\mu(\vec{\nabla} \times \vec{H}) = -jw\mu(\sigma + jw\epsilon)\vec{E}$

$\nabla^2\vec{E} - j\omega\mu(\sigma + j\omega\epsilon)\vec{E} = 0$ ……………..(5)

$\nabla^2 E - \gamma^2 E = 0$ ……………….. …(6)

$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ ………… …(7)

γ : *is called the propagtion constant*

Also, similarly can be find:

$\nabla^2\vec{H} - \gamma^2\vec{H} = 0$ ………(8)

These expressions are called vector Helmholtz's eqs.

$(\nabla^2 - \gamma^2)E_{(z)} = 0$, & $(\nabla^2 - \gamma^2)H_{(z)} = 0$

Since is γ a complex quantity we can express as:]

$\gamma = \alpha + j\beta$ −−−−− (9)

$\gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta$ …………… (10)

From eq.(7) & eq.(10) get:

$\alpha^2 - \beta^2 = -\omega\mu\epsilon$

$\beta^2 - \alpha^2 = \omega\mu\epsilon$ ……………… (11)

$2\alpha\beta = \omega\mu\sigma$ ……………………(12)

$\alpha = \omega\sqrt{\frac{\mu\in}{2}\sqrt{1 + (\frac{\sigma}{\omega\in})^2} - 1}$ …(13) , *Attenuation constant* $(\frac{Np}{m})$

$\beta = \omega\sqrt{\frac{\mu\in}{2}\sqrt{1 + (\frac{\sigma}{\omega\in})^2} + 1}$ … … (14), Phase constant (rad/m)

Therefore the simplified solution of EMW:

$E_{X=}\hat{i}E_{\circ}\ e^{-\alpha z} = iEe^{-(\gamma + j\beta)z + j\omega t}$ ……….(15)

$E_{X=}\hat{i}E_{\circ}\ e^{-\alpha z}e^{j(\omega t - \beta z)}$……….(16)

From eq(16)see that the propagated wave decreased or attenuated in amplitude by a factor $e^{-\alpha z}$, and the factor α :is known the attenuation coefficient of the medium .Its measure of spatial rate of wave decay in medium ,measure in Neper per meter. for free space σ =0 and therefore α=0. i.e the wave doesn't attenuated in free space. the quantity β is a measure of phase shift per unit length in radians per meter and called phase constant or wave number

Also: $\hat{H}_{y=}\hat{j}H_\circ\, e^{-\alpha z}e^{j(\omega t-\beta z)}$..........(17)

The complex intrinsic impedance(**η**) of the medium given by:

$$\boldsymbol{\eta} = \frac{\boldsymbol{E_\circ}}{\boldsymbol{H_\circ}} \text{ ....(18)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \dots\dots\dots\dots\dots..(19)$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\left(1+(\frac{\sigma}{\omega\epsilon})^2\right)^{\frac{1}{4}}} \dots\dots\dots\dots\dots\dots.(20)$$

$$tan2\theta_\eta = \frac{\sigma}{\omega\epsilon} \text{ ----------------(21)}$$

$0 \le \theta_\eta \le 45°$ ⟹ $\boldsymbol{\eta}=|\boldsymbol{\eta}|e^{i\theta\eta}$

$$\vec{E}_=\hat{\imath}E_\circ\, e^{-\alpha z}e^{j(\omega t-\beta z)}$$

$$\vec{H} = \hat{j}\frac{E_\circ}{|\eta|}\, e^{-\alpha z}e^{j(\omega t-\beta z-\theta\eta)}$$

It is evident that E and H are out of phase by (θ_η).

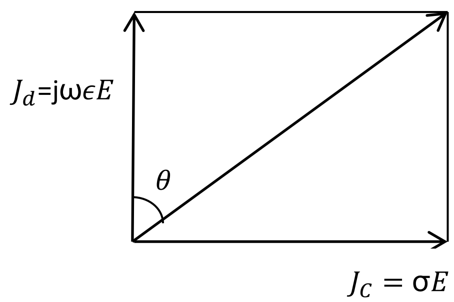
The phase velocity given by

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}+1}}\text{..................}(22)$$

The wave length: $\lambda = \frac{2\pi}{\beta}$

The ratio of conduction current density J_c to that of the displacement current J_d is :

$$\frac{|J_c|}{|J_d|} = \frac{|\sigma E|}{|j\omega\epsilon E|} = \frac{\sigma}{\omega\epsilon} = tan\theta$$



$$tan\theta = \frac{\sigma}{\epsilon\omega} \dots\dots\dots\dots\dots\dots..(23)$$

Loss tangent and θ is loss angle of the medium.

* Lossless or Perfect dielectric if $tan\theta$ is very small i.e(σ<<ωϵ).

* Good conductor if $tan\theta$ is large (σ>>ωϵ).*

In general ,for propagation of EMW, characteristics of any medium doesn't only depend on the parameters (σ, ϵ, & μ)but also on wave frequency(ω).

From definition of intrinsic impedance: $tan\ 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$ and for loss tangent $tan\theta = \frac{\sigma}{\omega\epsilon}$,therefore : $\theta = 2\theta_\eta$ ....(24)

$$\vec{\nabla} \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E}$$

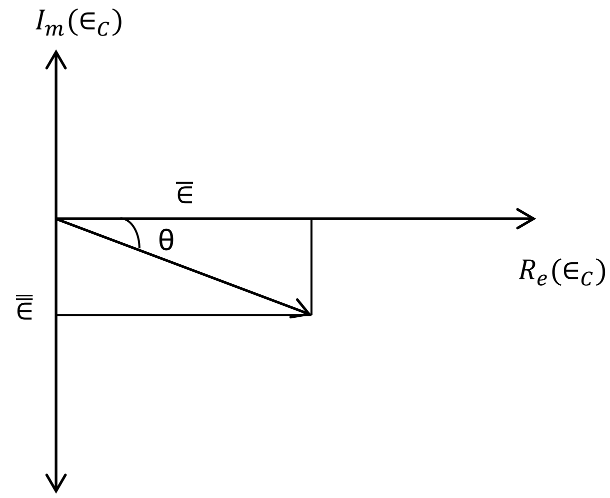
$$\vec{\nabla} \times \vec{H} = j\omega\epsilon\left(1 - \frac{j\sigma}{\omega\epsilon}\right)\vec{E} = j\omega\epsilon_c \vec{E}$$

$$\boxed{\epsilon_c = \epsilon(1 - \frac{j\sigma}{\omega\epsilon})}$$ ....*(25) Complex permittivity of the medium*

$$\epsilon_c \equiv \tilde{\epsilon} = \bar{\epsilon} - j\bar{\bar{\epsilon}}$$

Loss tangent is:

$$\tan\theta = \frac{\bar{\bar{\epsilon}}}{\bar{\epsilon}} = \frac{\sigma}{\epsilon w}$$



Measure of power loss:

$\bar{\epsilon} = \epsilon \quad \equiv \; real\ part$

$\bar{\bar{\epsilon}} = \frac{\sigma}{\omega} \equiv imagenary\ part$

Summary:

$$\vec{\nabla}.\vec{E} = 0\,,\ \vec{\nabla}.\vec{H} = 0,\ \vec{\nabla}\times\vec{H} = \vec{J} + \in\frac{\partial\vec{E}}{\partial t}\ \&\ \vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

Plan wave: $\vec{E} = \hat{\imath}E_\circ\, e^{j(\omega t - kz)}$ & $\vec{H} = \hat{\jmath}H_\circ\, e^{j(\omega t - kz)}$

$$\gamma = \alpha + j\beta$$

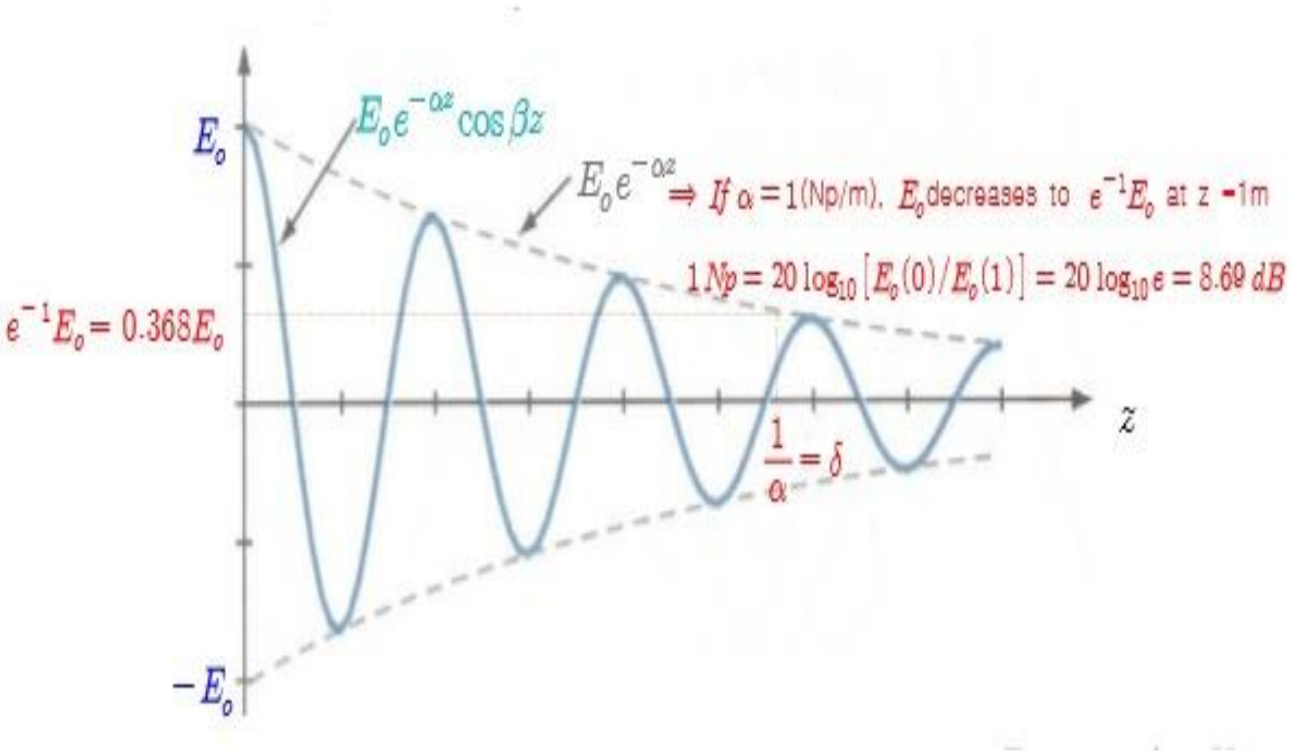
$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad , \quad |\eta| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\left(1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right)^{\frac{1}{4}}} \quad , tan2\theta_\eta = \frac{\sigma}{\omega\in}$$

loss tangent $tan\theta = \frac{\sigma}{\epsilon\omega} = \frac{\bar{\bar{\epsilon}}}{\bar{\epsilon}}$

$$v_p = \frac{\omega}{\beta} \quad ,\lambda = \frac{2\pi}{\beta} \quad ,\epsilon_c = \bar{\epsilon} - j\bar{\bar{\epsilon}} \quad ,\bar{\epsilon} = \epsilon \quad ,\bar{\bar{\epsilon}} = \frac{\sigma}{\omega}$$



$\delta = \frac{1}{\alpha}$, 1Np=20log($\frac{E(0)}{E(1)}$ $= 20\log(e) = 8.69\ dB$

$one\ period: Amplitude\ decrease = e^{-1}E_0 = E(1)$=E1

<u>Lossless dielectric medium</u>

$\sigma = 0$: and $\alpha = \omega\sqrt{\frac{\mu\in}{2}\sqrt{1+(\frac{\sigma}{\omega\in})^2} - 1}$

So $\alpha = 0$

$\beta = \omega\sqrt{\frac{\mu\in}{2}\sqrt{1+(\frac{\sigma}{\omega\in})^2} + 1}$

$\beta = \omega\sqrt{\mu\epsilon}$ ………(1)

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \qquad \dots\dots\dots\dots\dots\dots\dots (2)$$

$$\eta = \sqrt{\frac{\mu_\circ \mu_r}{\epsilon_\circ \epsilon_r}} = \eta_\circ \sqrt{\frac{\mu_r}{\epsilon_r}} \qquad \dots\dots\dots\dots\dots\dots\dots.. (3)$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}}$$

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots. (4)$$

$$v_p = \frac{1}{\sqrt{\mu_\circ \epsilon_\circ \mu_r \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \qquad \dots\dots\dots\dots\dots \quad (5)$$

<u>Free space (vacuum):</u>

$\sigma = 0\,, \quad \in = \epsilon_\circ, \quad \mu = \mu_\circ$

$$\alpha = 0 \qquad \dots\dots\dots\dots\dots\dots\dots\dots. (1)$$

$$\beta = \omega\sqrt{\mu_\circ \in_\circ} \qquad \dots\dots\dots\dots\dots\dots\dots\dots. (2)$$

$$\beta = \frac{\omega}{c}$$

$$v_p = \frac{1}{\sqrt{\mu_\circ \epsilon_\circ}} = c$$

Where $v_p = \frac{\omega}{\beta}$

$$\eta = \sqrt{\frac{\mu_\circ}{\epsilon_\circ}} = \eta_\circ = 120\pi = 377\Omega, \qquad \theta_\eta = 0$$

$$\lambda = \frac{2\pi c}{\omega} = \frac{2\pi}{\beta}$$

<u>Complex wave parameters:</u>

Complex permittivity

$\epsilon_c = \tilde{\epsilon} = \bar{\epsilon} - j\bar{\bar{\epsilon}}$ …………………(1)

$\vec{\nabla} \times \vec{H} = j\omega\tilde{\epsilon}\vec{E} = J + j\omega\epsilon\vec{E}$

$= \sigma\vec{E} + j\omega\epsilon\vec{E}$

$= (\sigma + j\omega\epsilon)\vec{E}$ … … … … … … … … … … . (2)

$= j\omega\left(\frac{\sigma}{j\omega} + \epsilon\right)\vec{E}$ … … … … … … … … … . (3)

$$\therefore\ \tilde{\epsilon} = \epsilon_c = \frac{\sigma}{j\omega} + \epsilon \qquad = \epsilon - j\frac{\sigma}{\omega} \text{ … … … … . (4)}$$

$$\therefore\ \tilde{\epsilon} = \epsilon \quad and \qquad \epsilon = \frac{\sigma}{\omega}$$

Similarly the permeability can also be complex

$\mu_c = \tilde{\mu} = \bar{\mu} - j\bar{\bar{\mu}}$

Complex wave number : $k = k_c = \bar{k} - j\bar{\bar{k}}$

$\vec{E} = E_\circ e^{-jkz} = E_\circ e^{-\bar{k}z} e^{-j\bar{\bar{k}}z}$

Where $e^{-\bar{k}z}$ *is attenation* and $e^{-j\bar{\bar{k}}z}$ *is oscillation*

Complex propagation constant $\gamma = \bar{\gamma} + j\bar{\bar{\gamma}}$

$$\boxed{\vec{E} = E_\circ\, e^{-\gamma z} = E_\circ\, e^{-\bar{\gamma} z}\, e^{-j\bar{\bar{\gamma}} z}}$$ Or $\vec{E} = E_\circ\, e^{-\bar{\alpha} z}\, e^{-j\beta z}$

The physical meaning of the real and imaginary part of the wave vector (k) and propagation constant (γ).

$k = \beta - j\alpha \quad , \quad \gamma = \alpha + j\beta \;\; and \;\; k = -j\gamma$

$\alpha = -Im\left(w\sqrt{\tilde{\mu}\tilde{\epsilon}}\right)$

$\beta = Re\left(w\sqrt{\tilde{\mu}\tilde{\epsilon}}\right)$

Calculating $\alpha \; and \; \beta \;\; from \; \mu, \epsilon, and \;\; \sigma$

$k = \beta - j\gamma$

$k = w\sqrt{\tilde{\mu}\tilde{\epsilon}}$

$\alpha = -Im\left(w\sqrt{\tilde{\mu}\tilde{\epsilon}}\right)$

$\beta = Re\left(w\sqrt{\tilde{\mu}\tilde{\epsilon}}\right)$

$\gamma = \alpha + j\beta$

α =collects all in to a single parameter loss information.

β = collects all phase single parameter loss information.

<u>Absorption coefficients</u> (α_p)

The absorption coefficients α_p describes how power decay as a function of position:

$$\boxed{p_{(z)} = p_\circ\, e^{-\alpha_p z}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1) \quad \text{power decay}$$

Previously defined the attenuation coefficient (α)that described how the field amplitude decays as a function of position :

$$\overrightarrow{E_{(z)}} = E_\circ e^{-\alpha z} e^{-j\beta z} \quad .\ldots\ldots\ldots.(2)$$

Given that P $\propto |E|^2$the attenuation coefficient α and absorption coefficient α_p are related through:

$$p_{(z)} = \frac{|E_{(Z)}|^2}{\eta} = \frac{E_\circ^2}{\eta} e^{-2\alpha z} .\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

From equation (1) and equation (3) get

$$\alpha_p = 2\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Complex impedance:

The wave impedance is in general a complex number.

$$\eta_C = R_0 + j X_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\eta_C = \sqrt{\frac{\tilde{\mu}}{\tilde{\epsilon}}} = \sqrt{\frac{\mu}{\epsilon + \frac{\sigma}{JW}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$|\eta_C| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\theta_\eta = \arg(\eta) = \frac{1}{2}\tan\left(\frac{\sigma}{\omega\epsilon}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

η_C =collected all amplitude and phase in formation between $\vec{E}$ and $\overrightarrow{H}$ into a single parameter. It is an unintuitive mix of the fundamental parameters.

Complex refractive index ($\tilde{n}$) :

Wave number recalls that:

$$k_c = k_\circ n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (1)$$

However, we now known that k_c is a complex number, so refractive index must be as well:

$$\tilde{n} = n_\circ - j\kappa \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$n_\circ = ordinary\ refractive\ index$

$\kappa = extinetion\ coefficient$

We can now relate the real and imaginary parts of refractive index to the real and imaginary parts of (k_c) as well as $(\alpha\ and\ \beta)$:

$$k_c = k_\circ \tilde{n}$$

$$\bar{k} - j\bar{\bar{k}} = k_\circ\,(n_\circ - j\kappa\ ) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3)$$

$$\beta - j\alpha = k_\circ\,(n_\circ - j\kappa\ ) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$n_\circ = \frac{R_l\,(k_c)}{k_\circ}$$

$$n_\circ = \frac{\beta}{k_\circ} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\kappa = \frac{I_m(k_c)}{k_\circ}$$

$$\kappa = \frac{\alpha}{k_\circ} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Electromagnetic Theory Questions and Answers – Lossy and Lossless Dielectrics:

This set of Electromagnetic Theory Multiple Choice Questions & Answers (MCQs) focuses on "Lossy and Lossless Dielectrics".

1. For a dielectric, the condition to be satisfied is?
   a) $\sigma/\omega\epsilon > 1$
   b) $\sigma/\omega\epsilon < 1$
   c) $\sigma = \omega\epsilon$
   d) $\omega\epsilon = 1$

Answer: b

Explanation: In a dielectric, the conductivity will be very less. Thus the loss tangent will be less than unity. This implies $\sigma/\omega\in < 1$ is true.

2. For a perfect dielectric, which parameter will be zero?
   a) Conductivity
   b) Frequency
   c) Permittivity
   d) Permeability

Answer: a

Explanation: The conductivity will be minimum for a dielectric. For a perfect dielectric, the conductivity will be zero.

3. Calculate the phase constant of a wave with frequency 12 rad/s and velocity 3×10^8 m/s(in 10^{-8} order)?

a) 0.5

b) 72

c) 4

d) 36

Answer: c

Explanation: The phase constant is given by $\beta = \omega\sqrt{\mu\epsilon}$ where ω is the frequency in rad/s and $\frac{1}{\sqrt{\mu\epsilon}}$ is the velocity of wave. On substituting $\frac{1}{\sqrt{\mu\epsilon}} = 3\times10^8$ m/s and ω = 12, we get $\beta = 12/(3\times10^8)$ = 4×10^{-8}m/s.

4. For a lossless dielectric, the attenuation will be?

a) 1

b) 0

c) -1

d) Infinity

Answer: b

Explanation: The attenuation is the loss of power of the wave during its propagation. In a lossless dielectric, the loss of power will not occur. Thus the attenuation will be zero.

5. Which of the following is the correct relation between wavelength and the phase constant of a wave?
   a) Phase constant = 2π/wavelength
   b) Phase constant = 2π x wavelength
   c) Phase constant = 1/(2π x wavelength)
   d) Phase constant = wavelength/2π

Answer: a

Explanation: The phase constant is the ratio of 2π to the wavelength λ. Thus $\beta = 2\pi/\lambda$ is the correct relation.

6. Skin depth phenomenon is found in which materials?
   a) Insulators
   b) Dielectrics
   c) Conductors
   d) Semiconductors

Answer: c

Explanation: Skin depth is found in pure conductors. It the property of the conductor to allow a small amount of electromagnetic energy into its skin, but not completely. This is the reason why EM waves cannot travel inside a good conductor.

<u>Electromagnetic Theory Questions and Answers–Loss Tangent</u>

This set of Electromagnetic Theory Multiple Choice Questions & Answers (MCQs) focuses on “Loss Tangent”.

1. The loss tangent refers to the?
   a) Power due to propagation in conductor to that in dielectric.
   b) Power loss
   c) Current loss
   d) Charge loss

Answer: a

Explanation: The loss tangent is the tangent angle formed by the plot of conduction current density vs displacement current density. It is the ratio of Jc by Jd. It represents the loss of power due to propagation in a dielectric, when compared to that in a conductor.

2. Calculate the conduction current density when the resistivity of a material with an electric field of 5 units is 4.5 units?
   a) 22.5
   b) 4.5/5
   c) 5/4.5
   d) 9.5

Answer: c

Explanation: The conduction current density is the product of the conductivity and the electric field. The resistivity is the reciprocal of the conductivity. Thus the required formula is $Jc = \sigma E = E/\rho = 5/4.5$ units.

3. The loss tangent is also referred to as?
   a) Attenuation
   b) Propagation
   c) Dissipation factor
   d) Polarization

Answer: c

Explanation: The loss tangent is the measure of the loss of power due to propagation in a dielectric, when compared to that in a conductor. Hence it is also referred to as dissipation factor.

4. The loss tangent of a wave propagation with an intrinsic angle of 20 degree is?
   a) Tan 20
   b) Tan 40
   c) Tan 60
   d) Tan 80

Answer: b

Explanation: The angle of the loss tangent δ is twice the intrinsic angle θn. Thus $\tan \delta = \tan 2\theta n = \tan 2(20) = \tan 40$.

5. The expression for the loss tangent is given by
   a) $\sigma/(\omega\epsilon)$
   b) $\omega\epsilon/\sigma$
   c) σ/ϵ
   d) ω/ϵ

Answer: a

Explanation: The conduction current density is $Jc = \sigma E$ and the displacement current density is $Jd = j\omega\epsilon E$. Its magnitude will be $\omega\epsilon E$. Thus the loss tangent $\tan \delta = Jc / Jd = \sigma/(\omega\epsilon)$ is the required expression.

6. Find the loss angle in degrees when the loss tangent is 1?
   a) 0
   b) 30
   c) 45
   d) 90

Answer: c

Explanation: The loss tangent is $\tan \delta$, where δ is the loss angle. Given that loss tangent $\tan \theta = 1$. Thus we get $\theta = \tan^{-1}(1) = 45$.

7. The complex permittivity is given by 2-j. Find the loss tangent?
   a) 1/2
   b) -1/2
   c) 2
   d) -2

Answer: a

Explanation: The loss tangent for a given complex permittivity of ϵ = $\bar{\epsilon}$– j$\bar{\bar{\epsilon}}$ is given by tan θ = $\bar{\bar{\epsilon}}$/ $\bar{\epsilon}$. Thus the loss tangent is 1/2.

Example 1: A lossy dielectric has an intrinsic impedance of $200\angle30^{\circ}\Omega$ at the particular frequency. If at that particular frequency a plane wave that propagate in a medium has a magnetic field given by :

$H = 10\, e^{-\alpha x} \cos\left(wt - \frac{x}{2}\right) \hat{j}\, \frac{A}{M}$.   Find E and ?

Solution:

From intrinsic impedance the magnitude of E field

$$\boldsymbol{\eta} = \frac{\boldsymbol{E}_\circ}{\boldsymbol{H}_\circ} = 200\angle30^{\circ} \qquad \boldsymbol{E}_\circ = 2000\angle30^{\circ}$$

It is seen the E field leads H field.

$\theta_\eta = 30^{\circ} = \frac{\pi}{6}$

Hence E =-$2000e^{-\alpha x} \cos\left(wt - \frac{x}{2} + \frac{\pi}{6}\right) \hat{k}$ (V/m)

To find α:

$$\frac{\alpha}{\beta} = \sqrt{\left[\frac{\sqrt{1+(\frac{\sigma}{w\epsilon})^2}-1}{\sqrt{1+(\frac{\sigma}{w\epsilon})^2}+1}\right]}$$

$$\sigma/_{w\epsilon} = tan2\theta_{\eta} = tan60^{\circ} = \sqrt{3}$$

$$\frac{\alpha}{\beta} = \sqrt{\left[\frac{2-1}{2+1}\right]} = \frac{1}{\sqrt{3}}$$

And we know $\beta = \frac{1}{2}$

$$\alpha = \frac{\beta}{\sqrt{3}} = 0.2887 \quad Np/m$$

Hence $E = -2000e^{-0.2887x}\cos\left(wt - \frac{x}{2} + \frac{\pi}{6}\right)\hat{k} \;\; (V/m)$

Example 2: A uniform plane wave propagate in a lossless dielectric in the +z direction. The electric field is given by :

$$\vec{E}_{(z,t)} = 377\cos\left(wt - \left(\frac{4\pi}{3}\right)z + \frac{\pi}{6}\right)\hat{\imath}\ (V/m)$$

The intrinsic impedance of the medium equal 188.5 Ω find:

1) Dielectric constant of the material if $\mu = \mu_{\circ}$.

2) Wave frequency.

3) Magnetic field equation.

Solution:

For lossless dielectric

1) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\sqrt{\mu_\circ}}{\in_r \epsilon_\circ}$

$$\sqrt{\epsilon_r} = \frac{1}{\eta} \sqrt{\frac{\mu_\circ}{\epsilon_\circ}} = 1.9986$$

$\epsilon_r = 4$

2) $\beta = \frac{4\pi}{3} = w\sqrt{\epsilon \mu_\circ}$

$$w = \frac{4\pi}{3\sqrt{\mu_\circ \epsilon}} = 2\pi f = 3.9946 * 10^{16}$$

$f = 99.93 * 10^6 \approx (100\ MH_Z)$

3) $\vec{E}_{(z,t)} = 377 \cos\left(wt - \left(\frac{4\pi}{3}\right) z + \frac{\pi}{6}\right) \hat{\imath} \quad (V/m)$

Magnetic field equation

$$\vec{H}_{(z,t)} = \frac{377}{\eta} \cos\left(wt - \left(\frac{4\pi}{3}\right) z + \frac{\pi}{6}\right) \hat{\jmath} \quad (A/m)$$

References

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