# 2015

## Digital Logic Design Lab

## Featuring EWB 5.12



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### [Lab 1]. Familiarization; Playing with EWB 5.12

#### Introduction to Electronics Workbench

Electronics Workbench is an electronics and digital logic lab inside a computer, modeled after a real electronics workbench. It is a design tool that provides you with components & instruments to create "virtual" board-level designs:

- No actual breadboards, components, or instruments needed.
- Click-and-drag schematic editing.
- It offers mixed analog & digital simulation and graphical waveform analysis.
- Circuit behavior simulated realistically.
- Results displayed on multimeter, oscilloscope, bode plotter, logic analyzer, etc.

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The main GUI interface of EWB

#### Using Electronics Workbench for Design

You may use EWB to:

- 1- Explore ideas and test preliminary circuits.
- 2- Refine circuits to full layout (If circuit requires parts of a previous design)
- 3- Export files in format used by PCB (Printed Circuit Board) layout packages as move from design to production.

#### **General EWB Functions**

#### Selecting

- To move a component or instrument need to select it selected item highlights: <u>components red</u>, <u>wires</u> thicken

– Clicking to Select

To select single item, click on it.

To select additional items, press CTRL+ click.

- <u>Selecting All</u>

Choose Edit/Select All.

- Dragging to Select

Place pointer above & to side of group of items. Press & hold mouse button & drag downward diagonally. Release mouse button when rectangle encloses everything desired.

#### – Deselecting

To deselect single item, press CTRL+click.

To deselect all selected items, click on empty spot in window.

#### **Setting Labels, Wiring**

#### Setting Labels, Values, Models & Reference IDs,

- To set labels, values (for simple components) & models (for complex components), select component and choose Circuit/Component Properties, choose desired tab, make any changes, and click OK.

- Can also invoke Circuit/Component Properties box by double-clicking on component.

\* Notes:

The Circuit/Component Properties box contains a number of tabs; depending on which component is selected an analog component has either a value or a model, not both.

#### Wiring Components

– Point to a component's terminal so it highlights; press & hold mouse button, and drag so a wire appears drag wire to a terminal on another component or to an instrument connection, when terminal on second component or instrument highlights, release mouse button

#### Inserting, Connecting, Editing

#### Inserting Components

– To insert component into existing circuit, place it on top of wire; it will automatically be inserted if there is room.

**Connecting Wires** 

– If drag a wire from a component's terminal to another wire, a **connector** is automatically created when you release mouse button.

Note: a connector button also appears in the Basic toolbar (to insert connectors into an existing circuit).
 <u>Deleting Wires</u>

- To delete a wire, select it & choose Edit/Delete

- Alternatively, disconnect wire by selecting one end of it & moving it to an open spot on circuit window.

#### <u>Changing Wire Color</u>

– To change a wire's color, double-click it & choose Schematic Options tab; click the Color button & choose a new color.

#### Straightening a Wire

- move wire itself.

- move component to which wire is attached.
- press ALT and move component to which wire is attached.
- select component and press appropriate arrow key to align it.
- If two wires cross in a way that makes them hard to follow, select one & drag it to new location
   \*Note:
  - the way a wire is routed sometimes depends on terminal from which wire was dragged; try disconnecting routed wire & then rewire from the opposite terminal.

#### Instruments



#### - Using an Instrument Icon

To display the Instruments toolbar, click the Instruments button on the Parts Bin toolbar.

To place an instrument on the circuit window, drag the desired button from the Instruments toolbar to the window. To attach an instrument to a circuit, point to a terminal on its icon so it highlights and drag a wire to a component. To remove an instrument icon, select it & choose Edit/Delete

#### <u>– Opening an Instrument</u>

Double-click the instrument's icon to see its controls

- To selection options, click buttons on the controls
- To change values or units, click the up/down arrows.

#### Simulation

– Turning on Power



Click the power switch to turn power on. Click switch again to turn power off. (Note: Turning off power erases data & instrument traces.

#### Lab Tasks

#### Task 1: Name the basic toolboxes of EWB



#### Task 2: Basic buttons in EWB toolboxes



#### Task 3 EWB Toolbar

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File Edit Circuit Analysis Window Help	
	ol
	Pause

#### Task 4: Simple circuit; playing with EWB

In the following circuit

🚰 Electronics Workbench	Liste	
Untitled +Vc	<u>-</u>	I Logic Gates     D
Ready 1.30 s	Temp: 27	

Draw the following circuit. After that make the following changes

- Connect the output of the converter to the red probe
- Connect the Vcc line to the input of the inverter
- Start simulating the circuit
- State your observation down:

#### **Observation**:



- In the same circuit above, stop the simulation and connect the ground to the input of the inverter. State your observation down:



#### **Observation**:

#### Task 5: Simple circuit; two inverters connected serially

Repeat Task 2 of this report and state down your observations.



#### Task 6: Simple circuit; a clock source with a red probe

Draw the following circuit and simulate it. Write down your observations. Notice that the **clock** (from *Sources* toolbox) frequency is 2 Hz.



**Note**: You can change the default values of the clock by doing mouse right clicking on the clock and click on the "Component Properties …" as shown below:

	(1000 Hz/50%)	Clock Properties	? <mark>- x</mark>
$\rightarrow$	Ŷ	Label Value Fault Display	
(i)1000 Hz/50%		Frequency (F): 1000	
Component Properties		Duty cycle (D): 50 A %	
Cut		Voltage (V): 5 V	
Сору			
Delete			
Rotate			
Flip Vertical			
Flip Horizontal			
Halp			
пер		ОК	Cancel

#### Task 7: Simple circuit; a clock source with two red probes

Draw the following circuit and simulate it. Write down your observations. Notice that the clock frequency is 2 Hz.



#### Task 8: EWB Menu

Name the following icons and state down their functions



### [Lab 2]. Basic logic Gates (AND, OR, and NOT gates)

#### **Objectives**

- 1- To study and understand the 3 basic gates.
- 2- Implement the basic gate in EWB.
- 3- The study the specifications of every gate when connected it with one input constant and the other is variable.

#### AND and NAND gates

This gate gives high output (1) if all the inputs are 1's. otherwise the output will be low (0).

Its Boolean algebra representation is: C=A.B

And it's truth table and schema as following:



The NAND gate works opposite to the AND gate. Its Boolean algebra representation is: C=(A.B)' And it's truth table and schema as following:



#### **OR and NOR gates**

This circuit will give high output (1) if any input is high (1).

Its Boolean algebra representation is: C=A+B and it's truth table and schema as following:



The NOR gate works opposite to the  $\overline{OR}$  gate. Its Boolean algebra representation is: C=(A+B)' And it's truth table and schema as following:



#### **NOT** gate

This is the simplest gate it just inverts the input, if the input is high the output will be low and conversely. So B=A'



#### Lab Tasks

#### Task 1: The AND and NAND gates

In EWB, draw the following two circuits and fill the truth table below



А	В	A.B	(A.B)'
0	0		
0	1		
1	0		
1	1		



#### Task 2: The AND-NOT combination

In EWB, draw the following circuit and fill the truth table



A	В	(A.B)'
0	0	
0	1	
1	0	
1	1	

#### Task 3: The OR and NOR gates

In EWB, draw the following two circuits and fill the truth table below



А	В	A+B	(A+B)'
0	0		
0	1		
1	0		
1	1		



#### Task 4: The NOR-NOT combination



#### Task 5: Finding the truth table of a gate using the logic converter

The logic converter can be found in the *Instruments* toolbox. It can be used to derive a truth table from a circuit schematic:

- 1. Attach the input terminals of the logic converter to up to eight input points in the circuit.
- 2. Connect the single output of the circuit to the output terminal on the logic converter icon.
- 3. Click the Circuit to Truth Table  $\rightarrow 101$  button.

The truth table for the circuit appears in the logic converter's display.

In the following circuit, we will be examining the AND gate. The two inputs of the gate are attached the A and B inputs of the logic converter. The circuit output C is connected to *Out* line of the logic converter.



After clicking on the Truth Table  $\longrightarrow$  101 button of the logic converter, the logic converter tries all possible combinations of the circuit input and derives its truth table.



Ī	Logic Converter											
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#### Task 6: Finding the truth table of a gate using the logic converter

Repeat what you did in task 5 for the NOR gate. Show your connections in the circuit below.

		•	
А	В	A+B	(A+B)'
0	0		
0	1		
1	0		
1	1		



#### Task 7: Finding the truth table of a three input gate using the logic converter

Repeat what you did in task 5 for a three-input AND gate. Show your connections in the circuit below. **Note**: you can obtain a three-input AND gate by drawing a regular two-input AND gate and then changing its *Number of Inputs* property as shown next.

1D-	2-Input AND Gate Properties	? 💌
	Label Models Fault Display Number of Inputs © 2 © 3 © 4 © 5 © 6 © 7 © 8	
	ОКС	ancel



0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

#### Task 8: Finding the truth table of a given circuit using the logic converter

Find the truth table of the following circuit:



0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

## [Lab 3]. Digital logic circuits analysis and converting Boolean expressions to digital circuits

#### **Objectives**

- To learn how to directly convert a Boolean expression to circuit.
- To learn how to analyze a given digital logic circuit by finding the Boolean expression that represents the circuit
- To learn how to analyze a given digital logic circuit by finding the truth table that represents the circuit.

Example:

 $Z = A + B \cdot C'$ 

The above function is implemented in the following digital logic Circuit



Now after drawing the circuit above using EWB we find that its truth table is as shown below (notice that logic 1 means connect the input to the Vcc line, and logic 0 means connecting the input to the ground)

Α	В	C	Ζ
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

#### Lab Tasks

#### Task 1: Converting Boolean expressions into circuits

Convert the following Boolean expression to a circuit, draw the circuit on EWB and simulate it to fill-in its truth table shown below.  $X = Y + Z \cdot Y'$ 

Draw the circuit in the space below



Now, fill-in the truth table of the circuit you drawn

Y	Z	Χ
0	0	
0	1	
1	0	
1	1	

#### **Task 2: Converting Boolean expressions into circuits**

Convert the following Boolean expression to a circuit, draw the circuit on EWB and simulate it to fill-in its truth table shown below.  $D = (A \cdot B) + (C' \cdot A)$ 



Α	B	С	D
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

#### Task 3: Digital logic circuit analysis - Finding the Boolean expression of a given circuit

Find the Boolean expression of the following circuit, draw the circuit on EWB and simulate it to fill-in its truth table shown below.

W =



**Note**: the logic converter tool from EWB to fill-in the following table. For that, you need to connect the A, B and C inputs of the logic converter to X, Y and Z lines, respectively. Further, you need to connect the 'out' line of the logic converter to W. As shown in the following diagram



Task 4: Digital logic circuit analysis – Finding the Boolean expression of a given circuit

Find the Boolean expression of the following circuit,  $\mathbf{D} =$ 



Draw the circuit on EWB and simulate it to fill-in its truth table shown below (use logic converter please).

Α	B	С	D
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

#### Task 5: Logic circuits with multiple outputs

Find the Boolean expression of the outputs of the following circuit,

**D** =

 $\mathbf{E} =$ 



Draw the circuit on EWB and simulate it to fill-in its truth table shown below (use logic converter please).

**Note**: You need to use the logic converter two times, once for the output D, and another time for the second output E.

Α	B	С	D	Ε
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

#### Task 6\*: Finding the Boolean expression of a given circuit using the logic converter

Draw the following circuit on EWB and then find its Boolean expression using the logic converter.



#### Task 7\*: Converting Boolean expressions to circuits using the logic converter

Use the logic converter to realize the following circuit using suitable logic gates: AB'C (BD + CDE) + AC'

## [Lab 4]. Boolean algebra and Simplification of Boolean expressions - I

#### **Objectives**

#### **Object**

- 1- To study DeMorgan's theory and implemented it.
- 2- Learn how to simplify Boolean logic equations using DeMorgan's theory.

#### DeMorgan's Theory – Background

Augustus De Morgan (27 June 1806 – 18 March 1871) was a British mathematician and

logician. He formulated De Morgan's laws.



In simple words, DeMorgan's Theory is used to convert AND/NAND gates to OR/NOR ones, and presented OR/NOR gates by AND/NAND gates by these 2-laws:

 $A + B = (A' \cdot B')'$  $A \cdot B = (A' + B')'$ 

#### **Basics of Boolean algebra**

**Boolean Postulates** 

P1: X = 0 or X = 1P2:  $0 \cdot 0 = 0$ P3: 1 + 1 = 1P4: 0 + 0 = 0P5:  $1 \cdot 1 = 1$ P6:  $1 \cdot 0 = 0 \cdot 1 = 0$ P7: 1 + 0 = 0 + 1 = 1

#### **Boolean Laws**

#### **T1 : Commutative Law**

(a) A + B = B + A(b) A B = B A

#### T2 : Associate Law

(a) (A + B) + C = A + (B + C)
(b) (A B) C = A (B C)

#### **T3 : Distributive Law**

(a) A (B + C) = A B + A C(b) A + (B C) = (A + B) (A + C)

#### **T4 : Identity Law**

(a) A + A = A(b) A A = A

#### T5 :

(a) 
$$AB + A\overline{B} = A$$
  
(b)  $(A+B)(A+\overline{B}) = A$ 

#### **T6 : Redundance Law**

(a) A + A B = A(b) A (A + B) = A

#### **T7**:

(a) 
$$0 + A = A$$
  
(b)  $0A = 0$ 

#### **T8 :**

(a) 
$$1 + A = 1$$
  
(b)  $1 A = A$ 

#### **T9**:

(a)  $\overline{A} + A = I$ (b)  $\overline{A} A = 0$ 

#### **T10**:

(a)  $A + \overline{A} B = A + B$ (b)  $A (\overline{A} + B) = A B$ 

#### **T11 : De Morgan's Theorem**

(a)  $(\overline{A+B}) = \overline{A} \ \overline{B}$ (b)  $(\overline{AB}) = \overline{A} + \overline{B}$ 

#### Simplifying Boolean logic functions





The Boolean expression that represents the above circuit is as follows



We can simplify the above Boolean expression as follows



This means that the above circuit can be replaced by the following one



#### Lab Tasks

#### Task 1: Circuit analysis

Find the Boolean expression that represents the outputs x and y shown in the following circuit.



According to the circuit above find the equation of X and Y, then fill the truth table.

X =

 $\mathbf{Y} =$ 

A	В	Х	Y
0	0		
0	1		
1	0		
1	1		

What do you notice?

#### Task 2: Circuit analysis

Find the Boolean expression that represents the outputs x and y shown in the following circuit.



According to the circuit above find the equation for X and Y, then fill the truth table.

 $\mathbf{X} =$ 

 $\mathbf{Y} =$ 

А	В	Х	Y
0	0		
0	1		
1	0		
1	1		

What do you notice?

#### Task 3: Simplifying Boolean functions

Simplify the following Boolean expression

 $F(A, B) = (A \cdot B) + A'(A+B)$ 

Draw the simplified and the original Boolean expression using EWB and make sure that they are booth equivalent by filling-in the following truth table.

A	В	F (A, B) (original)	Y (Simplified)
0	0		
0	1		
1	0		
1	1		

#### **Task 4: Simplifying Boolean functions**

Simplify the following Boolean expression

F(A, B, C) = (A+C') + C(C.A' + (B.A) + C
Draw the simplified Boolean expression using EWB. Find out the truth table of the circuit.

### Task 5: Simplifying Boolean functions in EWB using the logic converter

Simplify the following Boolean expression in EWB using the logic converter

F(A, B, C) = AB'C(BD + CD) + AC'

To draw the circuit after simplification, you need to click on the following button  $AB \rightarrow P$ , this will realize the simplified expression using basic gates.



# Task 6: Simplifying Boolean functions in EWB using the logic converter

Simplify the following Boolean expression in EWB using the logic converter

F(A, B, C) = AB'C' + A'B'C' + A'BC' + A'B'C

# [Lab 5]. DeMorgan's Theory and the Universal Gates

### **Objectives**

- 1- Practically show the correctness of DeMorgan's Theory.
- 2- Show how to represent any gate using NAND gates only or NOR gates only.
- 3- Universal gates NAND and NOR.
- 4- How to implement NOT, AND, and OR gate using NAND gates only.
- 5- How to implement NOT, AND, and OR gate using NOR gates only.
- 6- Equivalent gates.
- 7- Two-level digital circuit implementations using universal gates only.
- 8- Two-level digital circuit implementations using other gates.

# Background

The NAND gate represents the complement of the AND operation. Its name is an abbreviation of NOT AND. The graphic symbol for the NAND gate consists of an AND symbol with a bubble on the output, denoting that a complement operation is performed on the output of the AND gate.

The NOR gate represents the complement of the OR operation. Its name is an abbreviation of NOT OR. The graphic symbol for the NOR gate consists of an OR symbol with a bubble on the output, denoting that a complement operation is performed on the output of the OR gate.

A universal gate is a gate which can implement any Boolean function without need to use any other gate type. The NAND and NOR gates are universal gates.

In practice, this is advantageous since NAND and NOR gates are economical and easier to fabricate and are the basic gates used in all IC digital logic families.

In fact, an AND gate is typically implemented as a NAND gate followed by an inverter not the other way around!! Likewise, an OR gate is typically implemented as a NOR gate followed by an inverter not the other way around!!

### Implement any gate with NAND gates only

To build an inverter (**NOT** gate) using a NAND gate: All NAND input pins connect to the input signal A gives an output A'.

An **AND** gate can be replaced by NAND gates as shown in the figure (The AND is replaced by a NAND gate with its output complemented by a NAND gate inverter).

An **OR** gate can be replaced by NAND gates as shown in the figure (The OR gate is replaced by a NAND gate with all its inputs complemented by NAND gate inverters).

The following figure shows all cases presented above



#### Implement any gate with NOR gates only

To build an inverter (**NOT** gate) using a NOR gate: All NOR input pins connect to the input signal A gives an output A'.

An OR gate can be replaced by NOR gates as shown in the figure (The OR is replaced by a NOR gate with its output complemented by a NOR gate inverter)

An AND gate can be replaced by NOR gates as shown in the figure (The AND gate is replaced by a NOR gate with all its inputs complemented by NOR gate inverters)

The following figure shows all cases presented above



### **Equivalent Gates**

A NAND gate is equivalent to an inverted-input OR gate.



An AND gate is equivalent to an inverted-input NOR gate.



# Building Circuits using NAND and NOR gates only

# Example: Building Circuits using NAND gates only

Implement the following function using AND, OR gates





Re-implement the same function above using NAND gates only



### Example: Building Circuits using NOR gates only



# Lab Tasks

# Task 1: The Universal NAND gate

Use EWB to show that the following gates are equivalent



# Task 2: The Universal NOR gate

Use EWB to show that the following gates are equivalent



# Task 3: Implementing circuits using NAND gates only

Implement the following function using AND, OR gates F=(A+B).C'+A'D

Re-implement the same function above using NAND gates only

Show, using EWB, that both circuits are equivalent

# Task 3: Implementing circuits using NOR gates only

Implement the following function using AND, OR gates F=(A+B).C'+A'D

Re-implement the same function above using NOR gates only

Show, using EWB, that both circuits are equivalent

# Task 4: Implementing circuits using NAND gates only

Implement the following function using NAND gates (Use the logic converter in EWB) F=AB(A+C'D)+A'C'B

To do so, you need to write the Boolean algebra expression to implement and the press the  $AIB \rightarrow NAND$  button in the logic converter as shown next



The solution should look like as follows





# Task 5: Implementing circuits using NAND gates only

Implement the following function using NAND gates (Use the logic converter in EWB) F = CA' + B(A'.C'+D) + A'CB'

# [Lab 6]. Simplification of Boolean expressions - II

# **Objectives**

- 1- Study K-maps with 2, 3 and 4 inputs.
- 2- Simplify Boolean logic equations by using K-maps.

# Background

Check appendix #1 for details about k-maps.

# Lab Tasks

# Task 1: Simplifying two-input Boolean functions

Simplify the following Boolean expression using a k-map of size 2x2.

 $F(A, B) = (A \cdot B) + A'(A+B)$ 



Draw the simplified and the original Boolean expression using EWB and make sure that they are booth equivalent by filling-in the following truth table.

	А	В	F (A, B) (original)	Y (Simplified)
--	---	---	---------------------	----------------

0	0	
0	1	
1	0	
1	1	

# Task 2: Simplifying three-input Boolean functions

Simplify the following Boolean expression

A

в

$$F(A, B, C) = (A+C') + C(C.A' + (B.A) + C)$$





F

Draw the simplified	Boolean expression	using EWB.	Find out the truth	table of the circuit.
---------------------	--------------------	------------	--------------------	-----------------------

	А	В	С	F
1	0	0	0	
2	0	0	1	
3	0	1	0	
4	0	1	1	
5	1	0	0	
6	1	0	1	
7	1	1	0	
8	1	1	1	

# Task 3: Simplifying four-input Boolean functions

Simplify the following logic function using k-maps

 $F(A, B, C, D) = \Sigma(6, 8, 9, 10, 11, 12, 13, 14)$ 

Then draw the logic circuit that represents this function.

8 •

0 •

D

- A
- •

- F
  - •

•

Fill the truth table of the circuit above.

	А	В	С	D	F
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	

4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

# [Lab 7]. The Story of Minterms and Maxterms

### **Objectives**

Learn how implement logic functions using the standard forms: Sum of Products and Product of Sums.

### Background

We can write expressions in many ways, but some ways are more useful than others

A sum of products (SOP) expression contains: Only OR (sum) operations at the "outermost" level and each term that is summed must be a product of literals

The advantage is that any sum of products expression can be implemented using a three-level circuit

- literals and their complements at the first level
- AND gates at the second level
- a single OR gate at the third level

#### Example:

f(x,y,z) = y' + x'yz' + xz



Notice that the NOT gates are implicit and that literals are reused.

A minterm is a special product of literals, in which each input variable appears exactly once.

A function with n variables has  $2^n$  minterms (since each variable can appear complemented or not)

#### Example:

A three-variable function, such as f(x,y,z), has  $2^3 = 8$  minterms:

Each minterm is true for exactly one combination of inputs:

Those minterms are: x'y'z' x'y'z x'yz' x'yz xy'z' xy'z xyz' xyz A Minterm is true when:

Minterm	When the	When the minterm is True		
x'y'z'	x=0,	y=0,	z=0	m0
x'y'z	x=0,	y=0,	z=1	m1
x'yz'	x=0,	y=1,	z=0	m2
x'yz	x=0,	y=1,	z=1	m3
xy'z'	x=1,	y=0,	z=0	m4
xy'z	x=1,	y=0,	z=1	m5
xyz'	x=1,	y=1,	z=0	m6
xyz	x=1,	y=1,	z=1	m7

#### Sum of Minterms ( or Sum of Products)

Every function can be written as a sum of minterms, which is a special kind of sum of products form The sum of minterms form for any function is unique If you have a truth table for a function, you can write a sum of minterms expression just by picking out the rows of the table where the function output is 1.

#### Example

f = x'y'z' + x'y'z + x'yz' + x'yz + xyz'= m0 + m1 + m2 + m3 + m6 $= \Sigma m(0,1,2,3,6)$ 

#### The dual idea: products of sums

A product of sums (POS) expression contains: Only AND (product) operations at the "outermost" level, Each term must be a sum of literals.

Product of sums expressions can be implemented with three-level circuits

- literals and their complements at the first level
- OR gates at the first level
- a single AND gate at the second level
- Compare this with sums of products

#### Example

 $f(x, y, z) = y' \cdot (x'+y+z') \cdot (x+z)$ 



A maxterm is a sum of literals, in which each input variable appears exactly once.

A function with n variables has 2<sup>n</sup> maxterms

#### Example

A three-variable function f(x,y,z) has 8 maxterms

Each maxterm is false for exactly one combination of inputs:

Those materms are: x'+y'+z' x'+y'+z x'+y+z' x'+y+z' x+y+z' x+y+z' x+y+z' x+y+z' x+y+z' Maxterm Is false when:

Maxterm	When the maxterm is false	Maxterm ID
x + y + z	x=0, y=0, z=0	M0
x + y + z'	x=0, y=0, z=1	M1
x + y' + z	x=0, y=1, z=0	M2
x + y' + z'	x=0, y=1, z=1	M3
x' + y + z	x=1, y=0, z=0	M4
x' + y + z'	x=1, y=0, z=1	M5
x' + y' + z	x=1, y=1, z=0	M6
x' + y' + z'	x=1, y=1, z=1	M7

Every function can be written as a unique product of maxterms

If you have a truth table for a function, you can write a product of maxterms expression by picking out the rows of the table where the function output is 0. (Be careful if you're writing the actual literals!) f = (x' + y + z).(x' + y + z').(x' + y' + z')

= M4. M5.M7 =  $\Pi$ M(4,5,7)

 $\begin{aligned} f' &= (x + y + z).(x + y + z').(x + y' + z).(x + y' + z').(x' + y' + z) \\ &= M0. \ M1. \ M2. \ M3. \ M6 = \Pi M(0, 1, 2, 3, 6) \end{aligned}$ 

#### Minterms and maxterms are related

Any minterm  $m_i$  is the complement of the corresponding maxterm  $M_i$ For example m4' = M4 because (xy'z')' = x' + y + z

Minterm	Shorthand	Maxterm	Shorthand
x'y'z'	m0	x + y + z	M0
x'y'z	m1	x + y + z'	M1
x'yz'	m2	x + y' + z	M2
x'yz	m3	x + y' + z'	M3
xy'z'	m4	x' + y + z	M4
xy'z	m5	x' + y + z'	M5
xyz'	m6	x' + y' + z	M6
xyz	m7	x' + y' + z'	M7

#### Converting between standard forms

We can convert a sum of minterms to a product of maxterms

• In general, just replace the minterms with maxterms, using maxterm numbers that don't appear in the sum of minterms:

• The same thing works for converting from a product of maxterms to a sum of minterms

#### Example

From before  $f = \Sigma m(0,1,2,3,6)$ and  $f' = \Sigma m(4,5,7)$  = m4 + m5 + m7complementing (f')' = (m4 + m5 + m7)'so f = m4'. m5'. m7' [DeMorgan's law] = M4. M5. M7  $= \Pi M(4,5,7)$ 

# Lab Tasks

### Task 1: Three-input Boolean functions

Given the following truth table of a three-input logic circuit

А	В	С	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0



Draw a circuit that implements the above logic function (use minterms only)



Draw a circuit that implements the above logic function (use maxterms only)



### Task 2: Three-input Boolean functions

Simplify (using k-maps) the function presented in Task 1 of this lab. Draw the simplified form of the function on EWB. Use the Logic Converter of EWB to generate the truth table of the simplified circuit.

А	В	С	F (simplified)
0	0	0	
0	0	1	

0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

# Task 3: Four-input Boolean functions

Draw the following logic function using EWB

 $F(A, B, C, D) = \Sigma(6, 8, 9, 10, 11, 12, 13, 14)$ 



### Task 4: Four-input Boolean functions

Simplify (using k-maps) the function presented in Task 3 of this lab. Draw the simplified form of the function on EWB. Use the Logic Converter of EWB to generate the truth table of the simplified circuit.

	А	В	С	D	F
0	0	0	0	0	

1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

# Task 5: Simplifying 4-variable functions

Simplify and implement (using EWB) the following function F(a, b, c, d) = (a'+b'+d')(a+b'+c')(a'+b+d')(b+c'+d')

Draw you circuit below

# Task 6: Simplifying 4-variable functions: SOP

Draw a NAND logic diagram that implements the <u>complement</u> of the following function  $F(A, B, C, D) = \Sigma(0, 1, 2, 3, 4, 8, 9, 12)$ Draw you circuit below

# Task 7: Simplifying 4-variable functions: POS

Draw a logic diagram that implements the following function  $F(A, B, C, D) = \Pi(0, 1, 2, 3, 4, 8, 9, 12)$ Draw you circuit below

# [Lab 8]. XOR and XNOR gates: Basics and Applications

### **Objectives**

To learn how to build XOR gates from basic gates

To learn how to build a Half Adder and a Full Adder using XOR gates.

To learn how to build a parallel adder, subtracter, incrementer and decrementer using full adders.

# Background

The XOR gate (sometimes EOR gate, or EXOR gate) is a digital logic gate that implements an exclusive or; that is, a true output (1) results if one, and only one, of the inputs to the gate is true (1). If both inputs are false (0) or both are true (1), a false output (0) results. Next is the circuit representation of the XOR gate and its truth table.



Next is one way to build an XOR gate using NAND gates only



The XOR logic gate can be used as a one-bit adder (or a Half-Adder; HA)that adds any two bits together to output one bit (the sum) and another bit that represents the carry out. As shown below



The XOR logic gate can be used as a one-bit full adder that adds any three bits together to output one bit (the sum) and another bit that represents the carry out. As shown below

Inp	Inputs		Outputs	
A	B	$C_{ m in}$	$C_{\rm out}$	S
0	0	0	0	0
1	0	0	0	1
0	1	0	0	1
1	1	0	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0
1	1	1	1	



# Lab Tasks

# Task 1: XOR built from basic gates

Draw using EWB the following circuits then fill their truth tables:





#### What do you notice?

Each one of the above circuits can be replaced with one single logic gate that gives the same truth table, that's the Exclusive OR Gate or XOR.



А	В	F	No. of 1's
0	0	0	Even
0	1	1	Odd
1	0	1	Odd
1	1	0	Even

### Task 2: XNOR Gate

Draw using EWB the following circuit then fill its truth table:



The above circuit can be replaced with one single logic gate that gives the same truth table, that's the Exclusive NOR Gate or XNOR.



А	В	F	No. of 1's
0	0	1	Odd
0	1	0	Even
1	0	0	Even
1	1	1	Odd

# Task 3: 3-input XOR Gate

Draw using EWB a three-input XOR gate. Check the circuit using a Logic converter.



А	В	С	A⊕B⊕C
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

### Task 4: Half adder circuit

The following diagram represents the Half Adder (HA is a Logic Circuit that performs 1-bit binary addition). Given that P and Q are two 1-bit binary numbers, S is the 1-bit Sum of P and Q, and C is the CARRY bit.

(a) Find out the Boolean functions S and C, and write them in the corresponding blanks.

(b) Draw using EWB the HA circuit then find its truth table by using the logic converter.



Р	Q	S	С
0	0		
0	1		
1	0		
1	1		

# Task 5: Implementing HA circuit using EWB

Draw using EWB the HA circuit shown in the figure below then find its truth table by using the logic converter, compare the truth table obtained with the one in Task5, what do you notice?



# Task 6: Implementing FA circuit using EWB

Draw using EWB a full adder circuit; find out its truth table and Boolean functions.



Cin	P	Q	Sum	Cout
0	0	0		
0	0	1		
0	1	0		

0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

# Task 7: Implementing a 4-bit parallel adder using 4 FA's

Draw using EWB a 4-bit parallel adder circuit (the circuit below shows (6+3=9)) **Note**: you can find the decoded 7-segment under the "indicators" toolbar.



# Task 8: Implementing a 4-bit parallel subtracter using 4 FA's

Draw using EWB a 4-bit parallel adder circuit (the circuit below shows (b-6=5))



4-bit paralel subtracter

# Task 9: Implementing a 4-bit incrementer using 4 FA's

Draw using EWB a 4-bit incrementer circuit.

# Task 10: Implementing a 4-bit decrementer using 4 FA's

Draw using EWB a 4-bit decrementer circuit.

# [Lab 9]. Building logic circuits using Multiplexers

#### **Objectives**

• To learn how to build combinational logic circuits using multiplexers.

### Background

In a **Combinational Logic Circuit**, the output is dependant at all times on the combination of its inputs. Some examples of a combinational circuit include **Multiplexers**, **De-multiplexers**, **Encoders**, **Decoders**, **Full** and **Half Adders** etc.



A Multiplexer is a combination of logic gates resulting into circuits with two or more inputs (data inputs) and one output.

#### 4 Channel Multiplexer using Logic Gates

The following circuit shows a 4x1 mux. Based on the binary value placed at the inputs "a" and "b", what will appear at the circuit output Q is one of the following values: A, B, C, or D.



The circuit above is implemented based on the following truth table.

a	b	Q
0	0	А
0	1	В
1	0	С
1	1	D

#### **Drawing Multiplexers in EWB:**

Task: Draw the previous lab examples using EWB, follow the steps below to implement Multiplexers and Decoders.



Then choose 74151 (1-of-8 Data Sel/MUX) from the list:



You may also choose 74150 (1-of-16 Data Sel/Mux) as follows

1	F7	VCC	24
2	EG .	FQ	23
3			22
4		E 2	21
5		E10 E11	20
6		E11	19
7		E12	18
8		E13 E14	17
9,		C14 C15	16
10			15
11	n n	Р	14
12	GND	а С	13
		0	
	74150		

NOTE: the "A" line in the multiplexer is the least significant bit, while "C" is the most significant bit.

Data selector/multiplexer truth table:

Select		Strobe	Outputs		
С	B	Α	G'	W	Y
х	Х	Х	1	1	0
0	0	0	0	D0'	D0
0	0	1	0	D1'	D1
0	1	0	0	D2'	D2
0	1	1	0	D3'	D3
1	0	0	0	D4'	D4
1	0	1	0	D5'	D5
1	1	0	0	D6'	D6
1	1	1	0	D7'	D7

# Multiplexers can be used to synthesize logic functions

4-to-1 MUX can realize any 3-variable function, 8-to-1 MUX can realize a 3-variable or 4-variable function, in general  $2^{n}$ -to-1 MUX can realize an (n +1)-variable and n-variable function.

Example: realizing functions using Multiplexers

The function F=A'BC+AB'+AC' Can be implemented using an 8-1 mux as follows



#### **Example: realizing functions using Multiplexers**

The function F= A'C'+B'C'+C'D+ABCD'

Can be implemented using an 16-1 mux as follows



#### **Example: realizing a 4-variable function using 8-to-1Multiplexer** F(A, B, C, D) = A'B'C'D'+A'B'C'D+A'B'CD+A'BC'D'+ABC'D'+ABC'D+ABCD

Truth table:

	Α	В	С	D	F		
0	0	0	0	0	1	F=1	
1	0	0	0	1	1		
2	0	0	1	0	0	E-D	
----	---	---	---	---	---	--------------	--
3	0	0	1	1	1	Γ–D	
4	0	1	0	0	1	E-D'	
5	0	1	0	1	0	г–D	
6	0	1	1	0	0	E_0	
7	0	1	1	1	0	Г–0	
8	1	0	0	0	1	E-D'	
9	1	0	0	1	0	г–D	
10	1	0	1	0	0	E_0	
11	1	0	1	1	0	г=0	
12	1	1	0	0	1	<b>E_1</b>	
13	1	1	0	1	1	$\Gamma - 1$	
14	1	1	1	0	0	E-D	
15	1	1	1	1	1	г–D	

To implement this function using EWB, you draw the following circuit:



#### Task 1: Implementing single-output circuits using muxes

Implement the following function using one 8x1 multiplexer F(A, B, C, D) = A'B'C'D'+A'B'C'D+A'B'CD+A'BC'D'+ABC'D'+ABC'D'+ABC'D+ABCD

Note: this example has already been solved above. Just draw the circuit using EWB.

#### Task 2: Implementing single-output circuits using muxes

Implement the following function using one 16x1 multiplexer F(A, B, C, D) = A'C'B+AB'C'+B'C'D+ABCD'

1		200	24
2			23
3		LO LO	22
4	EU FA	にフ 〒10	21
5	52	L10 E11	20
6	LO F2	E12	19
7	СС F1	E12	18
8	FU	E10 F14	17
	GY GY	L17 F15	16
		~ ~ ~	15
11	n D	R R	14
12		ц С	13

74150

#### Task 3: Implementing single-output circuits using muxes

Implement the following function using one 8x1 multiplexer F(A, B, C, D) = A'C'B+AB'C'+B'C'D+ABCD'

	Α	В	С	D	F	
0	0	0	0	0		
1	0	0	0	1		
2	0	0	1	0		
3	0	0	1	1		
4	0	1	0	0		
5	0	1	0	1		
6	0	1	1	0		
7	0	1	1	1		
8	1	0	0	0		
9	1	0	0	1		
10	1	0	1	0		
11	1	0	1	1		
12	1	1	0	0		
13	1	1	0	1		
14	1	1	1	0		
15	1	1	1	1		

#### Task 4: Problems with verbal description

Design a combinational circuit (using two 8-to-1 multiplexers) with three inputs, and one output to implement the following function.

	Α	В	С	D	F
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1

i			1
1	כח	Vee	16
2		VUU DИ	15
3	DC	D4 D5	14
4			13
5		סע דת	12
6		D7 人	11
7 .	W CV	H D	10
8		а С	9
	עאט	U U	
	74151		

74151

			l
1	רא	$V \cap \cap$	16
2		ТИ	15
3		ר <del>י</del> דה	14
4			13
5		DU 107	12
6	1	D7 人	11
	∾ ⊑∕	н Ъ	10
8		ц С	9
	עאום		
•	74151		

## [Lab 10]. Building digital logic circuits using Decoders

#### **Objectives**

• To learn how to build combinational logic circuits using decoders.

#### Background

In a **Combinational Logic Circuit**, the output is dependant at all times on the combination of its inputs. Some examples of a combinational circuit include **Multiplexers**, **De-multiplexers**, **Encoders**, **Decoders**, **Full** and **Half Adders** etc.

A Decoder is a circuit with two or more inputs and one or more outputs. Its basic function is to accept a binary word (code) as an input and create a different binary word as an output.



#### Logic Functions Realized with Decoders:

**Example:** F = A + B'





#### **Drawing Decoders using EWB:**

Click on the voltant button on the toolbar, then drag a 741xx digital IC into your workspace. From the list, select either 74138 (3-8 decoder) or 74154 (4-16 decoder) as shown next.

🕈 🖯 🏥 🛱 🔃 🖌 👼
Digital ICs

741xx Series	×
74126 (Quad Bus BUFFER w/3-state Out) 74132 (Quad 2-In NAND (Schmitt)) 74133 (13-In NAND) 74134 (12-In NAND w/3-state Out) 74138 (3-to-8 Dec) 74139 (Dual 2-to-4 Dec/DEMUX)	Cancel



74138 (3-8 decoder)

			Sele	ct									
G2A'	G1	G2B'	C	В	А	Y0	Y1	Y2	Y3	Y4	Y5	Y6	Y7
Х	Х	1	Х	Х	Х	1	1	1	1	1	1	1	1
Х	0	Х	Х	Х	Х	1	1	1	1	1	1	1	1
0	1	0	0	0	0	0	1	1	1	1	1	1	1
0	1	0	0	0	1	1	0	1	1	1	1	1	1
0	1	0	0	1	1	1	1	0	1	1	1	1	1
0	1	0	0	1	1	1	1	1	0	1	1	1	1
0	1	0	1	0	0	1	1	1	1	0	1	1	1
0	1	0	1	0	1	1	1	1	1	1	0	1	1
0	1	0	1	1	0	1	1	1	1	1	1	0	1
0	1	0	1	1	1	1	1	1	1	1	1	1	0
1	1	0	x	x	x	Outp other	out cor rs 1	respon	iding t	o store	ed add	ress 0;	all

The 3-to-8 decoder truth table is shown next:



74154 (4-16 decoder)

## Example: drawing a 4-input function using 74154 (Sum of minterms)

Next is the function F(D, C, B, A)= $\Sigma 0$ , 7, 8, 10, 13, 15



#### Example: drawing a 4-input function using 74154 (Product of maxterms)

Next is the function  $F(D, C, B, A) = \Pi 11, 12, 13, 15$ 



#### Lab Tasks

#### Task 1: Implementing 3-variable Boolean expressions using 3-8 decoder

Implement the following function using 3-8 decoders.

	Α	B	С	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The above function can be implemented as shown next. Redraw this circuit using EWB.



#### Task 2: Implementing multiple 3-variable Boolean expressions using 3-8 decoder

Implement the following three functions using 3-8 decoders.

	Α	В	С	F1	F2	F3
0	0	0	0	0	1	1
1	0	0	1	1	1	0
2	0	1	0	0	0	1
3	0	1	1	1	1	0

4	1	0	0	0	1	1
5	1	0	1	1	0	0
6	1	1	0	1	0	0
7	1	1	1	0	0	1



#### Task 3: Problems with verbal description

Design a combinational circuit (using **one 4-16 decoder**) with four inputs, and one output to implement the following function.

	Α	В	С	D	F
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0

15	1	1	1	1	1
10	-	-	-	-	-



#### Task 4: Problems with verbal description

Design a combinational circuit (using **two 3-8 decoder**) with four inputs, and one output to implement the following function.

	Α	В	С	D	F
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1

1	A 1	VCC	16
2	н D	VUU VA	15
3	с С	10 V1	14
4	52A/	11 V2	13
	COR/	72 V2	12
	G1	VA	11
7		14 V5	10
8	L Z GNID		9
		10	
-			

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.			10
	A	VCC -	10
2	R	, ŭŭ	15
3	с С	+ 0 − √1	14
4			13
5	- ULH СОТ/		12
6	СС. С1	тэ VИ	11
7		14 V5	10
8			9
	UND	10	
-	74138		

Last updated on Monday, March 23, 2015 By Dr. Sulieman Bani-Ahmad

## [Lab 11]. Sequential Circuits??

## **Objectives**

• To learn how to build combinational logic circuits using decoders.

## Background



## [Lab 12]. Appendices

#### Appendix #1: K-Maps

Karnaugh Maps, or K-maps, are used for many small digital logic design problems. We will examine some Karnaugh Maps for two, three, and four variables. For two-input logic circuits. The truth table of the circuit is mapped into a 2x2 k-map. For three-input logic circuits. The truth table of the circuit is mapped into 4x2 or 2x4 k-maps. For four-input logic circuits. The truth table of the circuit is mapped into a 4x4 k-map. The ones (or zeros are grouped in twos, fours, or eights to do the simplification. Notice that the binary numbers in the horizontal and vertical edges of the k-map are gray codes.



K-maps of 2x2, 2x4, 4x2 and 4x4 sizes





Grouping 1's in a 4x4 k-map

The table appearing at the right side of this page shows the gray codes that we need for the K-maps of  $2x^2$ ,  $2x^4$ ,  $4x^2$  and  $4x^4$  sizes.

**Example** (two-input circuites):

Simplify the logic diagram below.



**Solution:** (Figure below)

- Write the Boolean expression for the original logic diagram as shown below
- Transfer the product terms to the Karnaugh map
- Form groups of cells
- Write Boolean expression for groups
- Draw simplified logic diagram

## Gray code

#### by <mark>bit</mark> width

2-bit	4-bit
00 01 11 10 3-bit	0000 0001 0011 0010 0110 0111 0101
000 001 011 010 110 111 101 100	0100 1100 1101 1111 1110 1010 1011 1001 1000



#### Example (three-input circuits):



Above we, place the 1's in the K-map for each of the product terms, identify a group of two, then write a *p*-*term* (product term) for the sole group as our simplified result.



Mapping the four product terms above yields a group of four covered by Boolean A'

#### Example (three-input circuits):



Mapping the four p-terms yields a group of four, which is covered by one variable C.

#### Example (three-input circuits):

Out =  $\overline{ABC} + \overline{ABC} + \overline{AB$ 

 $Out = \overline{A} + B$ 

After mapping the six p-terms above, identify the upper group of four, pick up the lower two cells as a group of four by sharing the two with two more from the other group. Covering these two with a group of four gives a simpler result. Since there are two groups, there will be two p-terms in the Sum-of-Products result A'+B

#### Example (three-input circuits):



The two product terms above form one group of two and simplifies to **BC Example (three-input circuits):** 



Mapping the four p-terms yields a single group of four, which is **B** 

#### Example (three-input circuits):



Mapping the four p-terms above yields a group of four. Visualize the group of four by rolling up the ends of the map to form a cylinder, then the cells are adjacent. We normally mark the group of four as above left. Out of the variables A, B, C, there is a common variable: C'. C' is a 0 over all four cells. Final result is C'.

#### Example (three-input circuits):



#### Example (four-input circuits):

 $Out = \overline{AB}CD + \overline{AB}CD + ABCD + A\overline{B}CD + A\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}D + ABC\overline{D}$ 



The above Boolean expression has seven product terms. They are mapped top to bottom and left to right on the K-map above. For example, the first P-term **A'B'CD** is first row 3rd cell, corresponding to map location **A=0**, **B=0**, **C=1**, **D=1**. The other product terms are placed in a similar manner. Encircling the largest groups possible, two groups of four are shown above. The dashed horizontal group corresponds the the simplified product term **AB**. The vertical group corresponds to Boolean CD. Since there are two groups, there will be two product terms in the Sum-Of-Products result of **Out=AB+CD**.

#### Example (four-input circuits):

Fold up the corners of the map below like it is a napkin to make the four cells physically adjacent.

 $Out = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D}$ 



The four cells above are a group of four because they all have the Boolean variables **B'** and **D'** in common. In other words, **B=0** for the four cells, and **D=0** for the four cells. The other variables (**A**, **B**) are **0** in some cases, **1** in other cases with respect to the four corner cells. Thus, these variables (**A**, **B**) are not involved with this group of four. This single group comes out of the map as one product term for the simplified result: **Out=B'C'** 

#### **Example (four-input circuits):**

For the K-map below, roll the top and bottom edges into a cylinder forming eight adjacent cells.



#### $Out = \overline{B}$

The above group of eight has one Boolean variable in common: B=0. Therefore, the one group of eight is covered by one p-term: **B'**. The original eight term Boolean expression simplifies to **Out=B'** 

#### Example (four-input circuits):

The Boolean expression below has nine p-terms, three of which have three Booleans instead of four. The difference is that while four Boolean variable product terms cover one cell, the three Boolean p-terms cover a pair of cells each.



The six product terms of four Boolean variables map in the usual manner above as single cells. The three Boolean variable terms (three each) map as cell pairs, which is shown above. Note that we are mapping p-terms into the K-map, not pulling them out at this point.

For the simplification, we form two groups of eight. Cells in the corners are shared with both groups. This is fine. In fact, this leads to a better solution than forming a group of eight and a group of four without sharing any cells. Final Solution is Out=B'+D'

#### Example (four-input circuits):

Below we map the unsimplified Boolean expression to the Karnaugh map.



 $Out = \overline{ABCD} + \overline{ABCD} + \overline{ABCD} + \overline{ABCD} + ABCD$ 

Above, three of the cells form into a groups of two cells. A fourth cell cannot be combined with anything, which often happens in "real world" problems. In this case, the Boolean p-term **ABCD** is unchanged in the simplification process. Result: **Out= B'C'D'+A'B'D'+ABCD** 

#### Example (four-input circuits):

Often times there is more than one minimum cost solution to a simplification problem. Such is the case illustrated below.

 $Out = \overline{ABCD} + \overline{ABCD} + \overline{ABCD} + \overline{ABCD} + \overline{ABCD}$  $+ ABCD + ABC\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D}$ CD 00 01 11 10 00 01  $11 \ 10$ в 00 00 6 D 1 01 D 01 ĥ a 6 1 11 1 11 /1 1 10 1 10 1 out= BCD + ACD + BCD + ACD  $Out = \overline{ABC}$  $+ \overline{A}BD + ABC + A\overline{B}\overline{D}$ 

Both results above have four product terms of three Boolean variable each. Both are equally valid *minimal cost* solutions. The difference in the final solution is due to how the cells are grouped as shown above. A minimal cost solution is a valid logic design with the minimum number of gates with the minimum number of inputs.

#### Example (four-input circuits):

Below we map the unsimplified Boolean equation as usual and form a group of four as a first simplification step. It may not be obvious how to pick up the remaining cells.



Pick up three more cells in a group of four, center above. There are still two cells remaining. the minimal cost method to pick up those is to group them with neighboring cells as groups of four as at above right.

On a cautionary note, do not attempt to form groups of three. Groupings must be powers of 2, that is, 1, 2, 4, 8 ...

#### Example (four-input circuits):

Below we have another example of two possible minimal cost solutions. Start by forming a couple of groups of four after mapping the cells.



The two solutions depend on whether the single remaining cell is grouped with the first or the second group of four as a group of two cells. That cell either comes out as either **ABC'** or **ABD**, your choice. Either way, this cell is covered by either Boolean product term. Final results are shown above.

## The biggest picture of all- Simplifying using K-maps, Boolean algebra. Example (four-input circuits):

Below we have an example of a simplification using the Karnaugh map at left or Boolean algebra at right. Plot C' on the map as the area of all cells covered by address C=0, the 8-cells on the left of the map. Then, plot the single ABCD cell. That single cell forms a group of 2-cell as shown, which simplifies to P-term ABD, for an end result of Out = C' + ABD.



Out=  $(A+B+C+\overline{D})(A+B+\overline{C}+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$  $(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+B+C+\overline{D})(\overline{A}+B+\overline{C}+D)$ 

	00	01	11	10	AB	00	01	11	10
00		0		0	0.0	1	0	1	0
01		0		0	01	1	0	1	0
11				0	11	1	1	1	0
10		0		0	10	1	0	1	0

Then fill in the implied 1s in the remaining cells of the map above right.

A	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	1			0
10	1	0	1 /	0

.

 $Out = \overline{CD} + CD + ABD$ 

#### **Example (four-input circuits – SOP and POS):**

Let us revisit a previous problem involving an SOP minimization. Produce a Product-Of-Sums solution. Compare the POS solution to the previous SOP.



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# Digital Logic Design Featuring EWB 5.12

