



**DEPARTMENT OF
ATMOSPHERIC SCIENCES**
قسم علوم الجو

**THE KINEMATIC AND DYNAMICAL ASPECTS IN
WEATHER FORECASTING**

الجوانب الحركية والديناميكية في التنبؤ بالطقس

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التنبؤ بالطقس يتطلب عملية دراسة وفهم للمنظومة الجوية وتتبع مراحل تطورها من خلال:

1- دراسة التغير الحاصل في مسار المنظومة (حقل الرياح، حقل الضغط) دون البحث في اسباب ذلك وهو ما يعرف بالكايناماتك **KINEMATICS** وهي كلمة اغريقية تعني الحركة.

2- دراسة القوى التي تسبب التغير في مسار المنظومة الجوية وحركتها وهذا ما يعرف بالداينمك **DYNAMIC**، والتي تتضمن نوعين من القوى:

- قوى اساسية والتي تؤثر على سرعة واتجاه المنظومة

قوة نحدار الضغط $\mathbf{p} = -\frac{1}{\rho} \nabla p$ ، قوة الجذب $\left| \frac{F_G}{m} \right| = |g| = 9.8 \text{ m}\cdot\text{s}^{-2}$ ، قوة الاحتكاك $\mathbf{F} = -\frac{1}{\rho} \frac{\partial \tau}{\partial z}$

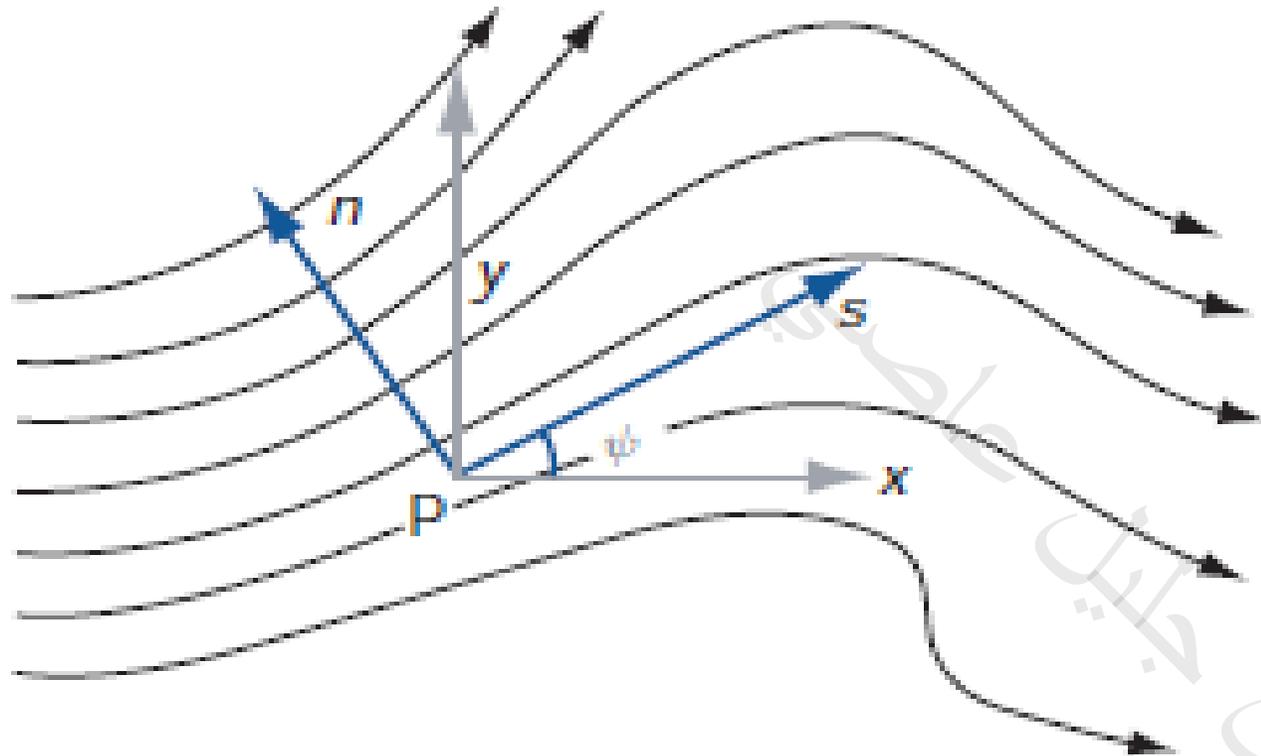
- قوى ظاهرية والتي تؤثر على اتجاه المنظومة فقط

قوة كوريولس $2\Omega \sin \phi$ ، قوة الطرد المركزي $\Omega^2 \vec{R}$

والتي تجتمع في معادلة الزخم التي تمثل قانون نيوتن الثاني في الحركة:

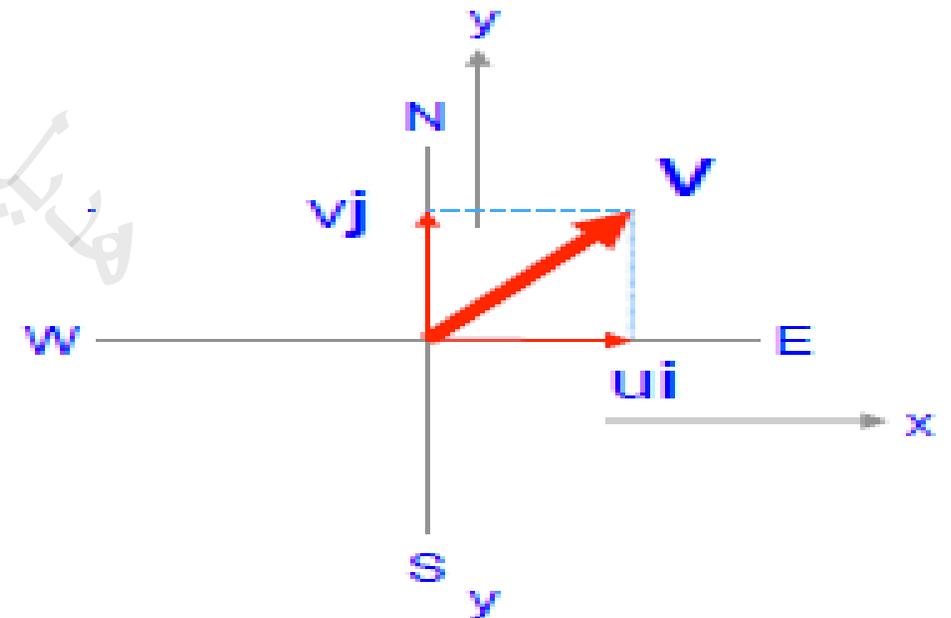
$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho} \nabla p - f \mathbf{k} \times \mathbf{V} + \mathbf{F}$$





ادناه تظهر المحاور الكارتيزية ومتجهاتها و التي سيتم من خلالها التمثيل الرياضي لتحديد مسار الحركة في مجال حقل الرياح وكما تظهر مركبات السرعة بالنسبة لهذه المحاور

اعلاه تظهر المحاور الطبيعية والمتمثلة بـ s وهو المحور الموازي لحركة الرياح والمحور n وهو المحور العمودي على اتجاه الرياح والتي سيتم خلاله تمثيل التغير الحاصل في حركة المنظومة نسبة للمحاور الكارتيزية



Gradient, Divergence and Curl Del Operator

Del Operator: is a vector differential operator denoted by the symbol $\vec{\nabla}$:

$$\vec{\nabla} = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \quad (1.11)$$

This operator can be used in three differential ways:

a. Gradient of Scalar (pressure)

$$\vec{\nabla} p = i \frac{\partial p}{\partial x} + j \frac{\partial p}{\partial y} + k \frac{\partial p}{\partial z} \quad (\text{vector}) \quad (1.12)$$

b. Divergence of a Vector (velocity)

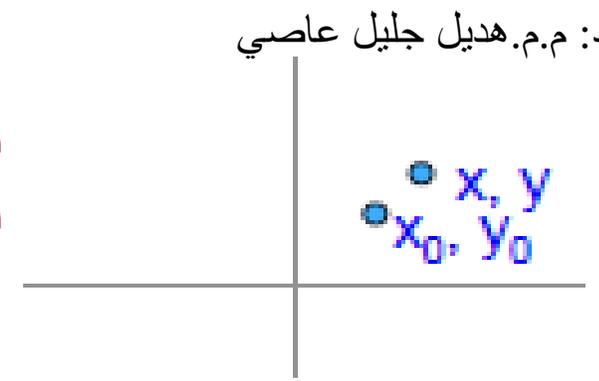
$$\vec{\nabla} \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad (\text{scalar}) \quad (\text{How?}) \quad (1.13)$$

c. Curl of a Vector (velocity)

$$\begin{aligned} \vec{\nabla} \times \vec{V} &= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \\ &= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) i + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) j + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) k \quad (\text{vector}) \quad (1.14) \end{aligned}$$



سيتم استخدام تقنية تايلور لحساب التغير الحاصل في حقل الرياح عند نقطة اعتبارية X, Y
بالنسبة لنقطة متجاورة X_0, Y_0



$$u = u_0 + \frac{1}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) x - \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) y + \frac{1}{2} \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) x + \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) y$$

$$v = v_0 + \frac{1}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) y + \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) x - \frac{1}{2} \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) y + \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) x$$

A



B



C



D



E

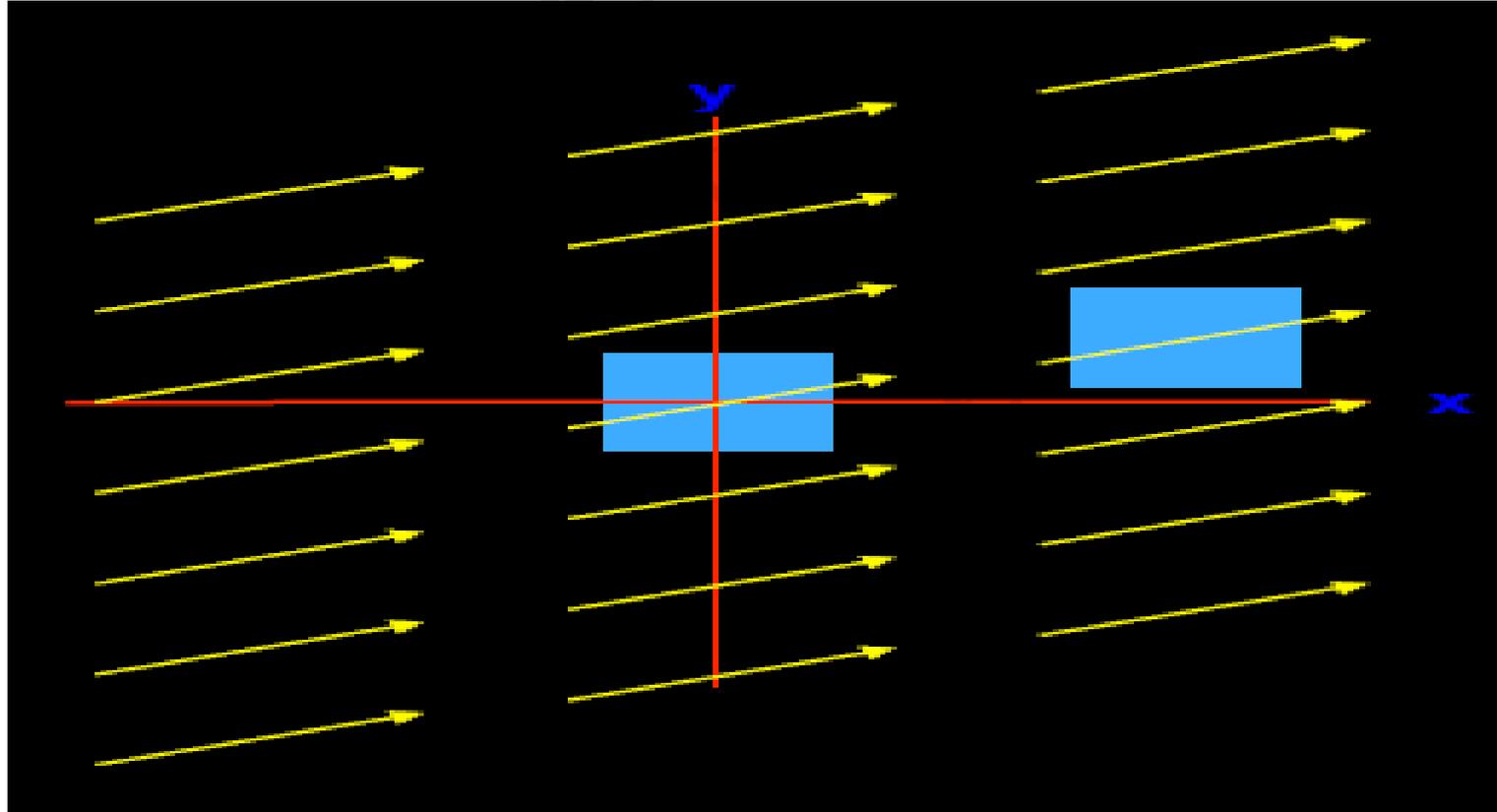


Translation Divergence

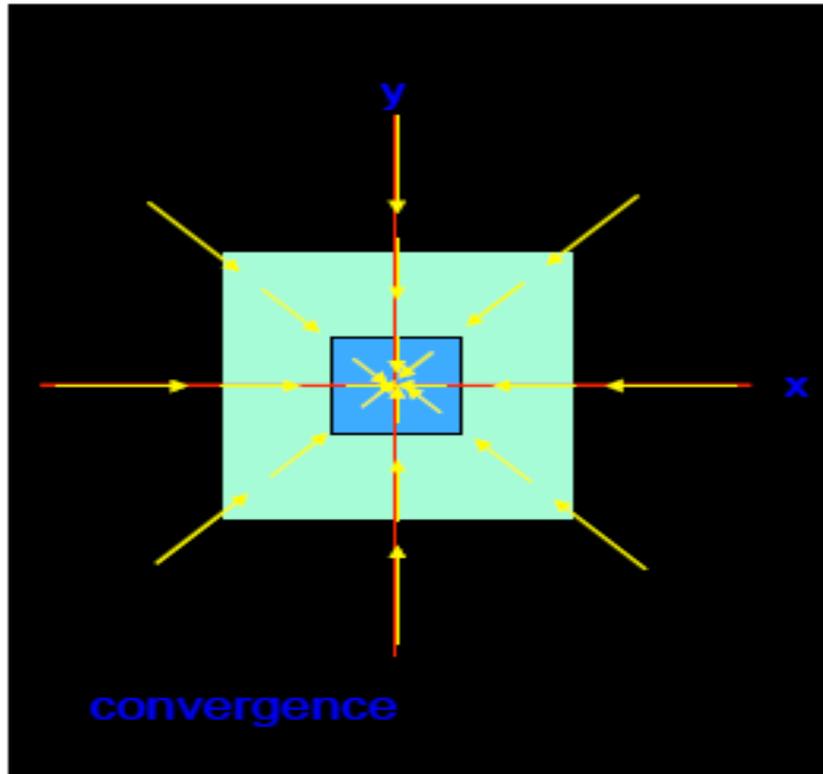
Relative
VorticityStretching
DeformationShearing
Deformation

اي حقل رياح يتحرك خطيا يمكن تمثيله بهذه الحدود الخمسة ،اما بالنسبة لحقل الرياح الذي يتحرك بشكل غير خطي ايضا يمكن تمثيله بشكل تقريبي بهذه الحدود الخمس:

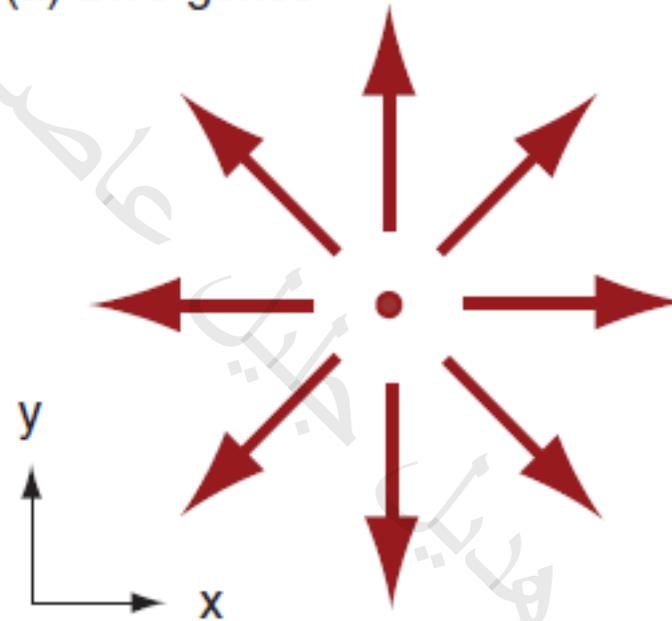
الحد الاول A: (Translation V_0, U_0) وتمثل انتقال حقل الرياح من مكان الى اخر.



الحد الثاني B : (Divergence D) التباعد ويمثل التغير في مساحة حقل الرياح نقصان او زيادة.



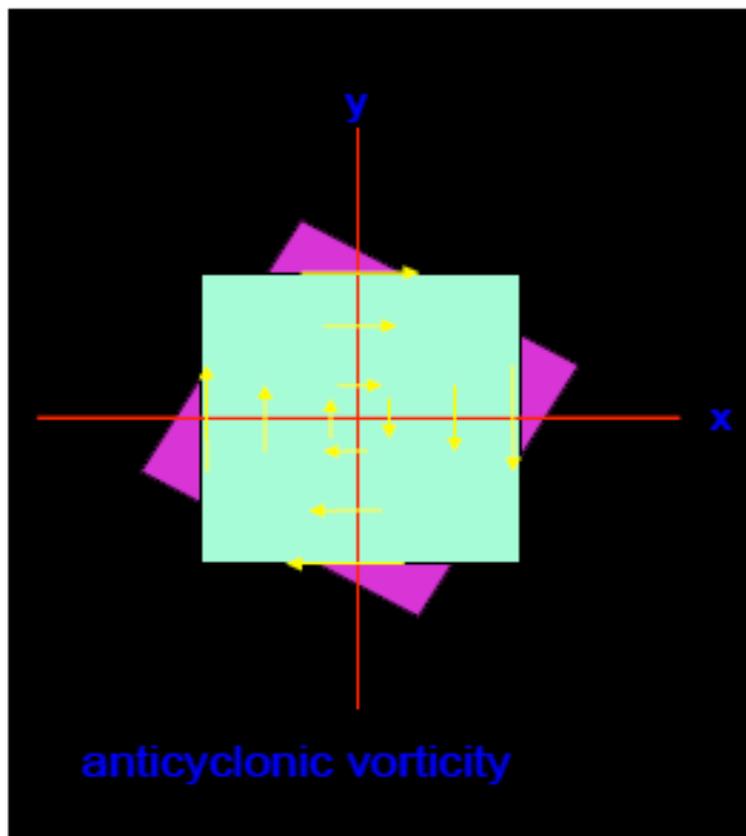
(a) Divergence



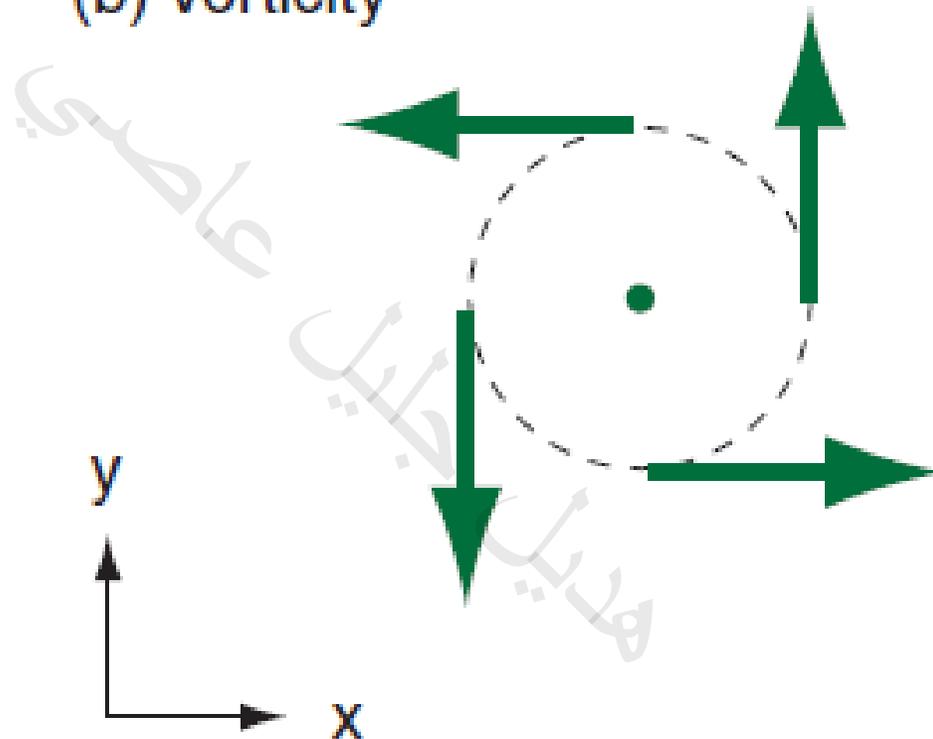
$$D = \frac{\Delta U}{\Delta x} + \frac{\Delta V}{\Delta y}$$



الحد الثالث **C** : (Relative vorticity) الدورانية النسبية ويمثل التغير في اتجاه حركة حقل الرياح مع عقرب الساعة او عكس عقرب الساعة .



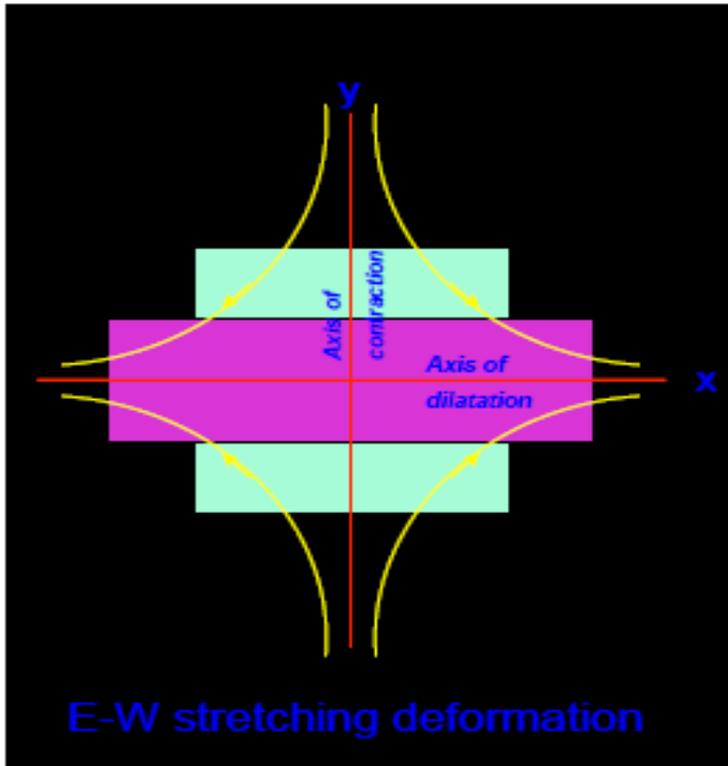
(b) Vorticity



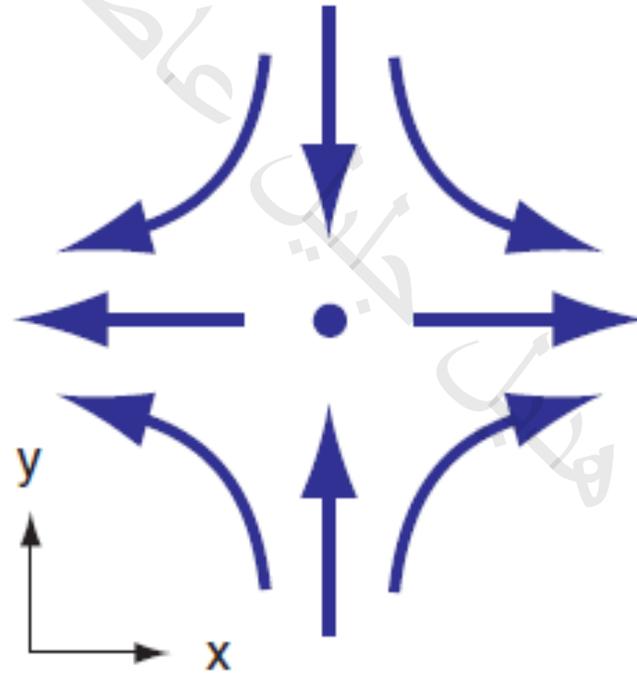
$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y}$$



الحد الرابع D : Stretching Deformation والذي يمثل التشويه الحاصل في شكل حقل الرياح.



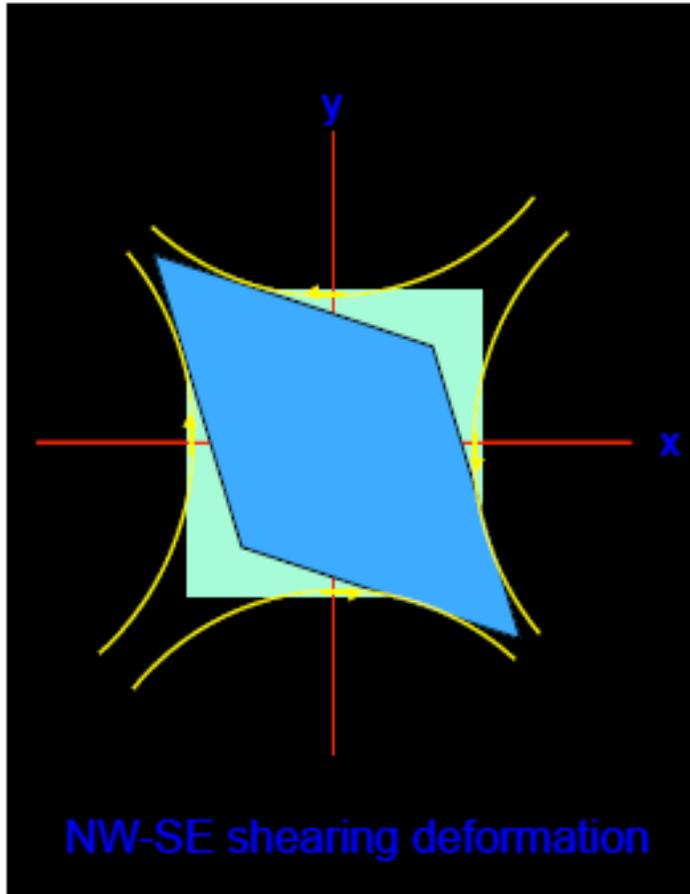
(c) Stretching Deformation



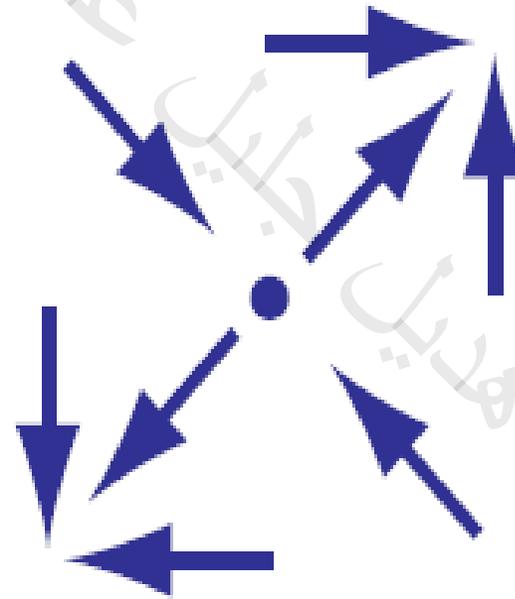
$$F_1 = \frac{\Delta U}{\Delta x} - \frac{\Delta V}{\Delta y}$$



الحد الخامس E : (Shearing Deformation) وهو التغير الحاصل في موقع حقل الرياح.



(d) Shearing Deformation

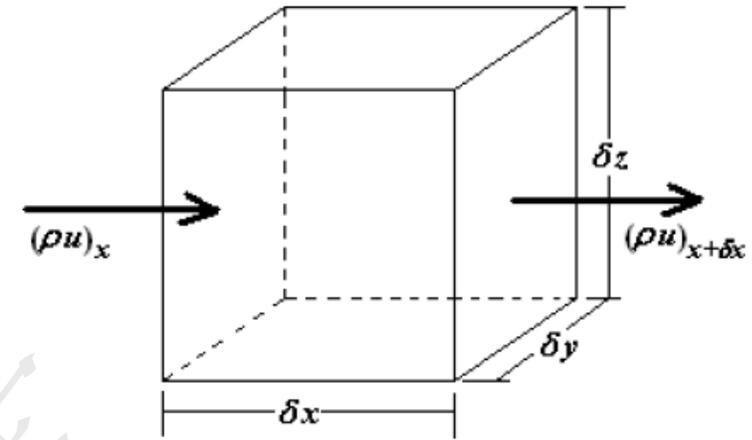
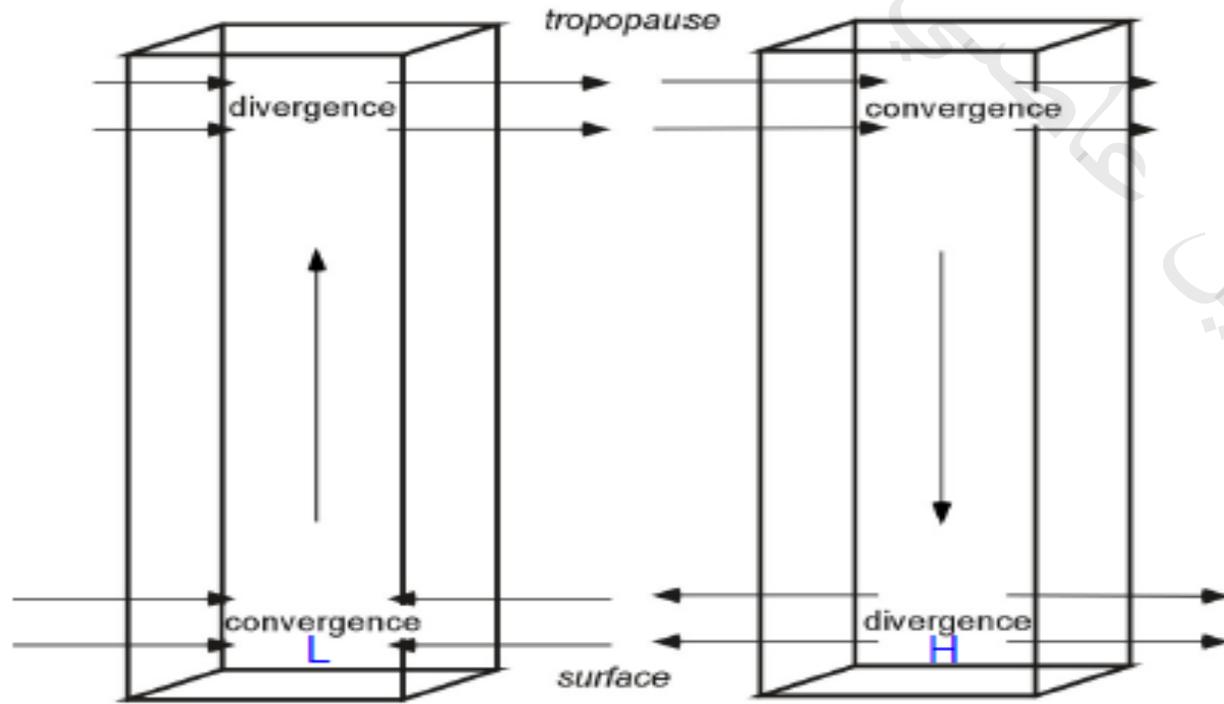


$$F_2 = \frac{\Delta V}{\Delta x} + \frac{\Delta U}{\Delta y}$$



تکمن اهمية دراسة هذه الحدود الخمس في تشخيص المدى السايونوتيكي للحركة الراسية ضمن عمود الهواء والتي تعتبر مهمة جدا في عملية التنبؤ بالطقس .

1- ايجاد العلاقة بين التباعد والحركة الراسية من خلال معادلة الاستمرارية (قانون حفظ الكتلة) :



$$\frac{\partial M_w}{\partial t} = \text{Inflow Rate} - \text{Outflow Rate.}$$



Divergence

العلاقة بين معادلة الاستمرارية والحركة الراسية

The Relation between continuity equation and vertical motion

that the net rate of mass accumulation in the cube is represented as

$$\frac{\partial M}{\partial t} = - \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \right] \delta x \delta y \delta z.$$

$$\frac{\partial \rho}{\partial t} = - \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \right] = -\nabla \cdot (\rho \vec{V}).$$

The expression above is known as the **mass divergence** form of the mass continuity equation. An alternative form of this expression arises by recalling that

$$\nabla \cdot (\rho \vec{V}) = \rho \nabla \cdot \vec{V} + \vec{V} \cdot \nabla \rho$$



Divergence

العلاقة بين معادلة الاستمرارية والحركة الراسية
 The Relation between continuity equation and vertical motion

The continuity equation states that

$$\nabla \cdot \mathbf{V} = 0 \quad \text{where} \quad \nabla = \frac{\partial}{\partial x} \hat{\mathbf{i}} + \frac{\partial}{\partial y} \hat{\mathbf{j}} + \frac{\partial}{\partial p} \hat{\mathbf{k}} \quad \&$$

$$\mathbf{V} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$$

We can also write

$$\nabla_H \cdot \mathbf{V}_H + \frac{\partial w}{\partial p} = 0 \quad \text{where} \quad \nabla_H = \frac{\partial}{\partial x} \hat{\mathbf{i}} + \frac{\partial}{\partial y} \hat{\mathbf{j}} \quad \&$$

$$\mathbf{V}_H = u\hat{\mathbf{i}} + v\hat{\mathbf{j}}$$



Divergence

العلاقة بين معادلة الاستمرارية والحركة الراسية

The Relation between continuity equation and vertical motion

$$\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} \quad \& \quad \nabla_H \cdot \mathbf{V}_H = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

3-D divergence

Horizontal divergence

لتشخيص الحركة الراسية يجب نكامل معادلة الاستمرارية لعمود الهواء من اعلى التروبوسفير P_0 حيث الحركة الراسية تساوي صفرا الى ارتفاع معين حيث يكون الضغط يساوي P

$$\omega(p) - \omega(p_0) = - \int_{p_0}^p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \partial p$$



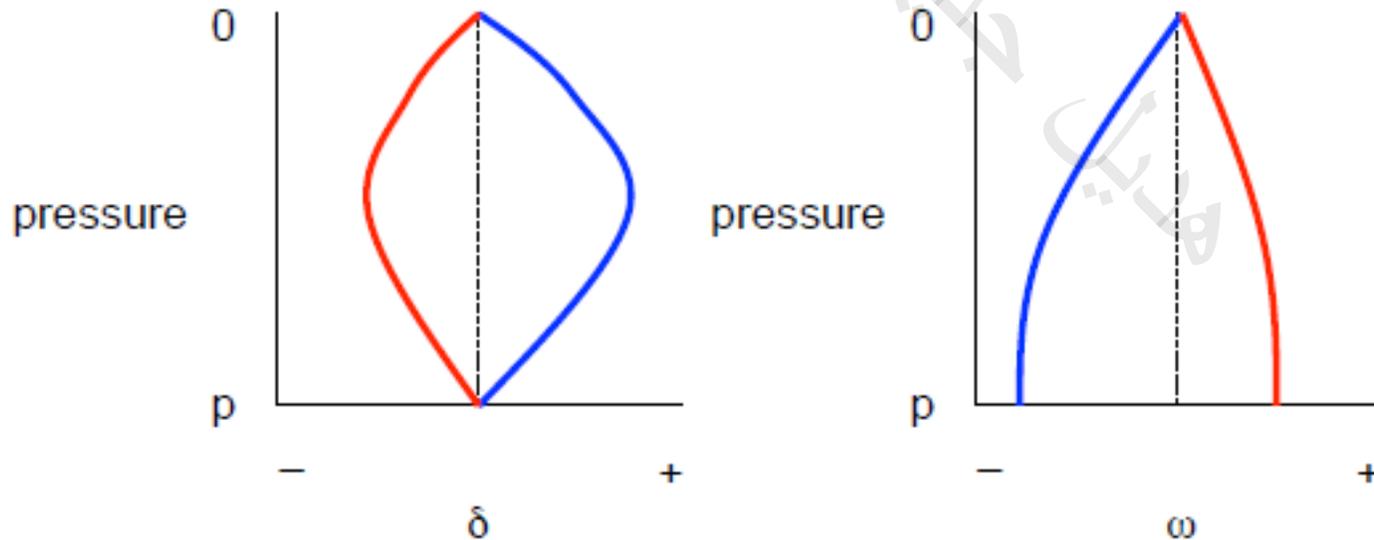
Divergence

العلاقة بين معادلة الاستمرارية والحركة الراسية The Relation between continuity equation and vertical motion

في اعلى الغلاف الجوي ايضا تكون الحركة الراسية تساوي صفر $W=0$

$$\omega(p) = - \int_{p_0}^p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dp$$

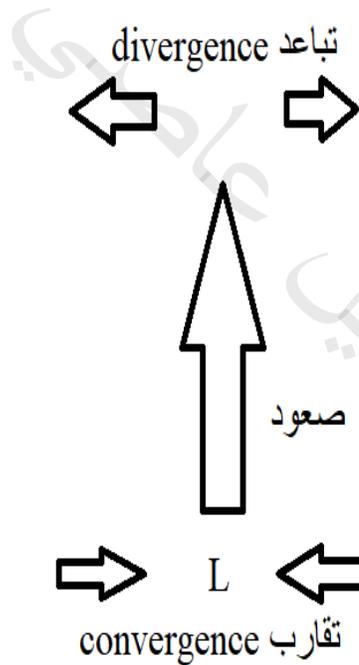
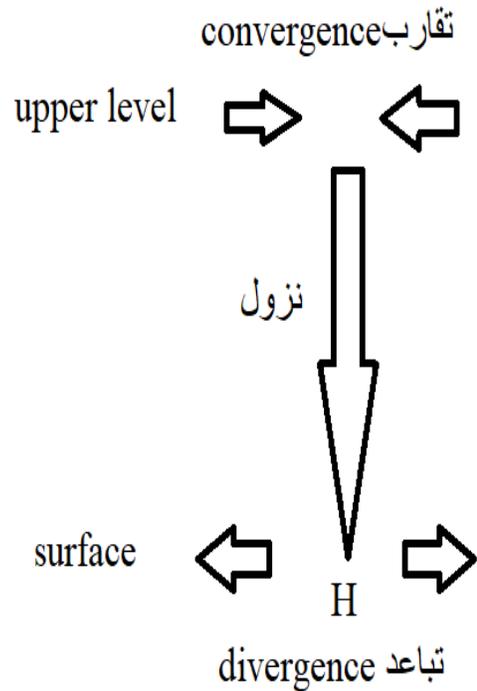
توضح هذه المعادلة ان قيمة الحركة الراسية W عند مستوى ضغطي معين تساوي تكامل قيمة التباعد او التقارب فوق هذا المستوى الضغطي.



Divergence

العلاقة بين معادلة الاستمرارية والحركة الراسية

The Relation between continuity equation and vertical motion



$$\omega(p) = -\overline{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}(p - p_0)$$

اعلى الغلاف الجوي P_0 تساوي صفر $P_0=0$

$$\omega(p) = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)p$$



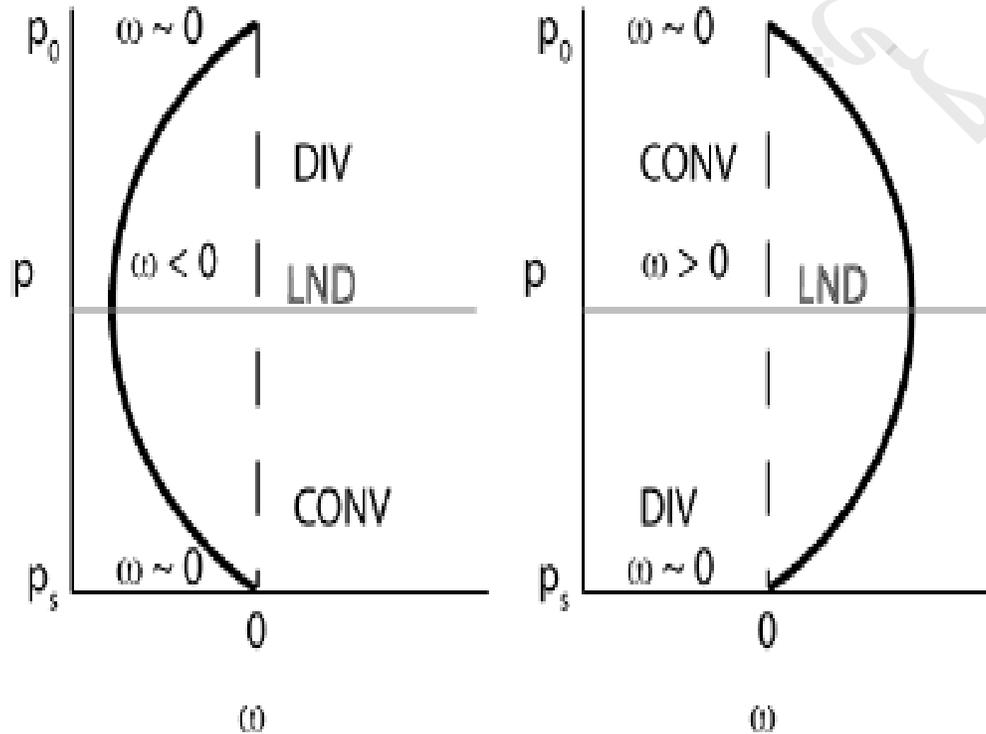
Divergence

العلاقة بين معادلة الاستمرارية والحركة الراسية

The Relation between continuity equation and vertical motion

نستنتج من المعادلة اعلاه انه اذا كانت قيمة التباعد العلوي موجبة تكون لدينا حركة راسية نحو الاعلى (**صعود**) اي قيمة W تكون سالبة. اذا كانت قيمة التباعد العلوي سالبة (**تقارب**) تكون لدينا حركة راسية نحو الاسفل (**نزول**) اي قيمة W تكون موجبة.

بما ان قيمة الحركة الراسية W عند السطح واعلى طبقة التروبوسفير تساوي صفر هذا يعني ان الحركة الراسية تبدأ بالازدياد صعودا حتى تصل اعلى قيمة لها او العكس تبدأ بقيمة سالبة حتى تصل اعلى قيمة لها بالسالب (**اقل قيمة لها**) عند منطقة معينة حيث تكون قيمة التباعد تساوي صفر حسب معادلة الاستمرارية تعرف هذه المنطقة بـ **Layer of Non Divergence (LND)** وهذا يتطلب ان تتغير قيمة التباعد على الاقل مرة خلال عمود الهواء وتتحول من تباعد الى تقارب او من تقارب الى تباعد.



Vorticity

Cartesian expression for vorticity

3D relative vorticity vector $\vec{v} = \nabla \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}$

Vertical component of vorticity vector (rotation in a horizontal plane)

$$\xi = \hat{k} \cdot \vec{v} = \hat{k} \cdot \nabla \times \vec{V} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Absolute vorticity (flow + earth's vorticity)

$$\eta = \hat{k} \cdot \vec{v}_a = \hat{k} \cdot \nabla \times \vec{V}_a$$

Absolute vorticity

$$\eta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) + f$$



Vorticity

The vorticity equation in height coordinates

$$\frac{du}{dt} - f v = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (1)$$

$$\frac{dv}{dt} + f u = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (2)$$

Expand total derivative

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + F_x \quad \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + f u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + F_y$$

Take $\frac{\partial(2)}{\partial x} - \frac{\partial(1)}{\partial y}$

write relative vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ as ζ

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} + (\zeta + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} = \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \left[\frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} \right]$$

$$\frac{d(\zeta + f)}{dt} = -(\zeta + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right]$$



Vorticity

$$\frac{d(\xi + f)}{dt} = -(\xi + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right]$$

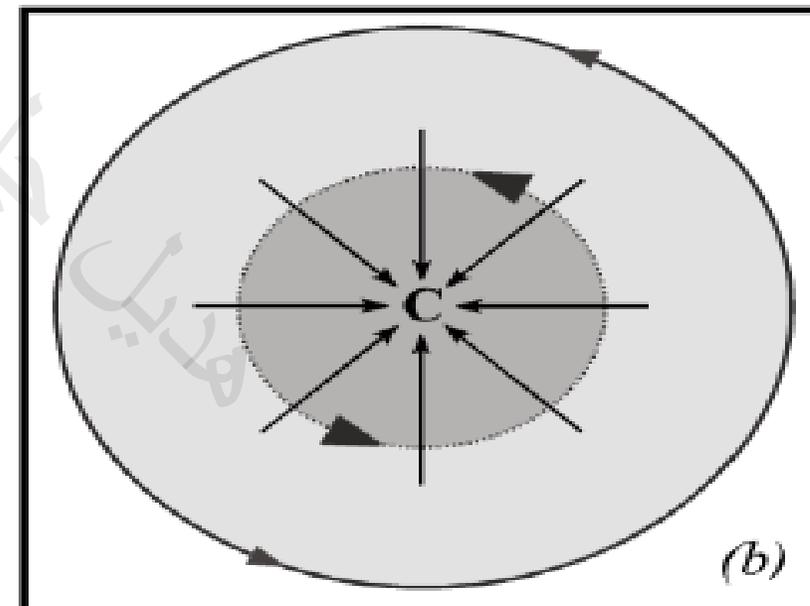
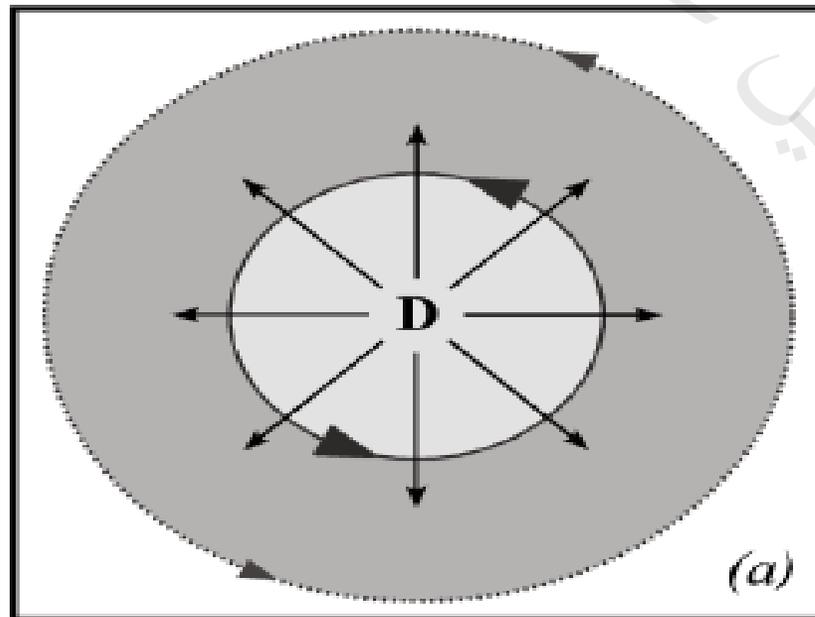
Rate of change
of relative vorticity
Following parcel

Divergence acting on
Absolute vorticity
(twirling skater effect)

Tilting of vertically
sheared flow

Pressure/density
solenoids

Gradients in force
Of friction



Vorticity

$$\frac{d(\xi + f)}{dt} = -(\xi + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right]$$

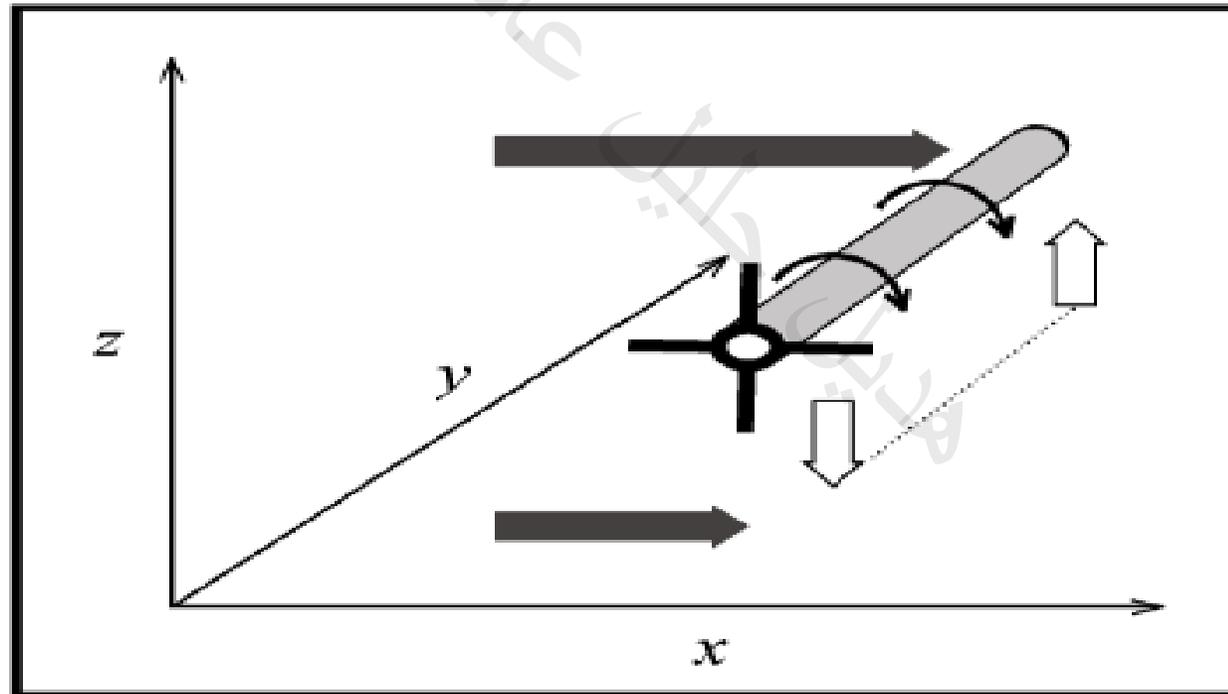
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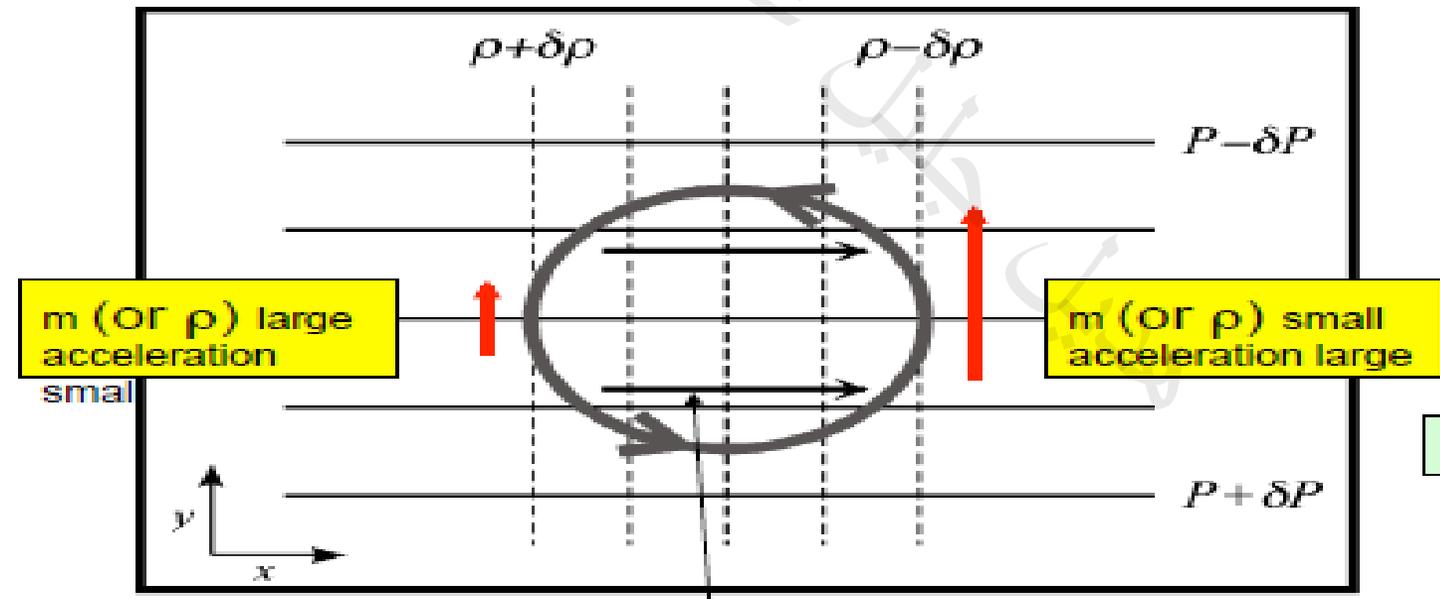


Vorticity

$$\frac{d(\xi + f)}{dt} = -(\xi + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right]$$

- Rate of change of relative vorticity Following parcel
- Divergence acting on Absolute vorticity (twirling skater effect)
- Tilting of vertically sheared flow
- Pressure/density solenoids
- Gradients in force Of friction

Solenoid: field loop that converts potential energy to kinetic energy



m (OR rho) large acceleration small

m (OR rho) small acceleration large

Cold advection pattern

$$F = ma$$

$$\frac{PGF}{m} = a$$

geostrophic wind



Vorticity

$$\frac{d(\xi + f)}{dt} = -(\xi + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right]$$

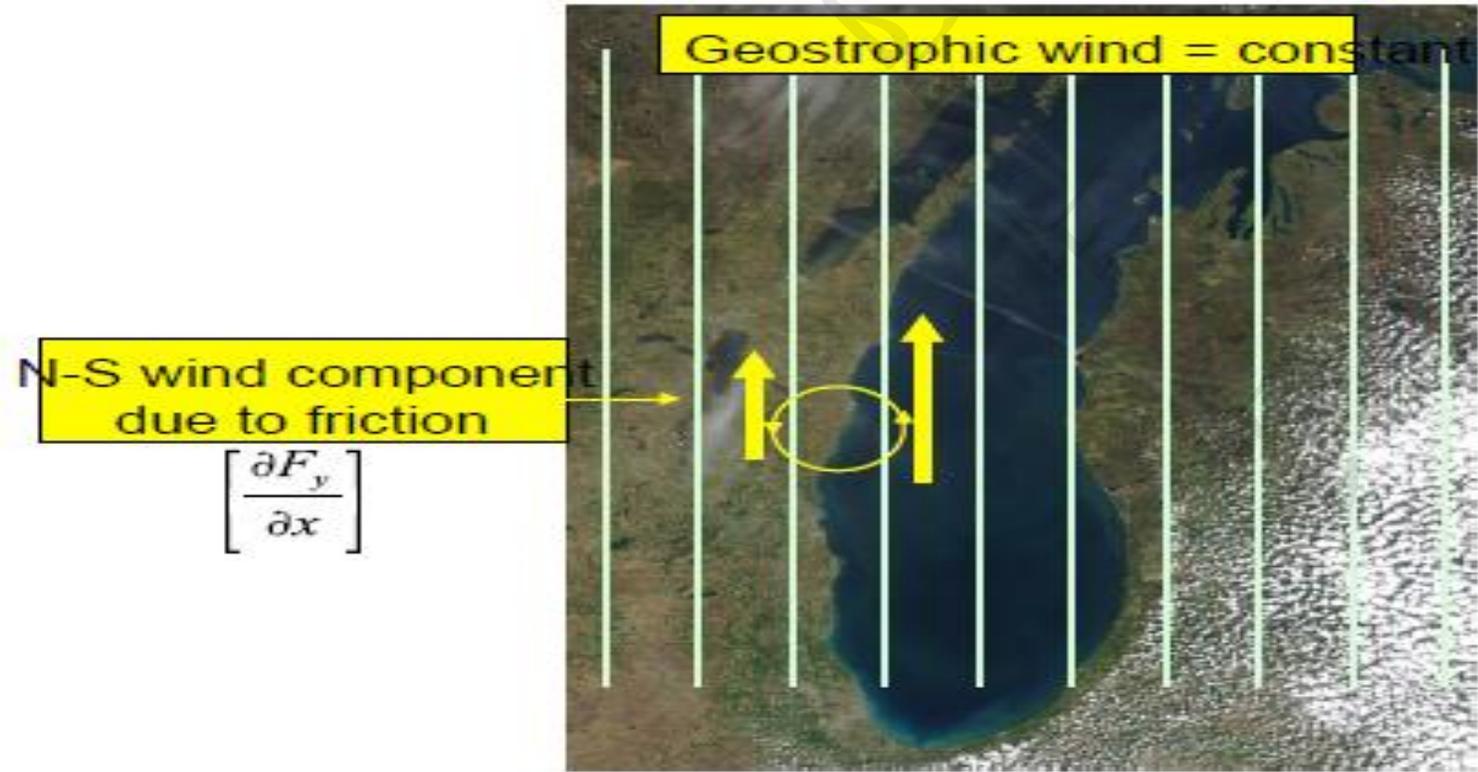
Rate of change of relative vorticity Following parcel

Divergence acting on Absolute vorticity (twirling skater effect)

Tilting of vertically sheared flow

Pressure/density solenoids

Gradients in force Of friction



Vorticity

The vorticity equation in pressure coordinates

$$\frac{du}{dt} = -\frac{\partial\phi}{\partial x} + f v + F_x \quad (1)$$

$$\frac{dv}{dt} = -\frac{\partial\phi}{\partial y} - f u + F_y \quad (2)$$

Expand total derivative

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial P} = -\frac{\partial\phi}{\partial x} + f v + F_x \quad \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial P} = -\frac{\partial\phi}{\partial y} - f u + F_y$$

Take $\frac{\partial(2)}{\partial x} - \frac{\partial(1)}{\partial y}$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) &= -u \frac{\partial}{\partial x} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f \right] - v \frac{\partial}{\partial y} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f \right] - \omega \frac{\partial}{\partial P} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] \\ &\quad - \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f \right] \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \left[\frac{\partial u}{\partial P} \frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial P} \frac{\partial \omega}{\partial x} \right] + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right] \end{aligned}$$

write relative vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ as ζ

$$\frac{\partial \zeta}{\partial t} = -u \frac{\partial}{\partial x} [\zeta + f] - v \frac{\partial}{\partial y} [\zeta + f] - \omega \frac{\partial \zeta}{\partial P} - [\zeta + f] \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \left[\frac{\partial u}{\partial P} \frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial P} \frac{\partial \omega}{\partial x} \right] + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right]$$



Vorticity

The vorticity equation

$$\frac{\partial \zeta}{\partial t} = -u \frac{\partial}{\partial x} [\zeta + f] - v \frac{\partial}{\partial y} [\zeta + f] + \omega \frac{\partial \zeta}{\partial P} - [\zeta + f] \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \left[\frac{\partial u}{\partial P} \frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial P} \frac{\partial \omega}{\partial x} \right] + \left[\frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} \right]$$

Local rate of change of relative vorticity
 Horizontal advection of absolute vorticity on a pressure surface
 Vertical advection of relative vorticity
 Divergence acting on Absolute vorticity (twirling skater effect)
 Tilting of vertically sheared flow
 Gradients in force of friction

In English: Horizontal relative vorticity is increased at a point if

- 1) positive vorticity is advected to the point along the pressure surface,
- 2) or advected vertically to the point,
- 3) if air rotating about the point undergoes convergence (like a skater twirling)
- 4) if vertically sheared wind is tilted into the horizontal due a gradient in verti
- 5) if the force of friction varies in the horizontal

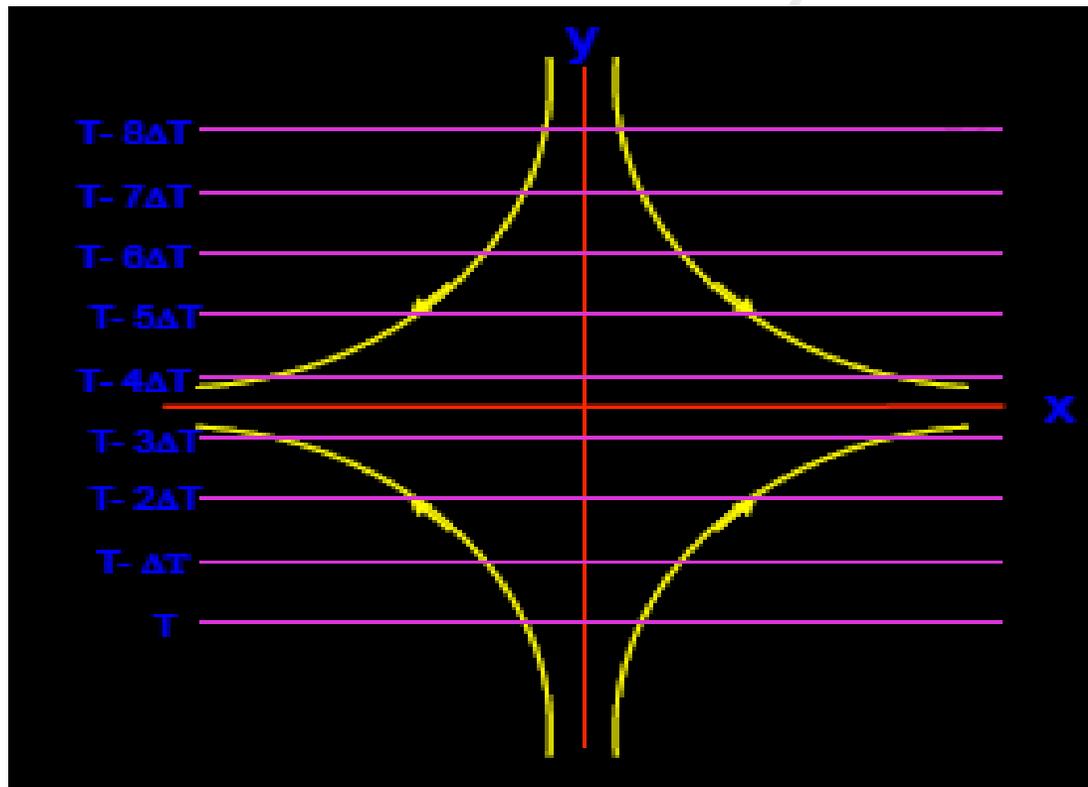
Solenoid terms disappear in pressure coordinates: we will work in P coordinate for



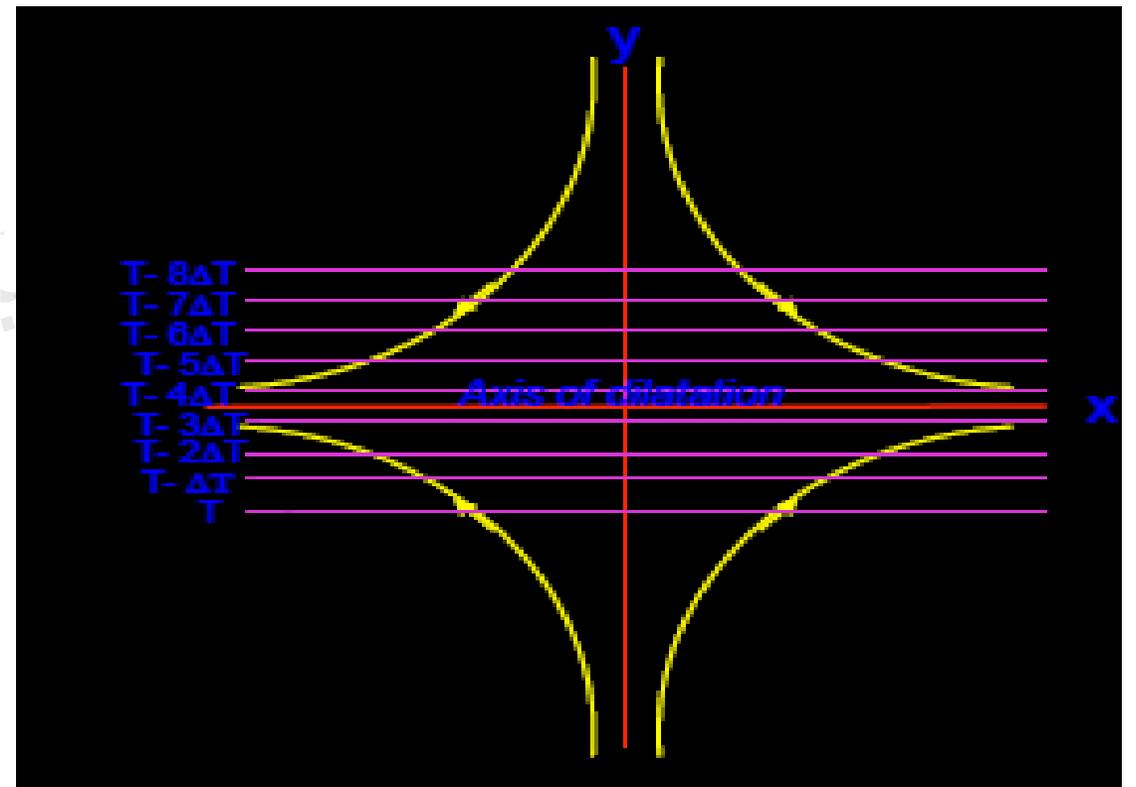
Deformation

Deformation flow is fundamental to the development of fronts

Time = t

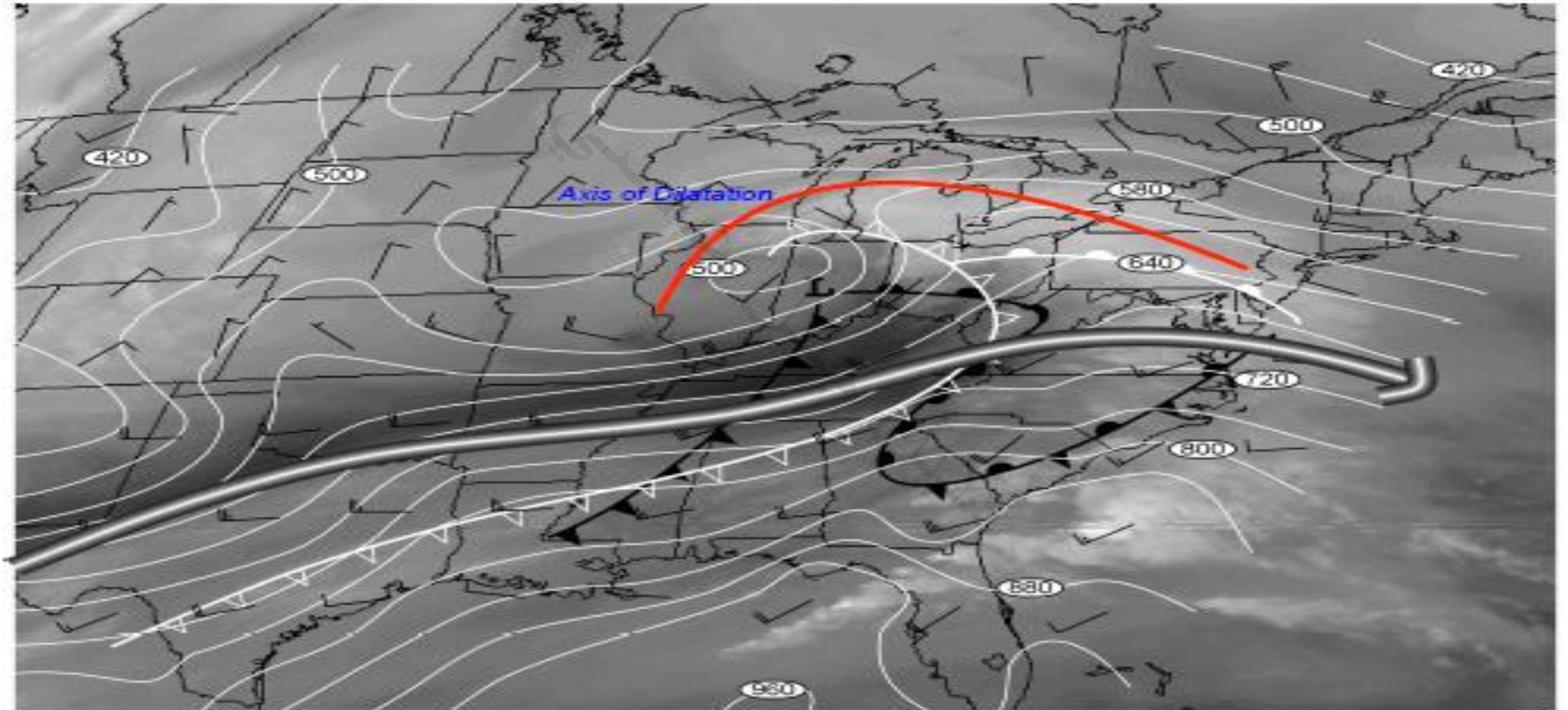


Time = $t + \Delta t$



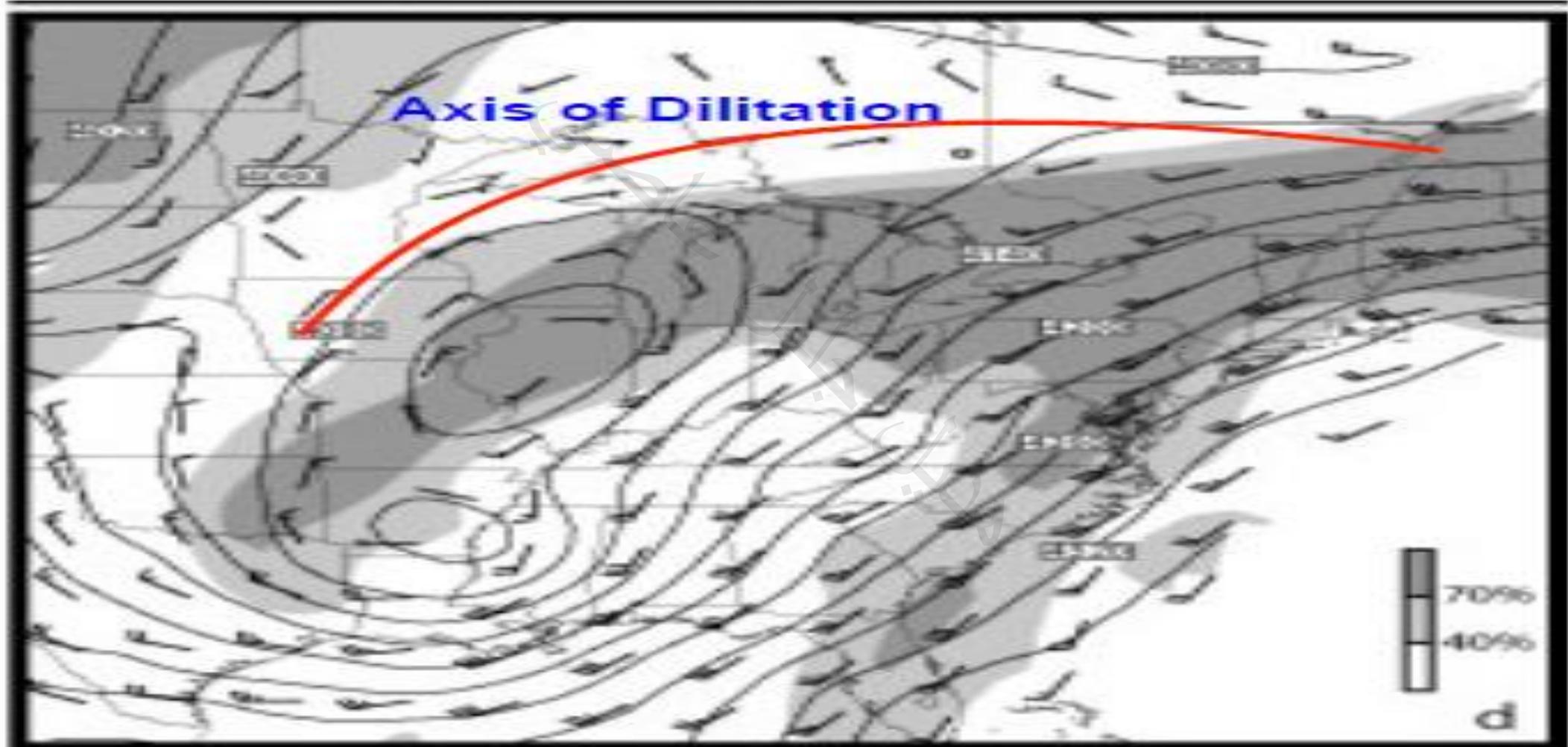
Deformation

EXAMPLES OF DEFORMATION



Deformation

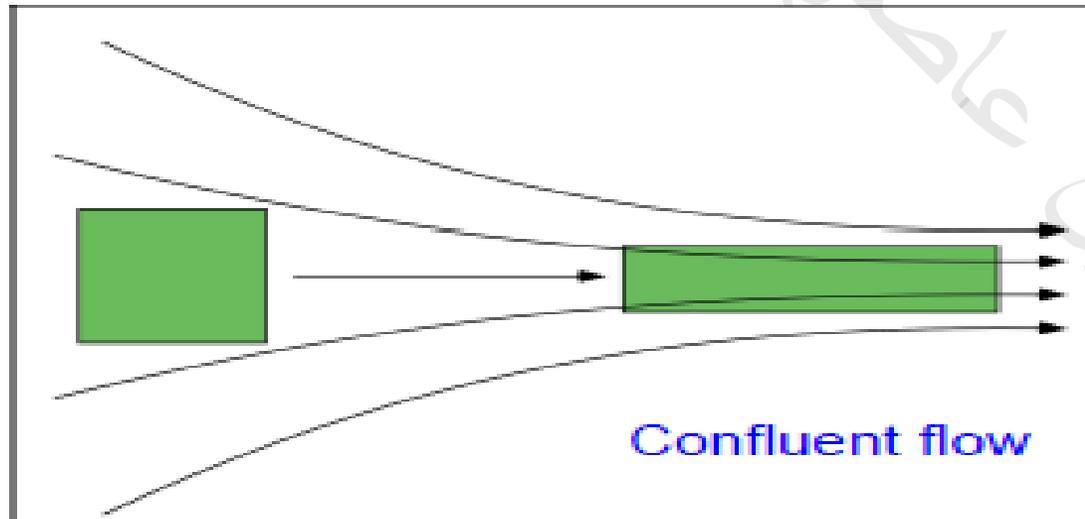
EXAMPLES OF DEFORMATION



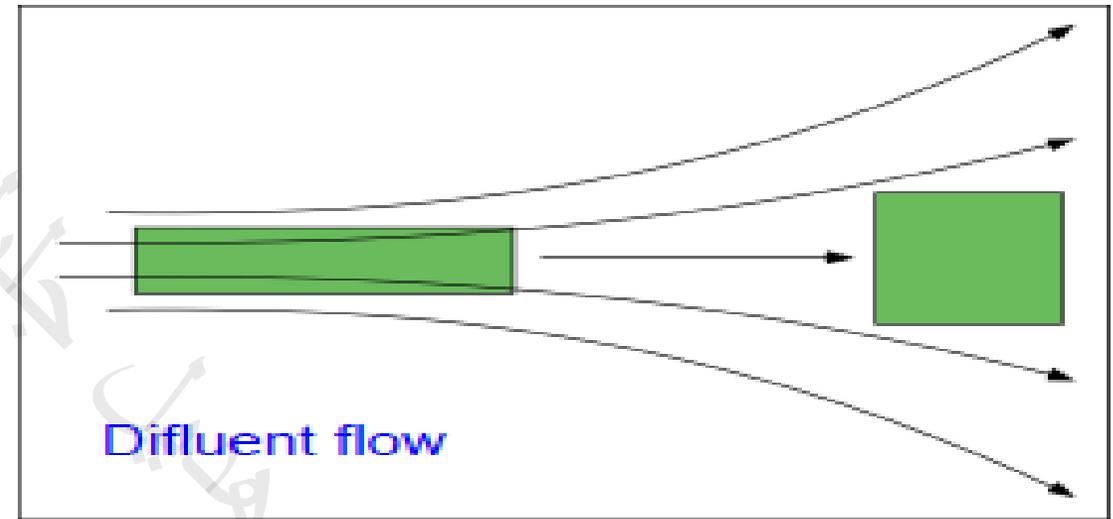
Deformation

CONFLUENT and DIFLUENT FLOW

Is this flow convergent?



Is this flow divergent?



**NO: The areas of the two boxes are identical.
The flow is a combination of translation and
deformation.**



Curvature

The terms for divergence, relative vorticity, and deformation strictly apply on a plane tangent to the earth's surface. If we take earth's curvature into account, we have to add an additional term.



Curvature

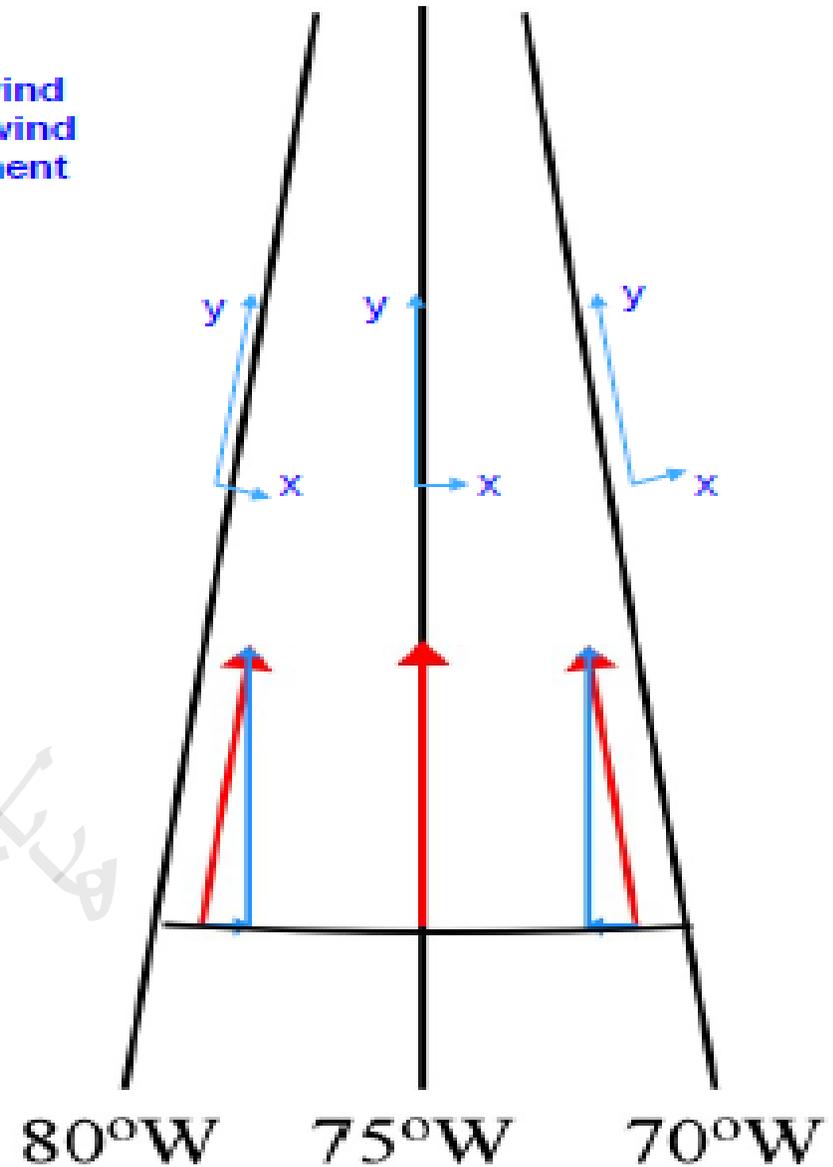
Suppose the wind is southerly and uniform. Is the wind convergent?

Yes!

$$\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{a} \tan \phi$$

Convergence of meridians toward north leads to convergence. This is the earth curvature term (the last term) in the expression for convergence (δ).

Red = wind
Blue = wind component



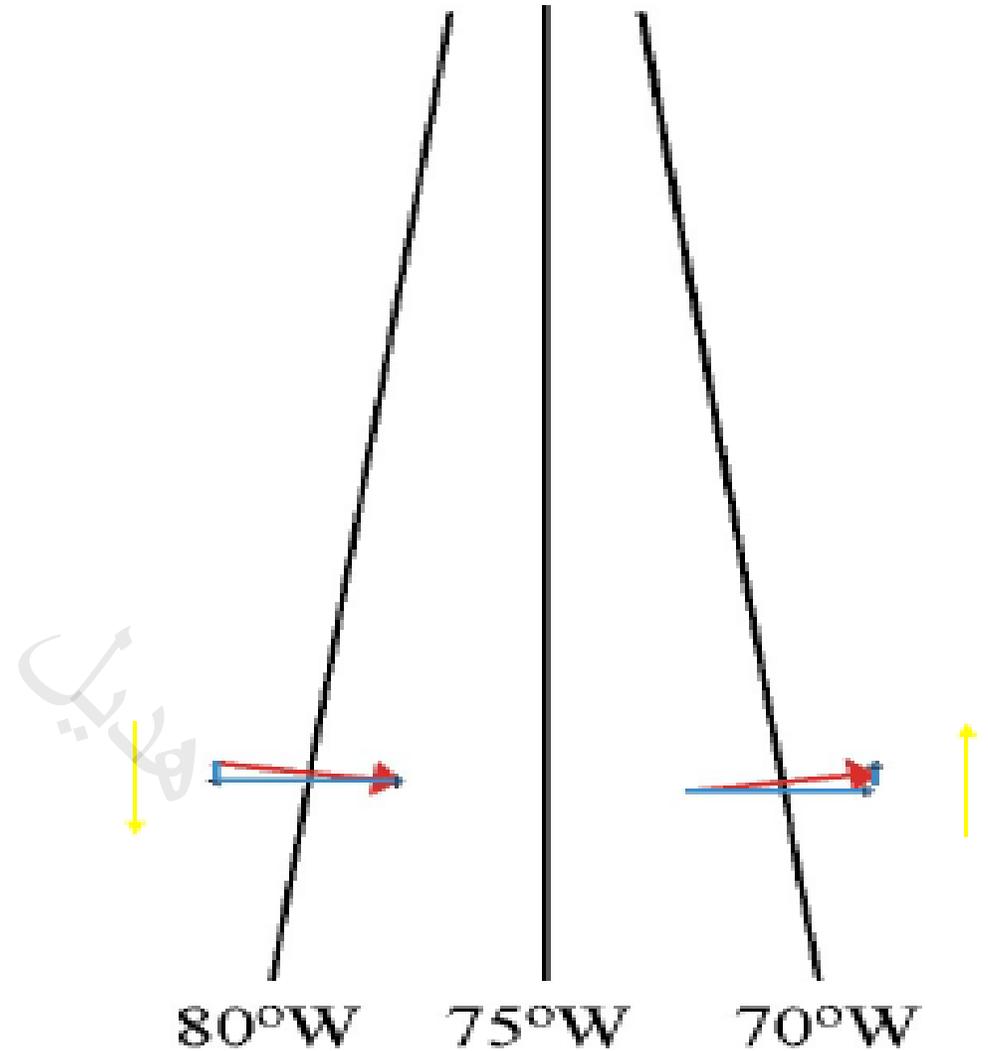
Curvature

Suppose the wind is westerly and uniform. Does vorticity exist?

Yes!

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + \frac{u}{a} \tan \phi$$

Convergence of meridians toward north creates vorticity. This is the earth curvature term (the last term) in the expression for vorticity (ζ).



Curvature

In a similar way, convergence of the earth's meridians toward the north leads to deformation in otherwise uniform flow

$$D_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \frac{v}{a} \tan \phi$$

$$D_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{u}{a} \tan \phi$$

Earth's curvature terms are an order of magnitude smaller than other terms, but cannot be ignored in models, at least in the middle and high latitudes.



References :

Jonathan. E. Martin., 2006, “ Mid-latitude Atmosphere Dynamics”, *whily*, PP.337.

هناك جليل
عاطي

