Mustansiriayah University / College of Science

Physics Department

Subject: Advance Optics

Lecture (4) for Ph.D.

EMW Propagation in dielectrics and conductors

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Plane waves in lossy media: ($\sigma \neq 0$)

$\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H}$

$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon}$

$\vec{\nabla}.\vec{B} = 0$

$\vec{\nabla} \times \vec{H} = (\sigma + j\omega\,\epsilon)\vec{E} = j\omega\epsilon_c\vec{E}$

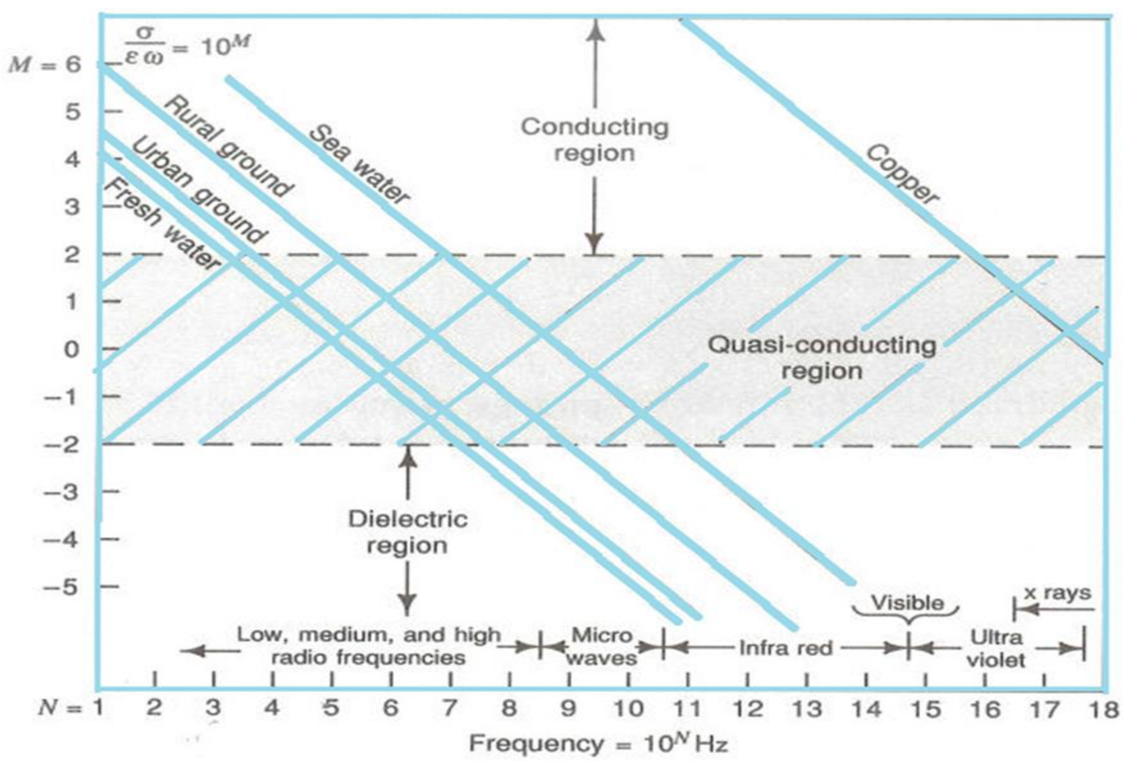
$\epsilon_c = \epsilon - j\frac{\sigma}{\omega} = \epsilon' - j\epsilon''$ ***including damping &ohmic losses***

Loss tangent $\tan\theta = \frac{\sigma}{\epsilon w} = \frac{\epsilon''}{\epsilon'}$

$\boldsymbol{\sigma \gg \epsilon\omega}$ ***good conductor***

$\boldsymbol{\sigma \ll \epsilon\omega}$ ***good insulator***

$\boldsymbol{\sigma = 0}$ ***lossless***



Wave propagation in low-loss dielectrics:

$\tan\theta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\epsilon\omega} \ll 10^{-2}$ dielectric

For a low-loss dielectric (like ordinary imperfect insulators)

$\frac{\sigma}{\epsilon\omega} \ll 1 \quad , \ \epsilon'' \ll \epsilon' \qquad\qquad \varepsilon_c = \epsilon' - j\epsilon'' = \epsilon - j\frac{\sigma}{\omega}$

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon'}\ (1- j\frac{\epsilon''}{\epsilon'})^{1/2}$$

$$\gamma = j\omega\sqrt{\mu\epsilon'}(1- j\frac{\epsilon''}{2\epsilon'} + \frac{1}{8}(\frac{\epsilon''}{\epsilon'})^2)$$

$$\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \qquad\qquad \text{NP/m}$$

$$\beta = \omega\sqrt{(\mu\epsilon'}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right) = \omega\sqrt{(\mu\epsilon}\ (1 + \frac{1}{8}(\frac{\sigma}{\epsilon\omega})^2)$$

$$\eta_c = \sqrt{\frac{\mu}{\epsilon'}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right) = \sqrt{\frac{\mu}{\epsilon}}\left(1 + j\frac{\sigma}{2\epsilon\omega}\right)$$

Phase velocity

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{(\mu\epsilon'}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)}$$

Wave propagation in good conductors:

$\tan\theta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\epsilon\omega} \gg 10^2$ Perfect conductor

Good conductor $\frac{\sigma}{\epsilon\omega} \gg 1$ & $\epsilon'' \gg \epsilon'$

$$\gamma = \alpha + j\beta = \omega\sqrt{\mu\epsilon}\,(1 - j\frac{\sigma}{\epsilon\omega})^{1/2}$$

$$\gamma = \sqrt{j}\sqrt{\omega\mu\sigma} = \frac{1+j}{\sqrt{2}}\sqrt{\omega\mu\sigma}$$

$$\gamma = (1+j)\sqrt{\pi f\mu\sigma}$$

When $\sqrt{j} = (e^{\frac{i\pi}{2}})^{\frac{1}{2}} = e^{\frac{i\pi}{4}} = \cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right) = \frac{1+j}{\sqrt{2}}$

$\alpha = \beta \cong \sqrt{\pi f\mu\sigma}$ ***good conductor***

$$\eta_c = \sqrt{\frac{\mu}{\epsilon'}}\left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{-1/2} = \sqrt{\frac{j\omega\mu}{\sigma}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}}$$

$$\eta_c = (1+j)\frac{\alpha}{\sigma}$$

$$\vec{E} = \hat{i}E_0 e^{-\alpha z}\, e^{-j\beta z}$$

$$\vec{H} = \frac{E}{\eta_c} = \hat{j}\frac{E_0}{\eta_c}e^{-\alpha z}\, e^{-j\beta z} = \hat{j}\frac{E_0}{\sqrt{\frac{j\omega\mu}{\sigma}}}e^{-\alpha z}\, e^{-j\beta z}$$

$\vec{H} = \hat{j}\frac{E_0}{\sqrt{\frac{\omega\mu}{\sigma}}}e^{-\alpha z}\, e^{-j(\beta+\frac{\pi}{4})}$ ***H(z) lags behind E(z) by $\frac{\pi}{4}$***

Phase velocity

$$v_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \ll c$$

$$v_p \downarrow \equiv \sigma \uparrow$$

Example 1: For copper (cu) with $\sigma = 5.8 \times 10^7 S/m$, and phase velocity ($v_p = 720\ m/s$) at f = 3 MHz:μ= μ_0 =4π*10^{-7} H/m.

Wavelength:

$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f} = 2\sqrt{\frac{\pi}{f\mu\sigma}}$=0.24mm

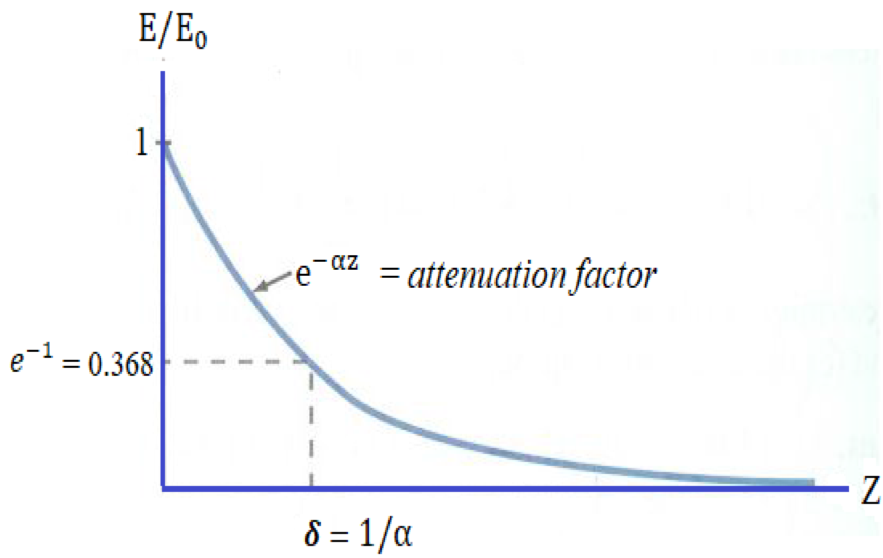
$\lambda \downarrow = \sigma \uparrow$

For $\lambda = 0.24\text{mm} \ll 100\text{m}$ in air at *f* = 3 MHz. Where $\lambda = \frac{c}{f} = 100m$

Skin depth δ = Depth of penetration of a good conductor

= Distance thru which the wave amplitude decrease by (e^{-1})

For $\vec{E} = \hat{\imath}E_0 e^{-\gamma z} = \hat{\imath}E_0 e^{-\alpha z}\ e^{-j\beta z}$



$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu\sigma}}$$

$$\delta = \frac{1}{\beta} = \frac{\lambda}{2\pi}$$

$\delta \downarrow$ as $\sigma \uparrow$ and/or $f \uparrow$

Cu $\delta = 0.038\ \text{mm}$ at $f = 3$ MHz

$\delta = 0.66\ \mu\text{m}$ at $f = 10$ GHz

$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{(\pi*50*4\pi*10^{-7}*5.8*10^{-7})^{\frac{1}{2}}} = ??\,\text{m}$ at $f = 50$ Hz

Example 2: A LP plane wave $\vec{E} = \hat{\imath}E(z,t)$ propagating along +z-direction in seawater($\epsilon_r = 72, \mu_r = 1, \sigma = 4\ S/m$) with $E_0 = \hat{\imath}100\cos(10^7\,\pi t)$ V/m at z=0. Find

a) $\alpha, \beta, \eta_c, v_p, \lambda$ & δ
b) Z_1 when $E_1 = 0.01E_0$
c) E (0.8, t) &H (0.8, t)

Solutions:

$\omega = 10^7\pi$

$f = \frac{\omega}{2\pi} = 5 \times 10^6 Hz$

$tan\theta = \frac{\sigma}{\epsilon\omega} = 200 \gg 1$ ***good conductor***

a) $\alpha = \beta = \sqrt{\pi f \mu\sigma} = 8.89\ rad/m$

$$\eta_c = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = \pi e^{\frac{j\pi}{4}}$$

$$v_p = \frac{\omega}{\beta} = 3.53 \times 10^6\ m/s$$

$$\lambda = \frac{2\pi}{\beta} = 0.707\ m$$

$$\delta = \frac{1}{\alpha} = 0.112\ m$$

b) $E_1 = 0.01E_0$

$E_1 = E_0 e^{-\alpha z_1}$

$0.01E_0 = E_0 e^{-\alpha z_1}$

$-\alpha z_1 = \ln(0.01)$

$$z_1 = \frac{-\ln(0.01)}{\alpha} = 0.518\ m$$

c) $E(z) = \hat{\imath}100e^{-\alpha z}\, e^{-j\beta z}$ in the phasor domain

$$E(z,t) = R_e\left(E(z)e^{j\omega t}\right) = \hat{\imath}100e^{-\alpha z}\cos(\omega t - \beta z)$$

$$\therefore E(0.8,t) = \hat{\imath}\ 0.082\cos(10^7\ \pi t - 7.11)$$

$$H(z,t) = R_e\left(\hat{\jmath}\frac{E(z)}{\eta_c}e^{j\omega t}\right) = R_e(\hat{\jmath}\frac{100e^{-\alpha z}\,e^{-j\beta z}}{\pi e^{\frac{j\pi}{4}}}e^{j\omega t})$$

$$H(z,t) = R_e(\hat{\jmath}\frac{100}{\pi}e^{-\alpha z}e^{j(\omega t - \beta z - \frac{\pi}{4})})$$

$$H(z,t) = \hat{\jmath}(\frac{100}{\pi}e^{-\alpha z}\cos(\ \omega t - \beta z - \frac{\pi}{4})$$

$$H(0.8,t) = \hat{\jmath}(\frac{100}{\pi}e^{-0.8\alpha}\cos(\ \omega t - 0.8\beta - \frac{\pi}{4}) \quad \textit{ignoring } \omega t \textit{ term}$$

$$H(0.8,t) = \hat{\jmath}\ 0.026\cos(10^7\ \pi t - 7.89)$$

$$H(0.8,t) = \hat{\jmath}\ 0.026\,cos(10^7\ \pi t - 2\pi - 1.61)$$

$$H(0.8,t) = \hat{\jmath}\ 0.026\,cos(10^7\ \pi t - 1.61) \qquad A/m$$

Note: $\boldsymbol{H = \frac{E}{\eta_c}}, \qquad \boldsymbol{\eta_c = \frac{E}{H}}$

EMW- Propagation states:

❖ ***Free space*** $\sigma = 0,\ \epsilon = \epsilon_0,\ \mu = \mu_0$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{(1+(\frac{\sigma}{\omega\epsilon})^2} - 1\right)}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\epsilon_0\mu_0} = \frac{\omega}{c}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\epsilon_0\mu_0}} = c$$

E & H in phase & amplitude does not decay

❖ ***Lossless dielectric medium*** $\sigma = 0,\ \epsilon\ \&\ \mu$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\epsilon\mu} = \frac{\omega}{v}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}$$

$$v = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

E & H in phase & amplitude still does not decay

❖ *Lossy media: ($\sigma \neq 0$)*

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}(\sqrt{(1+(\frac{\sigma}{\omega\epsilon})^2} - 1)}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}(\sqrt{(1+(\frac{\sigma}{\omega\epsilon})^2} - 1)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\frac{\mu\epsilon}{2}(\sqrt{(1+(\frac{\sigma}{\omega\epsilon})^2} - 1)}}$$

$$v = \sqrt{\frac{2}{\mu\epsilon(\sqrt{(1+(\frac{\sigma}{\omega\epsilon})^2} - 1)}}$$

E & H out of phase & amplitude decay

❖ *Good conductor* $\sigma \gg \omega\epsilon$, *Skin depth* ($\delta = \frac{1}{\alpha}$)

Strong attenuation $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$|\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$$

$\arg(\eta) = \frac{\pi}{4}$

E & H are out of phase

Very strong attenuation, wave tend to reflect from good conductor so often do not experience the loss & E leads H by $\frac{\pi}{4}$.

Refractive index (dielectric):

The refractive index of dielectric medium given by:

n = $\sqrt{\mu_r \epsilon_r}$ … … … … . (1) $n = \frac{C}{V}$

n > 1 & v < c

For non-magnetic material (μ_r = 1)

η = $\sqrt{\epsilon_r}$ … … … … … … . (2)

Power flow & energy density:

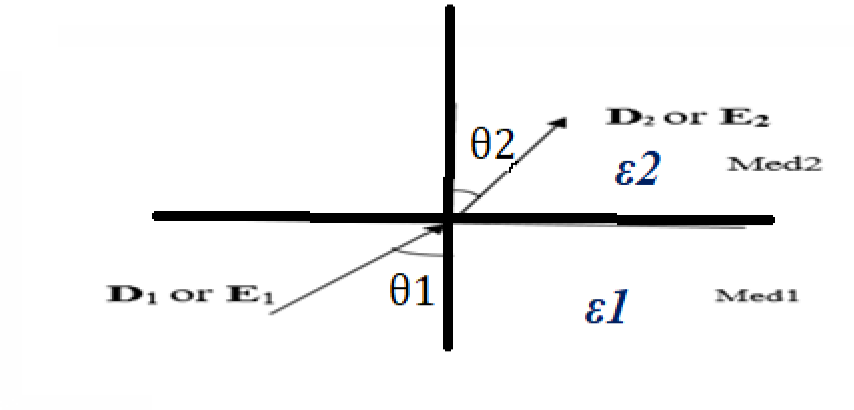
$$\frac{U_e}{U_m} = \frac{\frac{1}{2}\in E^2}{\frac{1}{2}\mu H^2} = \frac{\in E^2}{\mu H^2} = \frac{\in}{\mu}\eta^2 \qquad \text{where } \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\frac{U_e}{U_m} = \frac{\in}{\mu} \cdot \frac{\mu}{\in} = 1 \quad \Longrightarrow \quad U_e = U_m$$

$$< s > \; = \frac{E_{rms}^2}{\eta} \qquad \text{Energy flow}$$

Refraction of E & μ fields:

To determine the refraction of E-field across the interface between two mediums, used boundary conditions (B. C. S)



B.C.S.

$E_{t1} = E_{t2}$ ………. (1)

$D_{n1} = D_{n2}$ …….. (2)

$E_1 \sin\theta_1 = E_2 \sin\theta_2$ …….. (3)

$D_1 \operatorname{Cos}\theta_1 = D_2 \operatorname{Cos}\theta_2$ …….. (4)

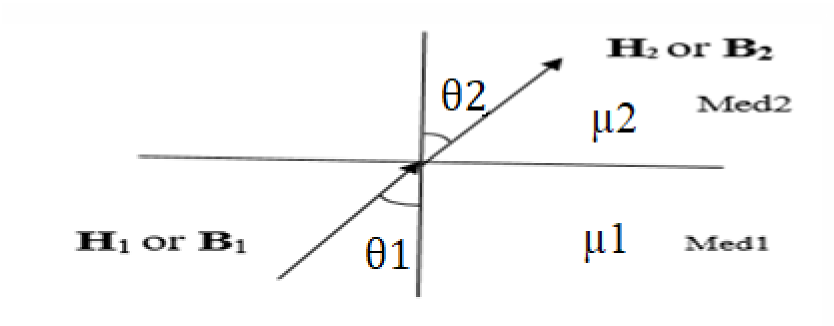
$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$ …... (5)

From eq (5) & eq (3) get

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

Law of refraction of the E-field at the boundary free of charge (ρ = 0)

Now used B.C.S to determine the refraction of H- field a cross the interface between two mediums.



B.C.S

$H_{t1} = H_{t2}$ ………. (1)

$B_{U1} = B_{U2}$ ……… (2)

$B_1 \cos\theta_1 = B_2 \cos\theta_2$ …….. (3)

$H_1 \sin\theta_1 = H_2 \sin\theta_2$ ……… (4)

$$\frac{B_1}{\mu_1}\sin\theta_1 = \frac{B_2}{\mu_2}\sin\theta_2$$

From eq. (5) & eq. (3) get

$$\boldsymbol{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}}}$$

Law of refraction of the magnetic flux lines of the boundary with no current density flow (J = 0)

The relation between $\vec{E}$ & $\vec{H}$:

$\widehat{u_E}$: unit vector along $\vec{E}$

$\widehat{u_H}$: unit vector along $\vec{H}$

$\overrightarrow{\vec{E} \times H} = \vec{S}$ Pointing vector

Pointing vector is the same direction of propagation($\widehat{u_S}$) or ($\widehat{u_K}$):

$\widehat{u_E} \times \widehat{u_H} = \widehat{u_S} = \widehat{u_K}$ …. …. …. (1)

$\widehat{u_K} \times \widehat{u_H} = -\widehat{u_E}$ …. …. …. (2)

$\widehat{u_K} \times \widehat{u_E} = \widehat{u_H}$ …. …. …. (3)

$\vec{H} = \frac{1}{\eta}\, \widehat{u_K} \times \vec{E}$ …. …. …. (4)

$\vec{E} = -\eta\, \widehat{u_K} \times \vec{H}$ …. …. …. (5)

If a uniform plan wave travel in the +z-direction may have x- & y-compounds:

$\vec{E}_{(z)} = \hat{\imath}\vec{E}_x^+(z) + \hat{\jmath}\vec{E}_y^+(z)$ …. …. (6)

The associated H- field is

$\vec{H}_{(z)} = \hat{\imath}\vec{H}_x^+(z) + \hat{\jmath}\vec{H}_y^+(z)$ …. …. (7)

The exact expression of magnetic field in terms of electrical field will be:

$$\vec{H}_{(z)} = \frac{1}{\eta}\hat{k} \times \vec{E}_{(z)} = -\hat{\imath}\,\frac{\vec{E}_y^+(z)}{\eta} + \hat{\jmath}\,\frac{\vec{E}_x^+(z)}{\eta}$$

$$\vec{H}_x^+(z) = -\frac{\vec{E}_y^+(z)}{\eta}$$

$$\vec{H}_x^+(z) = \frac{\vec{E}_x^+(z)}{\eta}$$

Where $\widehat{u_S} = \widehat{u_K} = \hat{k}$

Summary:

	Any medium	*Lossless medium* $\sigma = 0$	*Low-loss medium* $\frac{\epsilon''}{\epsilon'} \ll 1$	*Good conductor* $\frac{\epsilon''}{\epsilon'} \gg 1$	*Unit*
$\alpha =$	$\omega\sqrt{\frac{\mu\epsilon'}{2}\left(\sqrt{\left(1+\left(\frac{\epsilon''}{\epsilon'}\right)^2\right.} - 1\right)}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sqrt{\pi f \mu \sigma}$	$\frac{Np}{m}$
$\beta =$	$\omega\sqrt{\frac{\mu\epsilon'}{2}\left(\sqrt{\left(1+\left(\frac{\epsilon''}{\epsilon'}\right)^2\right.} - 1\right)}$	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$	$\sqrt{\pi f \mu \sigma}$	$\frac{rad}{m}$
$\eta_c =$	$\sqrt{\frac{\mu}{\epsilon'}}\left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\epsilon}}$	$\sqrt{\frac{\mu}{\epsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	Ω
$v_p =$	$\frac{\omega}{\beta}$	$\frac{1}{\sqrt{\mu\epsilon}}$	$\frac{1}{\sqrt{\mu\epsilon}}$	$\sqrt{\frac{2\pi f}{\mu\sigma}}$	$\frac{m}{s}$
$\lambda =$	$\frac{2\pi}{\beta} = \frac{v_p}{f}$	$\frac{v_p}{f}$	$\frac{v_p}{f}$	$\frac{v_p}{f}$	m

Notes:

$\epsilon' = \epsilon \quad \& \quad \epsilon'' = \frac{\sigma}{\omega}$

$$\epsilon' = \frac{\beta^2 - \alpha^2}{\omega^2 \mu}$$

$$\epsilon'' = \frac{2\alpha\beta}{\omega^2 \mu}$$

In free space $\epsilon = \epsilon_0, \ \mu = \mu_0$

Low-loss medium $\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} < 0.01$

Good conductor $\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} > 100$

Loss tangent $\tan\theta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\epsilon\omega}$

In general $\eta_c = \sqrt{\frac{\mu}{\epsilon'}}(1 - j\tan\theta)^{-1/2}$

dB scale

Power intensity ratio in log scales not a unit:

dB = $10 \log \left(\frac{I}{I_0}\right) = 10 \log\left(\frac{P}{P_0}\right) = 20 \ \log\left(\frac{V}{V_0}\right)$ > 0 gain

↓ intensity ↓ power ↓ voltage < 0 loss

Attenuation:

$E_{(Z)} = E_0 \, e^{-\gamma z} = E_0 \, e^{-\alpha z} e^{-j\beta z}$

$A_{(Z)} = 20 \log \left(\frac{E_{(z)}}{E_{(0)}}\right) = 20 \log|e^{-\alpha z}|$

$A(z) = \frac{-20 \, \alpha \, z}{\ln(10)} \, dB$

Example3: If E-field intensity going through a medium attenuates at a rate of (0.4 dB/ m) what is α.

Solution:

$-0.4 = -8.686\ \alpha + \text{lm}$

$$\alpha = \frac{0.4}{8.686} = 0.046\frac{nepers}{m}$$

Note: α positive number for attention.

Example4: The sinusoidal electric field with E_0 = 250 v/ m and frequency f= 1 G Hz exists in a lossy dielectric medium with E_r = 2.5 and loss tangent of 0.001. Find the average power dissipated in the medium per cubic meter.

Solution:

$$\tan\theta = 0.001 = \frac{\sigma}{\omega\epsilon} = \frac{\sigma}{\omega \in_0 \in_r} = \frac{\sigma}{2\,\pi\ x\ 2\ x\ 10^9 \frac{10^{-9}}{36\pi}\ x\ 2.5}$$

$\therefore \sigma = 1.39 \text{ x } 10^{-4}\ {}^{S}/_{m}$

The average power P_{ave} dissipated per unit volume V:

$$\frac{P_{ave}}{V} = \frac{1}{2}\vec{J}.\vec{E} = \frac{1}{2}\,\sigma\,E^2 = \frac{1}{2}\,(1.39\ x\ 10^{-4})\ x\ (250)^2$$

$= 4.34\ \text{w/m}^2$

Note:

$$P_{ave} = \frac{1}{2}\frac{v^2}{R} = \frac{1}{2}\frac{(El)^2}{2\rho l/A} = \frac{1}{2}\sigma\,E^2(lA)$$

$$v: volt \quad , R: electrical\ resistance\ of\ the\ uniform\ specimen$$

$$l : length\ of\ the\ specimen$$

$$A : Area\ cros - section\ of\ the\ specimen$$

$$\rho: electrical\ resistivity$$

Skin depth:

$\delta = \frac{1}{\propto}\sqrt{\frac{2}{\mu\sigma\omega}}$ ……….. (1

E (z) = $E_0\, e^{-\propto z}\, e^{-j\beta z}$ ……… (2

At z= $\delta \Longrightarrow$ |E| decreases to ($\frac{1}{e}$) or (0.63 drop)

A (z) = 20 log $|\frac{E(z)}{E(0)}|$ = 20 log $|e^{-\propto z}|$

A (z) = - 8.686 ∝z dB

at z= δ , |E| decrease by (-8.7) dB

at z = 2 δ , |E| decrease by (- 17.3) dB .

Example5: The skin depth or non-magnetic conducting medium is (2µ m) at f= 5 GHz, Find phase velocity v_p in this medium, then find the attenuation in dB , when the wave penetrates (10 µm) into the material?

Solution:

$v_p = \frac{\omega}{\beta}$

For conductor $\propto = \beta = \frac{1}{\delta}$

$v_p = \omega\delta = 2\pi \text{ x } 5 \text{ x } 10^9 \text{ x } 2 \text{ x } 10^{-6} = 6.28 \text{ x } 10^4$ m/s

A (z) = 20 log $|\frac{E(z)}{E(0)}|$ = 20 log $|e^{-\propto z}|$ = -8.686 ∝z

A (z) = -8.686 z/ δ = -8.686 . $\frac{10}{2}$ = - 43.4 dB high loss

There are only surface current on conductors.

Example6:

a) ***Calculate the diclectric loss (in dB) of an_EM ware propagating through (100 m) of teflon at f=(MHz)***
b) ***at f= 10 GHz . ϵ_rr = 2.08 , tan θ = 0.0004 at 25 c^0 assuming frequency independeue***

Solution:

a) $\tan\theta = \frac{\sigma}{\omega\epsilon}$

$\sigma = \omega\epsilon_0 \boldsymbol{\epsilon_r} \tan\theta = 2\pi \times 10^6 \times \frac{10^{-9}}{36\pi} 2.08 \times 0.0004$

$\sigma = 4.6 \times 10^{-8}$ s/m

$\propto = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{\sigma\,\eta_\circ}{2\sqrt{\epsilon_r}} = \frac{4.6 \times 10^{-8} \times 377}{2 \times \sqrt{2.08}}$

$\propto = 6.04 \times 10^{-6}$ Np/m

A (z) = -8.686 $\propto$ z

= -8.686 x 6.04 x 6.04 x 10^{-6} x 100 = - 0.005 dB

b) $\sigma = \omega\epsilon_0\, \epsilon\text{r} \tan\theta = 4.6 \times 10^{-4}$ s/m

$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{\sigma\eta_\circ}{2\sqrt{\epsilon_r}} = 6.04 \times 10^{-2}$ Np /m

A (z) = - 8.68 $\propto$z = -8.68 x 6.04 x 10^{-2} x 100

= - 50 dB

Coaxial cable works well at low frequency (Tv to antenna) but not so well at high frequency.

Example7: In a non – magnetic , lossy , dielectric medium a plane was of frequency f=30 MHZ characterized by the magnetic field phasor :

$H=(\hat{\imath} - j4\hat{k})\, e^{-2y}\, e^{-j9y}$ A/m . Determine the time domain expressions for the electric and magntic field vectors .

Solution:

$H(r,t) = R_e\ (\hat{\imath} - j4\hat{k})\ e^{-2y} \cdot e^{j(wt-9y)}$.

$H(r,t) = \hat{\imath} e^{-2y} \cos(\omega t - 9y) + \hat{k} 4 e^{-2y} \sin(wt - 9y)$

$\propto = 2$, & $\beta = 9$

$-\omega^2 \mu \bar{\epsilon} = \propto^2 - \beta^2$

$\omega^2 \mu \bar{\bar{\epsilon}} = 2 \propto \beta$

$\frac{\bar{\bar{\epsilon}}}{\bar{\epsilon}} = \frac{2\propto\beta}{\beta 2 - \propto^2} = \frac{2X\ 2\ X9}{9^2 - 2^2} = 0.468 = \tan\theta$

$\epsilon_r = \frac{\bar{\epsilon}}{\epsilon_0} = \frac{\beta^2 - \alpha^2}{\omega^2 \mu_0 \epsilon_0} = \frac{77C^2}{\omega^2} = \frac{77\ X\ (3\ X\ 10^8))^2}{(2\ X\ 300\ X\ 10^6)^2)}$

$\epsilon_r = 1.95$

$\eta_c = \sqrt{\frac{\mu}{\bar{\epsilon}}}\ (1 - j\tan\theta)^{-1/2} = \frac{\eta_\circ}{\sqrt{\epsilon_r}}\ (1 - j\tan\theta)^{-1/2}$

$\eta_c = \frac{377}{\sqrt{1.95}}\ (1 - j\,0.468)^{-1/2} = 257\ e^{j\,0.22}$

$\mathbf{E} = \hat{k}\, \eta_c \cos(\omega t - 9y) - \hat{\imath}\, 4\, \eta_c\, e^{-2y} \sin(\omega t - 9y)$

$= \hat{k} 257\ e^{-2y} \cos(\omega t - 9y + 0.22) - \hat{\imath}\ 1028\ e^{-2y} \sin(\omega t - 9y + 0.22)$.

Example8:

A uniform plane wave propagate in a lossless dielectric in the ^+z direction. The electric field is given by: $\overrightarrow{E(z,t)} = 377 Cos(\omega t - \frac{4\pi}{3} z + \frac{\pi}{6}) \hat{\imath}$ V/m

The average power density measured was 377 W/m^2, Find:

(i) Dielectric constant of the material if $\mu = \mu_0$

(ii) Wave frequency

(iii) Magnetic field equation

Solution:

(i) Average power:

$$P_{ave} = \frac{1}{2}\frac{E^2}{\eta} = 377$$

$$\frac{1}{2}\frac{(377)^2}{\eta} = 377$$

$$\rightarrow \eta = 377/2 = 188.5\Omega$$

For lossless dielectric:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}}$$

$$\sqrt{\varepsilon_r} = \frac{1}{\eta}\sqrt{\frac{\mu_0}{\varepsilon_0}} = 1.9986$$

$$\rightarrow \varepsilon_r = 4.0$$

(ii) Wave frequency:

$$\beta = 4\pi/3 = \omega\sqrt{\mu_0 \varepsilon}$$

$$\omega = \frac{4\pi}{3\sqrt{\mu_0 \varepsilon}}$$

$$2\pi f = 3.9946 \times 10^{16}$$

$$\rightarrow f = 99.93 \times 10^6 \approx (100 MHz)$$

iii) Magnetic field equation:

$$\vec{H}(z,t) = \frac{377}{\eta} Cos(\omega t - \frac{4\pi}{3}z + \frac{\pi}{6})\hat{j} = 2Cos(\omega t - \frac{4\pi}{3}z + \frac{\pi}{6})\hat{j}$$

Example9:

In a lossless medium for which $\eta = 60\pi$, $\mu_r = 1$ and $\mathbf{H} = -0.1\cos(\omega t - z)\,\mathbf{a}_x + 0.5\sin(\omega t - z)\mathbf{a}_y$ A/m, calculate ε_r, ω , and $\mathbf{E}$.

Solution:

In this case , $\sigma=0$, $\alpha=0$, and $\beta=1$, so

$$\eta=\sqrt{\mu/\varepsilon}=\sqrt{\frac{\mu_o}{\varepsilon_o}}\sqrt{\frac{\mu_r}{\varepsilon_r}}=\frac{120\pi}{\sqrt{\varepsilon_r}} \quad or \quad \sqrt{\varepsilon_r}=\frac{120\pi}{\eta}=\frac{120\pi}{60\pi}=2 \ \rightarrow \ \varepsilon_r=4$$

$$\beta=\omega\sqrt{\mu\varepsilon}=\omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r}=\frac{\omega}{c}\sqrt{4}=\frac{2\omega}{c}$$

$$or \qquad \omega=\frac{\beta c}{2}=\frac{1\left(3\times10^8\right)}{2}=1.5\times10^8 \text{ rad/s}$$

$$\nabla\times\mathbf{H}=\sigma\mathbf{E}+\varepsilon\frac{\partial\mathbf{E}}{\partial t}\rightarrow\mathbf{E}=\frac{1}{\varepsilon}\int\nabla\times\mathbf{H}\ dt$$

where $\sigma=0.$

$$\text{But} \quad \nabla\times\mathbf{H}=\begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial z} \\ \mathbf{H}_x(z) & \mathbf{H}_x(z) & 0 \end{vmatrix}=-\frac{\partial\mathbf{H}_y}{\partial z}\mathbf{a}_x+\frac{\partial\mathbf{H}_x}{\partial z}\mathbf{a}_y$$

$$=0.5\cos(\omega t-z)\,\mathbf{a}_x-0.1\sin(\omega t-z)\,\mathbf{a}_y$$

Hence

$$\mathbf{E}=\frac{1}{\varepsilon}\int\nabla\times\mathbf{H}dt=\frac{0.5}{\varepsilon\omega}\sin(\omega t-z)\,\mathbf{a}_x+\frac{0.1}{\varepsilon\omega}\sin(\omega t-z)\,\mathbf{a}_y$$

$$=94.25\sin(\omega t-z)\,\mathbf{a}_x+18.85\cos(\omega t-z)\mathbf{a}_y \quad \text{V/m}$$

Example10:

A uniform plane wave propagating in a medium has
$E=2e^{-\alpha z}\sin(10^8 t-\beta z)\mathbf{a}_y$ V/m. If the medium is characterized by
$\varepsilon_r=1, \mu_r=20$, and $\sigma=3$ mhos/m, find α, β, and $\mathbf{H}$.

Solution:

We need to determine the loss tangent to be able to tell whether the medium is a lossy dielectric or a good conductor.

$$\frac{\sigma}{\omega\varepsilon} = \frac{3}{10^8 \times 1 \times \frac{10^{-9}}{36\pi}} = 3393 >> 1$$

showing that the medium may be regarded as a good conductor at the frequency of operation. Hence, $\alpha = \beta = \sqrt{\frac{\mu\omega\sigma}{2}} = \left[\frac{4\pi\times10^{-7}\times20(10^8)(3)}{2}\right]^{1/2} = 61.4$

$$\alpha = 61.4 \text{ Np/m} \quad , \quad \beta = 61.4 \text{ rad/m}$$

Also, $|\eta| = \sqrt{\frac{\mu\omega}{\sigma}} = \left[\frac{4\pi\times10^{-7}\times20(10^8)}{3}\right]^{1/2} = \sqrt{\frac{800\pi}{2}}$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 3393 \rightarrow \theta_\eta = 45^o = \frac{\pi}{4}$$

Hence

$$\mathbf{H} = H_o e^{-\alpha z} \sin\left(\omega t - \beta z - \frac{\pi}{4}\right)\mathbf{a}_H$$

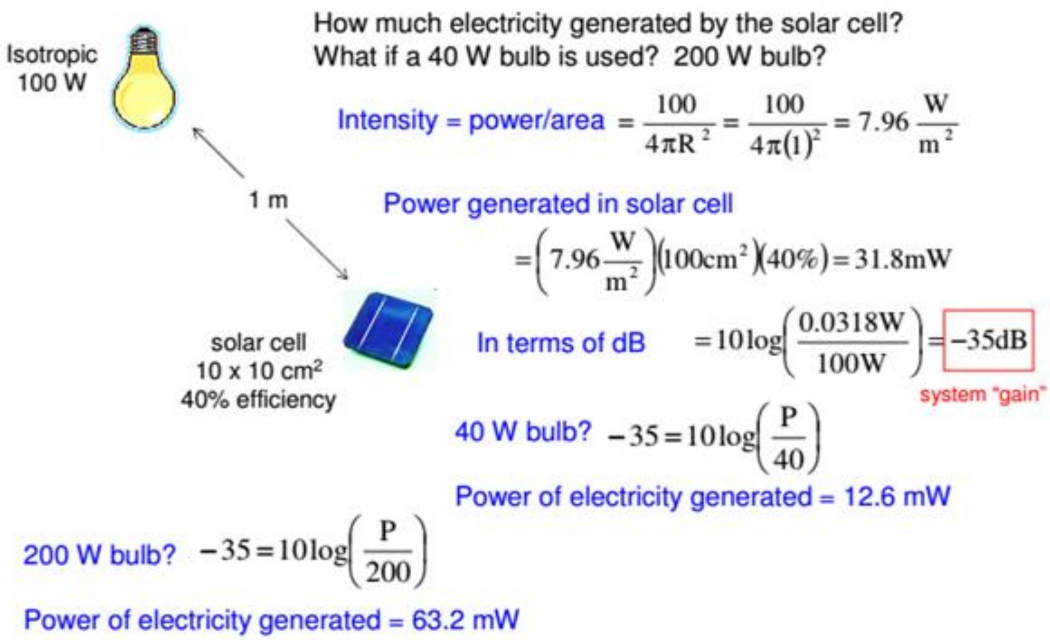
$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_y = -\mathbf{a}_x$$

and

$$H_o = \frac{E_o}{|\eta|} = 2\sqrt{\frac{3}{800\pi}} = 69.1\times10^{-3}$$

Thus $\mathbf{H} = -69.1\times10^{-3} e^{-61.4z} \sin\left(10^8 t - 61.42z - \frac{\pi}{4}\right)\mathbf{a}_x$ mA/m

Example11:



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